New Simplified Formulas for ELF Subsurface-to-Subsurface Propagation

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Abstract—New formulas for the electric and magnetic fields produced by the four elementary dipole antennas have been developed for the subsurface-to-subsurface propagation case. The formulas are of rather simple, but useful, form and are completely general (i.e., the air can easily be replaced by the sea bottom). They are valid at any frequency and at any range beyond a certain minimum distance for the flat-earth case. They are especially applicable at extremely low frequency (ELF) for the air/sea and sea/sea seabed propagation cases. The main restrictions on these formulas are 1) the square of the index of refraction is greater than or equal to 10 and 2) the horizontal separation is greater than or equal to 3 times the sum of the depths of burial of the transmitting and receiving point sources. With these new formulas, computer evaluation can be reduced to fractions of a minute, compared with hours for the complete numerical evaluation of the exact Sommerfeld integrals. The formulas also exhibit an interference pattern set up under certain close-range conditions. The three waves (direct, modified mirror image, and lateral) may interfere, either constructively or destructively, with each other.

I. INTRODUCTION

IN THE PAST, many investigators have erroneously believed that the field-strength equations tabulated in Kraichman [1, ch. 3] are only valid when the conduction currents in the water or earth are much greater than the displacement currents (i.e., $\sigma_1 \gg \omega\epsilon_1$). Indeed, as long as $|n^2| \gg 1$, the displacement currents can be included simply by replacing σ_1 with $\sigma_1 + i\omega\epsilon_1$ in the field-strength equations. Thus Kraichman's tabulated results are considerably more general than they are stated to be. (The index of refraction n is equal to γ_1/γ_0 , where $\gamma_1 (= \sqrt{i\omega\mu_0(\sigma_1 + i\omega\epsilon_1)})$ is the propagation constant in the water or earth, and $\gamma_0 (= i\omega\sqrt{\mu_0/\epsilon_0})$ is the propagation constant in the air.)

It is the purpose of this article to extend the range of validity of Kraichman's results and present new formulas for horizontal electric dipole (HED), horizontal magnetic dipole (HMD), vertical electric dipole (VED), and vertical magnetic dipole (VMD) subsurface-to-subsurface propagation. These formulas have been obtained complete from previously derived results. The main restrictions on their use are 1) the square of the index of refraction ($|n^2|$) is greater than or equal to 10 and 2) the horizontal separation is greater than or equal to three times the sum of the depths of burial of the transmitting and receiving point sources. An additional restriction must also be applied to the lateral-wave components. The quantity $|\gamma_1\rho^2/(z+h)|$ must be greater than or equal

to $4c_1$, where $c_1 = 3, 6, 9, 15$, or 25 , depending on the particular field-strength component.

In this article, the four dipole antennas (VED, VMD, HED, and HMD) are situated at depth h ($h \geq 0$) with respect to a cylindrical coordinate system (ρ, ϕ, z) and are assumed to carry a constant current I . The axes of the VED and HED (of dipole moment p) are oriented in the z and x directions, respectively, and the axes of the VMD and HMD (of dipole moment m) are oriented in the z and y directions, respectively. The earth or water occupies the lower half-space (medium 1, $z > 0$) while the air occupies the upper half-space (medium 0, $z < 0$). The magnetic permeability of the earth is assumed to equal μ_0 , the permeability of free space. The International System (SI) of units is employed and the suppressed time factor of $\exp(i\omega t)$ is assumed.

In three recent papers, Wu and King [2]–[4] have derived new simple formulas for the six field components generated by a horizontal electric dipole in a half-space of water or earth near its boundary with air. They indicate that their formulas are valid for $\rho^2 \gg (z+h)^2$ and $|n^2| \gg 1$. On examining their results, the author has discovered that they could have been obtained almost by inspection from the previously derived results of Wait [5], Weaver [6], Bannister [7], and Bannister and Hart [8] (most of which are summarized in Kraichman [1]). Also, five out of the six field component expressions derived by Wu and King involve the unphysical distance $z+h+\rho$. Their use of this unphysical distance leads to errors in the range where the various waves interfere, either constructively or destructively, with each other. These errors are discussed in more detail in the Appendix.

II. DERIVATION PROCEDURE

As we shall soon see, the unphysical distance $z+h+\rho$ can be avoided. When both the transmitting and receiving dipoles are located below the earth's surface (h and $z \geq 0$), the HED hertz vector potential Π_x may be expressed as (Wait [5]):

$$\Pi_x = \frac{p}{4\pi(\sigma_1 + i\omega\epsilon_1)} [P_0 - P_1 + I] \quad (1)$$

where

$$P_0 = \frac{e^{-\gamma_1 R_0}}{R_0} \quad (2)$$

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is the direct-wave contribution and

$$P_1 = \frac{e^{-\gamma_1 R_1}}{R_1} \quad (3)$$

is the mirror-image contribution. The remaining term is

$$I = 2 \int_0^\infty \frac{e^{-u_1(z+h)}}{u_1 + u_0} J_0(\lambda \rho) \lambda d\lambda$$

$$= \frac{2}{(\gamma_1^2 - \gamma_0^2)} \int_0^\infty (u_1 - u_0) e^{-u_1(z+h)} J_0(\lambda \rho) \lambda d\lambda \quad (4)$$

where $R_0^2 = \rho^2 + (z - h)^2$, $R_1^2 = \rho^2 + (z + h)^2$, $u_0^2 = \lambda^2 + \gamma_0^2$, and $u_1^2 = \lambda^2 + \gamma_1^2$. When the measurements distance is much less than a free-space wavelength, (1) reduces to

$$\Pi_x \approx \frac{p}{4\pi(\sigma_1 + i\omega\epsilon_1)} \left[P_0 - P_1 + \frac{2}{(\gamma_1^2 - \gamma_0^2)} \int_0^\infty (u_1 - \lambda) e^{-u_1(z+h)} J_0(\lambda \rho) \lambda d\lambda \right] \quad (5)$$

Wait [5] has shown that this equation is equivalent to

$$\Pi_x \approx \frac{p}{4\pi(\sigma_1 + i\omega\epsilon_1)} \left[P_0 - P_1 + \frac{2}{(\gamma_1^2 - \gamma_0^2)} \left(\frac{\partial^2 P_1}{\partial z^2} - \frac{\partial^3 N}{\partial z^3} + \gamma_1^2 \frac{\partial N}{\partial z} \right) \right] \quad (6)$$

where

$$N = I_0 \left\{ \frac{\gamma_1}{2} [R_1 - (z + h)] \right\} K_0 \left\{ \frac{\gamma_1}{2} [R_1 + (z + h)] \right\} \quad (7)$$

and that, when $|\gamma_1 \rho| \gg 1$ and $\rho \gg (z + h)$

$$N \approx \frac{e^{-\gamma_1(z+h)}}{\gamma_1 \rho} \quad (8)$$

To quote Wait [5], "The manner in which the exponential factor $\exp[-\gamma_1(z + h)]$ occurs is rather interesting. It is only in the integral N that this factor emerges."

If we substitute (8) into (5) and take the indicated derivatives, (5) reduces to

$$\Pi_x \approx \frac{p}{4\pi(\sigma_1 + i\omega\epsilon_1)} \left\{ \frac{e^{-\gamma_1 R_0}}{R_0} + \frac{2e^{-\gamma_1(z+h)}}{(\gamma_1^2 - \gamma_0^2)\rho^3} - \frac{e^{-\gamma_1 R_1}}{R_1} - \frac{2e^{-\gamma_1 R_1}}{(\gamma_1^2 - \gamma_0^2)R_1^3} [(1 - 3 \sin^2 \psi_1) \cdot (1 + \gamma_1 R_1) - \gamma_1^2 R_1^2 \sin^2 \psi_1] \right\} \quad (9)$$

where $\sin \psi_1 = (z + h)/R_1$. For $\rho^2 \gg (z + h)^2$, we can set R_0 and R_1 equal to ρ everywhere except in the exponents. Thus

$$\Pi_x \approx \frac{p}{4\pi(\sigma_1 + i\omega\epsilon_1)} \left\{ \frac{e^{-\gamma_1 R_0}}{\rho} + \frac{2e^{-\gamma_1(z+h)}}{(\gamma_1^2 - \gamma_0^2)\rho^3} - \frac{e^{-\gamma_1 R_1}}{\rho} \left[1 + \frac{2}{(\gamma_1^2 - \gamma_0^2)\rho^2} (1 + \gamma_1 \rho) \right] \right\} \quad (10)$$

Here, we see that Π_x has only three components: 1) a direct-wave component, 2) a modified mirror-image component, and 3) a lateral-wave component. Unlike Wu and King's results [3], [4], there is no false component that depends on the unphysical distance $z + h + \rho$.

At first glance, this procedure appears to be considerably more complicated than the procedure employed by Wu and King. Luckily, however, the general quasi-static range field components have already been derived for the four elementary dipoles (Wait [5], Weaver [6], Bannister [7], Bannister and Hart [8], Wait and Campbell [9], [10], and Sinha and Bhattacharya [11]) as have the lateral-wave expressions for the quasi-near, near-field, and far-field ranges [1, tables 3.2, 3.7, and 3.16]. Although displacement currents were ignored in most of these analyses, they can be included simply by replacing σ_1 by $\sigma_1 + i\omega\epsilon_1$ (as long as $|n^2| \gg 1$). For the sake of convenience, the direct and modified mirror-image contribution to each dipole field-strength component is listed in Bannister [12, appendix]. For some components, these expressions are of very simple form (H_ϕ^{VE}), while for other components, these expressions are quite complicated (H_z^{VM}).

Thus the hardest part of the problem has already been solved. We can now use these previously derived results to obtain adequate formulas for the fields produced by submerged dipole sources subject to the conditions $|n^2| \gg 1$ and $\rho^2 \gg (z + h)^2$.

The derivation procedure that we will follow is 1) to take the previously derived direct and modified mirror-image results and let $\rho \geq 3(z + h)$, remembering not to replace R_0 and R_1 by ρ in the exponents, and 2) for the lateral-wave expressions, to let

$$f(\rho, z + h) = f(\rho, 0^+) e^{-\gamma_1(z+h)} \quad (11)$$

where 0^+ refers to an infinitesimal distance below the surface of the earth or water.

The quasi-static range ($|\gamma_0 \rho| \ll 1$) lateral-wave functions $f(\rho, 0^+)$ can be obtained from Bannister [7] or [1, table 3.8] (with σ_1 replaced by $\sigma_1 + i\omega\epsilon_1$). For the sake of convenience, the lateral-wave functions $f(\rho, 0^+) \exp[-\gamma_1(z + h)]$ are presented in Table I. Note that, for some of the field-strength components, these expressions are identical to the quasi-near range ($|\gamma_0 \rho| \ll 1 \ll |\gamma_1 \rho|$) results as given in [1, table 3.16]. Also, note that some of the signs are different than those of Bannister [7] and Kraichman [1]. This is because in this article we have inverted the coordinate system so that z and h are positive depths.

TABLE I
 $f(\rho, 0^+)e^{-\gamma_1(z+h)}$ LATERAL-WAVE FORMULAS WHEN $|\gamma_0\rho| \ll 1$ [$n^2 \geq 10$,
 $\rho^2 \gg (z+h)^2$]

Dipole Type	E_ρ	E_ϕ	E_z	H_ρ	H_ϕ	H_z
VED	$\frac{\gamma_1 p e^{-\gamma_1(z+h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)n^2\rho^2}(\gamma_1\rho I_1 K_1)$	0	$-\frac{p e^{-\gamma_1(z+h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)n^2\rho^3}$	0	$\frac{p e^{-\gamma_1(z+h)}}{2\pi n^2\rho^2}$	0
VMD	0	$-\frac{3i\omega\mu_0 m}{2\pi(\gamma_1^2 - \gamma_0^2)\rho^4}e^{-\gamma_1(z+h)}$	0	$\frac{m}{2\pi\gamma_1\rho^4}\left(\frac{\gamma_1\rho T}{2}\right)$	0	$-\frac{9m}{2\pi(\gamma_1^2 - \gamma_0^2)\rho^5}e^{-\gamma_1(z+h)}$
HED	$\frac{p \cos \phi e^{-\gamma_1(z+h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3}$	$\frac{p \sin \phi e^{-\gamma_1(z+h)}}{\pi(\sigma_1 + i\omega\epsilon_1)\rho^3}$	$-\frac{\gamma_1 p \cos \phi e^{-\gamma_1(z+h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)n^2\rho^2}(\gamma_1\rho I_1 K_1)$	$-\frac{p \sin \phi e^{-\gamma_1(z+h)}}{2\pi\gamma_1\rho^3}(\gamma_1\rho W)$	$\frac{p \cos \phi e^{-\gamma_1(z+h)}}{2\pi\gamma_1\rho^3}(\gamma_1\rho I_1 K_1)$	$\frac{3p \sin \phi e^{-\gamma_1(z+h)}}{2\pi(\gamma_1^2 - \gamma_0^2)\rho^4}$
HMD	$-\frac{i\omega\mu_0 m \cos \phi}{2\pi\gamma_1\rho^3}(\gamma_1\rho I_1 K_1)$ $\times e^{-\gamma_1(z+h)}$	$-\frac{i\omega\mu_0 m \sin \phi}{2\pi\gamma_1\rho^3}(\gamma_1\rho W)$ $\times e^{-\gamma_1(z+h)}$	$\frac{i\omega\mu_0 m \cos \phi}{2\pi n^2\rho^2}e^{-\gamma_1(z+h)}$	$\frac{m \sin \phi}{\pi\rho^3}\left(1 - \frac{6}{\gamma_1^2\rho^2}\right)e^{-\gamma_1(z+h)}$	$-\frac{m \cos \phi}{2\pi\rho^3}\left(1 - \frac{3}{\gamma_1^2\rho^2}\right)e^{-\gamma_1(z+h)}$	$-\frac{m \sin \phi}{2\pi\gamma_1\rho^4}\left(\frac{\gamma_1\rho T}{2}\right)$

*Argument of modified Bessel functions is $\gamma_1\rho/2$.

Approximately half of the field-component formulas as listed in Table I involve products of modified Bessel functions of argument $\gamma_1\rho/2$. The functions T and W are defined by

$$T = \{16I_1K_1 + \gamma_1^2\rho^2[I_1K_1 - I_0K_0] + 4\gamma_1\rho[I_1K_0 - I_0K_1]\} \quad (12)$$

and

$$W = \left[3I_1K_1 - \frac{\gamma_1\rho}{2}(I_0K_1 - I_1K_0)\right]. \quad (13)$$

Numerical values for these functions have been provided by Bannister [7]. When $|\gamma_1\rho| \geq 4$, the function $\gamma_1\rho I_1K_1 \approx 1$, while when $|\gamma_1\rho| \geq 6$, the function $\gamma_1\rho W \approx 2$. Furthermore, when $|\gamma_1\rho| \geq 10$, the function $\gamma_1\rho T/2 \approx 3$.

As an example of our derivation procedure, consider the HED E_ρ component. The lateral-wave E_ρ component for the quasi-static range can be obtained from Table I or [5, eq. (67)]. It is equal to

$$E_\rho^{HE} \approx \frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} e^{-\gamma_1(z+h)}. \quad (14)$$

The lateral-wave E_ρ component for the nearfield range can be obtained from [1, table 3.7] (with σ_1 replaced by $\sigma_1 + i\omega\epsilon_1$), or from [5, eqs. (23) and (32)]. Therefore

$$E_\rho^{HE} \approx \frac{p \cos \phi e^{-\gamma_1(z+h)} e^{-\gamma_0\rho}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} (1 + \gamma_0\rho + \gamma_0^2\rho^2). \quad (15)$$

The lateral-wave E_ρ component for the farfield range can be

obtained from [1, table 3.2] (with σ_1 replaced by $\sigma_1 + i\omega\epsilon_1$), or from [5, eqs. (13) and (32)]. Thus

$$E_\rho^{HE} \approx \frac{p \cos \phi e^{-\gamma_1(z+h)} e^{-\gamma_0\rho}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} (\gamma_0^2\rho^2 F) \quad (16)$$

where

$$F = F(w_0) = 1 - i(\pi w_0)^{1/2} e^{-w_0} \operatorname{erfc}(i w_0^{1/2}) \quad (17)$$

is the Sommerfeld surface-wave attenuation function and

$$w_0 = -\frac{\gamma_0\rho}{2n^2} \quad (18)$$

is the Sommerfeld numerical distance. For small numerical distances $F(w_0) \approx 1$, while for larger numerical distances $F(w_0) \approx -1/(2w_0)$. Various approximate expressions for $F(w_0)$ are provided by Norton [13], [14] and Li [15].

When $|n^2| \gg 1$, the range of validity (14) and (15) overlap when $|\gamma_0\rho| \ll 1$. Similarly, when $|w_0| \ll 1$ and $|\gamma_0\rho| \gg 1$, the range of validity of (15) and (16) overlap. Therefore, we can simply combine (14), (15), and (16) to obtain an expression for the E_ρ lateral-wave component valid from the quasi-static to the farfield range. Therefore, for $|n^2| \gg 1$ and $\rho^2 \gg (z+h)^2$

$$E_\rho^{HE} \approx \frac{p \cos \phi e^{-\gamma_1(z+h)} e^{-\gamma_0\rho}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} (1 + \gamma_0\rho + \gamma_0^2\rho^2 F). \quad (19)$$

The direct and modified mirror-image contribution to each dipole field-strength component is listed in Bannister [12, appendix]. These expressions are valid for $|n^2| \gg 1$. When

$\rho \geq 3(z+h)$, the HED E_ρ expression reduces to

$$E_\rho^{HE} \approx \frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left\{ (1 + \gamma_1\rho)e^{-\gamma_1 R_0} - (3 + 3\gamma_1\rho^2 + \gamma_1^2\rho^3) \left[\frac{(z-h)^2}{2\rho^2} e^{-\gamma_1 R_0} + \frac{(z+h)^2}{2\rho^2} e^{-\gamma_1 R_1} \right] \right\} \quad (20)$$

We can now combine (19) and (20) and obtain an expression for the HED E_ρ component valid at almost any range from the source subject to the conditions that $|n^2| \gg 1$ and $\rho \geq 3(z+h)$. The final expression is

$$E_\rho^{HE} \approx \frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left\{ (1 + \gamma_0\rho + \gamma_0^2\rho^2 F)e^{-\gamma_0\rho}e^{-\gamma_1(z+h)} + (1 + \gamma_1\rho)e^{-\gamma_1 R_0} - (3 + 3\gamma_1\rho + \gamma_1^2\rho^2) \left[\frac{(z-h)^2}{2\rho^2} e^{-\gamma_1 R_0} + \frac{(z+h)^2}{2\rho^2} e^{-\gamma_1 R_1} \right] \right\} \quad (21)$$

When $\rho^2 \gg (z+h)^2$, the last two terms of (21) are negligible compared with the first two, resulting in

$$E_\rho^{HE} \approx \frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left[(1 + \gamma_0\rho + \gamma_0^2\rho^2 F)e^{-\gamma_0\rho}e^{-\gamma_1(z+h)} + (1 + \gamma_1\rho)e^{-\gamma_1 R_0} \right] \quad (22)$$

which is identical to Wu and King's result [2] for $|n^2| \gg 1$ and $\rho^2 \gg (z+h)^2$. Wu and King have shown that this simple formula is in excellent agreement with the exact results obtained from the numerical integration of the Sommerfeld integrals.

New formulas for the electric and magnetic fields produced by the four elementary dipole antennas are presented in Tables II and III for the subsurface-to-subsurface propagation case. All of these formulas have been obtained from previous derived results according to the procedure outlined above and are strictly valid for $|n^2| \gg 1$ and $\rho^2 \gg (z+h)^2$. (However, for most cases, the requirements that $|n^2| \gg 10$ and $\rho \geq 3(z+h)$ are sufficient.)

It should be noted that for many (but not all) cases, the range of validity of the formulas presented in Tables II and III can be extended down to $\rho \approx (z+h)$ if the direct and modified mirror-image terms in these equations are replaced by the equations listed in the appendix of Bannister [12]. For example, the HED E_ρ expression (21) would be replaced

by

$$E_\rho^{HE} \approx \frac{p \cos \phi}{4\pi(\sigma_1 + i\omega\epsilon_1)} \left\{ \frac{2}{\rho^3} (1 + \gamma_0\rho + \gamma_0^2\rho^2 F)e^{-\gamma_0\rho}e^{-\gamma_1(z+h)} + [(3 \cos^2 \psi_0 - 1)(1 + \gamma_1 R_0) - \gamma_1^2 R_0^2 \sin^2 \psi_0] \frac{e^{-\gamma_1 R_0}}{R_0^3} - (3 + 3\gamma_1 R_1 + \gamma_1^2 R_1^2) \sin^2 \psi_1 \frac{e^{-\gamma_1 R_1}}{R_1^3} \right\} \quad (23)$$

where $\sin \psi_0 = (z-h)/R_0$, $\cos \psi_0 = \rho/R_0$ and $\sin \psi_1 = (z+h)/R_1$.

It should also be noted that the two media can be inverted and the air (medium 0) replaced by the earth's crust (of conductivity σ_2 and dielectric constant ϵ_2). The same equations (Tables I-III) can be utilized, as long as $|n_2^2| = |\gamma_1^2/\gamma_2^2| \gg 10$ and $\rho \geq 3(z+h)$, simply by replacing $i\omega\epsilon_0$ by $\sigma_2 + i\omega\epsilon_2$.

In the ELF band, for the air/sea case, the measurement distance will be much less than a free-space wavelength (i.e., $|\gamma_0\rho| \ll 1$). Therefore, the lateral-wave portion of the equations presented in Tables II and III can be replaced by those listed in Table I, resulting in even further simplification (e.g., the HED E_ρ lateral-wave formula (19) can be replaced by (14)). However, for the ELF sea/seabed case, the wavelength in the sea bed will usually be comparable to the measurement distance, and the general formulas presented in Tables II and III are applicable.

III. RANGE OF VALIDITY OF LATERAL-WAVE FORMULAS

As we have previously mentioned, the attenuation factor $\exp[-\gamma_1(z+h)]$ for the lateral wave in the quasi-static range emerges only from the integral N , as given in (7).

When $|\gamma_1\rho| \gg 1$ and $\rho \gg (z+h)$, Wait and Campbell [10] have shown that the modified Bessel functions may be replaced by only the first terms in their respective asymptotic expansions to obtain adequate HMD expressions for the quasi-near range. On the other hand, Sinha and Bhattacharya [11] have shown, for the VMD case, the first two terms of the modified Bessel function's asymptotic expansion must be employed. This indicates that the range of validity of the lateral-wave expressions for the quasi-static range will not be the same for all field components.

As an example, consider the HED E_ρ lateral-wave component for the quasi-static range, which can be expressed as

$$E_\rho^{LW} \approx - \frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho} \frac{\partial}{\partial \rho} \int_0^\infty e^{-u_1(z+h)} J_0(\lambda\rho) d\lambda \quad (24)$$

When $|\gamma_1\rho| \gg 1$ and $\rho \gg (z+h)$, the usual procedure is to replace u_1 in the exact integral expressions by γ_1 , the propa-

TABLE II
ELECTRIC-FIELD SUBSURFACE-TO-SUBSURFACE PROPAGATION FORMULAS [$|n^2| \geq 10$, $\rho \geq 3(z+h)$]

Dipole Type	E_p	E_ϕ	E_z	
VED	$\frac{p}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^2} \left(\frac{\gamma_1}{n^2} \right) (\gamma_1 \rho I_1 K_1 + \gamma_0 \rho F) e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} - \frac{1}{2\rho^2} (3 + 3\gamma_1 \rho + \gamma_1^2 \rho^2) \left[(z+h) e^{-\gamma_1 R_1} - (z-h) e^{-\gamma_1 R_0} \right]$	0	$-\frac{p}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left(\frac{1}{n^2} \right) (1 + \gamma_0 \rho + \gamma_0^2 \rho^2 F) e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} + \frac{1}{2} (1 + \gamma_1 \rho + \gamma_1^2 \rho^2) \left(e^{-\gamma_1 R_0} - e^{-\gamma_1 R_1} \right)$	
VMD	0	$-\frac{i\omega\mu_0 m}{2\pi(\gamma_1^2 - \gamma_0^2)\rho^4} \left\{ (3 + 3\gamma_1 \rho + \gamma_0^2 \rho^2) e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} - \left[(3 + 3\gamma_1 \rho + \gamma_1^2 \rho^2) - \frac{(z+h)^2}{\rho^2} (15 + 15\gamma_1 \rho + 6\gamma_1^2 \rho^2 + \gamma_1^3 \rho^3) \right] e^{-\gamma_1 R_1} + \frac{(\gamma_1^2 - \gamma_0^2)\rho^2}{2} (1 + \gamma_1 \rho) \left(e^{-\gamma_1 R_0} - e^{-\gamma_1 R_1} \right) \right\}$	0	
HED	$\frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left\{ (1 + \gamma_0 \rho + \gamma_0^2 \rho^2 F) e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} + (1 + \gamma_1 \rho) e^{-\gamma_1 R_0} - (3 + 3\gamma_1 \rho + \gamma_1^2 \rho^2) \times \left[\frac{(z-h)^2}{2\rho^2} e^{-\gamma_1 R_0} + \frac{(z+h)^2}{2\rho^2} e^{-\gamma_1 R_1} \right] \right\}$	$\frac{p \sin \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left\{ [2 + \gamma_0 \rho (1 + F)] e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} - \left[(1 + \gamma_1 \rho) - \frac{(z+h)^2}{\rho^2} (3 + 3\gamma_1 \rho + \gamma_1^2 \rho^2) \right] e^{-\gamma_1 R_1} + \frac{1}{2} (1 + \gamma_1 \rho + \gamma_1^2 \rho^2) \left(e^{-\gamma_1 R_0} - e^{-\gamma_1 R_1} \right) \right\}$	$-\frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left(\frac{\gamma_1}{n^2} \right) (\gamma_1 \rho I_1 K_1 + \gamma_0 \rho F) e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} - \frac{1}{2\rho^2} (3 + 3\gamma_1 \rho + \gamma_1^2 \rho^2) \left[(z-h) e^{-\gamma_1 R_0} + (z+h) e^{-\gamma_1 R_1} \right]$	
HMD	$-\frac{i\omega\mu_0 m \cos \phi}{2\pi\gamma_1 \rho^3} \left\{ (\gamma_1 \rho I_1 K_1 + \gamma_0 \rho + \gamma_0^2 \rho^2 F) e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} - \frac{\gamma_1(z+h)}{\gamma_1^2 \rho^2} (3 + 3\gamma_1 \rho + \gamma_1^2 \rho^2) e^{-\gamma_1 R_1} + \frac{(1 + \gamma_1 \rho)}{2} \left[\gamma_1(z-h) e^{-\gamma_1 R_0} - \gamma_1(z+h) e^{-\gamma_1 R_1} \right] \right\}$	$-\frac{i\omega\mu_0 m \sin \phi}{2\pi\gamma_1 \rho^3} \left\{ (\gamma_1 \rho W + \gamma_0 \rho (1 + F)) e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} - \frac{\gamma_1(z+h) e^{-\gamma_1 R_1}}{\gamma_1^2 \rho^2} \left[(12 + 12\gamma_1 \rho + 4\gamma_1^2 \rho^2) - \frac{(z+h)^2}{\rho^2} (15 + 15\gamma_1 \rho + 6\gamma_1^2 \rho^2 + \gamma_1^3 \rho^3) \right] - \frac{(1 + \gamma_1 \rho)}{2} \left[\gamma_1(z-h) e^{-\gamma_1 R_0} + \gamma_1(z+h) e^{-\gamma_1 R_1} \right] \right\}$	$\frac{i\omega\mu_0 m \cos \phi}{2\pi\rho^2} \left(\frac{1}{n^2} \right) (1 + \gamma_0 \rho F) e^{-\gamma_0 \rho} e^{-\gamma_1(z+h)} + \frac{1}{2} (1 + \gamma_1 \rho) \left(e^{-\gamma_1 R_0} - e^{-\gamma_1 R_1} \right)$	

*Argument of modified Bessel function is $\gamma_1 \rho/2$.

TABLE III
MAGNETIC-FIELD SUBSURFACE-TO-SUBSURFACE PROPAGATION FORMULAS [$|\mu^2| \geq 10, \rho \geq 3(z+h)$]

Dipole Type	H_z	H_θ	H_ϕ
VED	0	$\frac{\rho}{2\pi\gamma_1\mu^3} \left[\left(\frac{1}{\mu^2} \right) (1 + \gamma_1\rho^2) e^{-\gamma_1\rho} e^{-\gamma_1(z+h)} + \frac{1}{2} (1 + \gamma_1\rho) (e^{-\gamma_1 R_0} - e^{-\gamma_1 R_1}) \right]$	
VHD	$\begin{aligned} & - \frac{\mu}{2\pi\gamma_1\mu^3} \left[\left(\frac{\gamma_1\rho}{2} + 3\gamma_1\rho + \gamma_1^2\rho^2 \right) e^{-\gamma_1\rho} e^{-\gamma_1(z+h)} \right. \\ & - \frac{\gamma_1(z+h)}{\gamma_1^2\rho^2} e^{-\gamma_1 R_1} \left[(45 + 45\gamma_1\rho + 18\gamma_1^2\rho^2 + 3\gamma_1^3\rho^3) \right. \\ & - \frac{(z+h)^2}{\rho^2} (105 + 105\gamma_1\rho + 45\gamma_1^2\rho^2 + 10\gamma_1^3\rho^3 + \gamma_1^4\rho^4) \\ & \left. \left. + \frac{(3 + 3\gamma_1\rho + \gamma_1^2\rho^2)}{2} \left[\gamma_1(z-h) e^{-\gamma_1 R_0} - \gamma_1(z+h) e^{-\gamma_1 R_1} \right] \right] \right] \end{aligned}$	0	
HED	$\begin{aligned} & - \frac{\rho}{2\pi\gamma_1\mu^3} \left[\gamma_1\mu^3 (1 + F) e^{-\gamma_1\rho} e^{-\gamma_1(z+h)} \right. \\ & - \frac{\gamma_1(z+h)}{\gamma_1^2\rho^2} e^{-\gamma_1 R_1} \left[(12 + 12\gamma_1\rho + 4\gamma_1^2\rho^2) \right. \\ & - \frac{(z+h)^2}{\rho^2} (15 + 15\gamma_1\rho + 6\gamma_1^2\rho^2 + \gamma_1^3\rho^3) \\ & \left. \left. + \frac{(1 + \gamma_1\rho)}{2} \left[\gamma_1(z-h) e^{-\gamma_1 R_0} - \gamma_1(z+h) e^{-\gamma_1 R_1} \right] \right] \right] \end{aligned}$	$\begin{aligned} & \frac{\rho}{2\pi\gamma_1\mu^3} \left[\gamma_1\mu^3 (1 + F) e^{-\gamma_1\rho} e^{-\gamma_1(z+h)} \right. \\ & - \frac{\gamma_1(z+h)}{\gamma_1^2\rho^2} (3 + 3\gamma_1\rho + \gamma_1^2\rho^2) e^{-\gamma_1 R_1} \\ & \left. - \frac{(1 + \gamma_1\rho)}{2} \left[\gamma_1(z-h) e^{-\gamma_1 R_0} + \gamma_1(z+h) e^{-\gamma_1 R_1} \right] \right] \end{aligned}$	
HD	$\begin{aligned} & - \frac{\mu}{2\pi\gamma_1\mu^3} \left[\left(\frac{\gamma_1\rho}{2} + 3\gamma_1\rho + \gamma_1^2\rho^2 \right) e^{-\gamma_1\rho} e^{-\gamma_1(z+h)} \right. \\ & + (1 + \gamma_1\rho) e^{-\gamma_1 R_0} e^{-\gamma_1^2\rho^2} \left[(12 + 12\gamma_1\rho + \gamma_1^2\rho^2) \right. \\ & - \frac{(z+h)^2}{\rho^2} (105 + 105\gamma_1\rho + 45\gamma_1^2\rho^2 + 10\gamma_1^3\rho^3 + \gamma_1^4\rho^4) \\ & \left. \left. - \frac{(3 + 3\gamma_1\rho + \gamma_1^2\rho^2)}{2} \left[\gamma_1(z-h) e^{-\gamma_1 R_0} - \gamma_1(z+h) e^{-\gamma_1 R_1} \right] \right] \right] \end{aligned}$	$\begin{aligned} & - \frac{\mu}{2\pi\gamma_1\mu^3} \left[\left(\frac{\gamma_1\rho}{2} + 3\gamma_1\rho + \gamma_1^2\rho^2 \right) e^{-\gamma_1\rho} e^{-\gamma_1(z+h)} \right. \\ & - \frac{\gamma_1(z+h)}{\gamma_1^2\rho^2} e^{-\gamma_1 R_1} \left[(45 + 45\gamma_1\rho + 18\gamma_1^2\rho^2 + 3\gamma_1^3\rho^3) \right. \\ & - \frac{(z+h)^2}{\rho^2} (105 + 105\gamma_1\rho + 45\gamma_1^2\rho^2 + 10\gamma_1^3\rho^3 + \gamma_1^4\rho^4) \\ & \left. \left. + \frac{1}{2} (1 + \gamma_1\rho + \gamma_1^2\rho^2) (e^{-\gamma_1 R_0} - e^{-\gamma_1 R_1}) \right] \right] \end{aligned}$	

*Argument of modified Bessel functions is $\gamma_1\rho/2$.

gation constant in the earth, resulting in

$$E_{\rho}^{LW} \approx -\frac{p \cos \phi e^{-\gamma_1(z+h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho} \frac{\partial}{\partial \rho} \int_0^{\infty} J_0(\lambda\rho) d\lambda \quad (25)$$

Since this integral is equal to $1/\rho$ (Erdélyi [16])

$$E_{\rho}^{LW} \approx \frac{p \cos \phi e^{-\gamma_1(z+h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} = f(\rho, 0^+) e^{-\gamma_1(z+h)}. \quad (26)$$

So far, we do not know exactly what values $|\gamma_1\rho|$ and $\rho/(z+h)$ must have in order to utilize (26) or any of the other quasi-static range lateral-wave formulas presented in Table I.

As a first-order approximation, we will let

$$u_1 = (\lambda^2 + \gamma_1^2)^{1/2} \approx \gamma_1 + \frac{\lambda^2}{2\gamma_1} \quad (27)$$

so that

$$e^{-u_1(z+h)} \approx e^{-\gamma_1(z+h)} e^{-\lambda^2(z+h)/2\gamma_1} \approx e^{-\gamma_1(z+h)} \left[1 - \frac{\lambda^2(z+h)}{2\gamma_1} \right]. \quad (28)$$

Inserting (28) into (24) results in

$$E_{\rho}^{LW} \approx -\frac{p \cos \phi e^{-\gamma_1(z+h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho} \frac{\partial}{\partial \rho} \left[\int_0^{\infty} J_0(\lambda\rho) d\lambda - \frac{(z+h)}{2\gamma_1} \int_0^{\infty} \lambda^2 J_0(\lambda\rho) d\lambda \right]. \quad (29)$$

Since the second integral is equal to $-1/\rho^3$ (Erdélyi [16])

$$E_{\rho}^{LW} \approx \frac{p \cos \phi e^{-\gamma_1(z+h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left[1 + \frac{3(z+h)}{2\gamma_1\rho^2} \right]. \quad (30)$$

It can easily be shown that the error incurred in neglecting the second term is less than 1 dB if the quantity

$$\left| (\gamma_1\rho) \left(\frac{\rho}{z+h} \right) \right| \geq 4c_1 \quad (31)$$

where $c_1 = 3$ for HED E_{ρ} component. That is, to a first-order approximation, it is the product of $|\gamma_1\rho| \times \rho/(z+h)$ that must exceed a specified number to accurately utilize (26) or any of the other quasi-static range lateral-wave formulas presented in Table I. For example, if $\rho = 5(z+h)$, we require $|\gamma_1\rho| \geq 2.4$ to use (26).

The most severe restriction will be for the VMD H_z component. Following the same procedure as in the derivation of the HED E_{ρ} component, we can express the VMD H_z compo-

nent as

$$H_z^{VM} \approx -\frac{9me^{-\gamma_1(z+h)}}{2\pi(\gamma_1^2 - \gamma_0^2)\rho^5} \left[1 + \frac{25(z+h)}{2\gamma_1\rho^2} \right]. \quad (32)$$

Again, it can easily be shown that the error incurred in neglecting the second term is less than 1 dB if (31) is satisfied, with $c_1 = 25$ for the VMD H_z component. For example, if $\rho = 5(z+h)$, then we require $|\gamma_1\rho| \geq 20$ to employ the VMD H_z lateral-wave formula for the quasi-static range presented in Table I.

The values of c_1 and range where $f(\rho, z+h)$ can be replaced by $f(\rho, 0^+) \exp[-\gamma_1(z+h)]$, i.e., the range where the quasi-static range lateral-wave formulas presented in Table I can be used) are presented in Table IV for each field-strength component. Here, we see that for the HED and HMD E_{ρ} , E_z , and H_{ϕ} components and the VED E_{ρ} and H_{ϕ} components ($c_1 = 3$), the condition $|\gamma_1\rho^2/(z+h)| \geq 12$ must be satisfied, while for the HED H_z and VMD E_{ϕ} and H_{ρ} components ($c_1 = 15$), the condition $|\gamma_1\rho^2/(z+h)| \geq 60$ must be satisfied.

This restriction (31) also applies to Wu and King's results [2]-[4] and to the quasi-near and nearfield range subsurface-to-subsurface propagation equations tabulated in Kraichman [1].

To a first-order approximation, the range of validity of the equations listed in Table I can be extended by multiplying the field-component expression by the quantity

$$1 + \frac{c_1(z+h)}{2\gamma_1\rho^2} \quad (33)$$

where the value of c_1 for each component is given in Table IV. For example, the HED H_z component listed in Table I is

$$H_z^{HE} \approx \frac{3p \sin \phi e^{-\gamma_1(z+h)}}{2\pi(\gamma_1^2 - \gamma_0^2)\rho^4}. \quad (34)$$

Since $c_1 = 15$ for this component (Table IV), then the extended form is

$$H_z^{HE} \approx \frac{3p \sin \phi e^{-\gamma_1(z+h)}}{2\pi(\gamma_1^2 - \gamma_0^2)\rho^4} \left[1 + \frac{15(z+h)}{2\gamma_1\rho^2} \right]. \quad (35)$$

IV. ANALYTICAL CONFIRMATION OF VMD H_z NULL FOUND BY FRASER-SMITH AND BUBENIK

A few years ago, Fraser-Smith and Bubenik [17] numerically evaluated the exact Sommerfeld integrals and found a rather deep null in the VMD H_z component for the subsurface-to-surface propagation case [17, fig. 6]. For their particular situation, the frequency was 100 Hz, the VMD source depth of burial was 100 m, and the null occurred at a distance $\rho \approx 250$ m. Since the skin depth δ in seawater at 100 Hz is ≈ 25 m, $\rho/\delta \approx 10$ and $h/\delta \approx 4$. To our knowledge, this null has not been analytically confirmed. The subsurface-to-surface VMD H_z component is equal to (from Table III,

TABLE IV
RANGE OF VALIDITY FOR QUASI-STATIC RANGE LATERAL-WAVE FORMULAS OF TABLE I
(where $f(\rho, z+h) = f(\rho, 0)^+ e^{-\gamma_1(z+h)}$)

Field-Strength Components			c_1	$ (\gamma_1 \rho) \left(\frac{\rho}{z+h} \right) $
HED and HMD	VED	VMD		
E_ρ, E_z, H_ϕ	E_ρ, H_ϕ	--	3	≥ 12
E_ϕ, H_ρ	--	--	6	≥ 24
--	E_z	--	9	≥ 36
H_z	--	E_ϕ, H_ρ	15	≥ 60
--	--	H_z	25	≥ 100

with $z = 0$)

$$H_z^{VM} \approx -\frac{m}{2\pi(\gamma_1^2 - \gamma_0^2)\rho^5} \left\{ (9 + 9\gamma_0\rho + 4\gamma_0^2\rho^2 + \gamma_0^3\rho^3)e^{-\gamma_0\rho}e^{-\gamma_1 h} - e^{-\gamma_1 D} \left[(9 + 9\gamma_1\rho + 4\gamma_1^2\rho^2 + \gamma_1^3\rho^3) - \frac{h^2}{\rho^2} (90 + 90\gamma_1\rho + 39\gamma_1^2\rho^2 + 9\gamma_1^3\rho^3 + \gamma_1^4\rho^4) \right] \right\} \quad (36)$$

where $D^2 = \rho^2 + h^2$.

For $f = 100$ Hz and $\rho = 250$ m, $|\gamma_0\rho| \ll 1$ and the $\gamma_0\rho$ terms in the lateral-wave portion of (36) are negligible. Furthermore, since the quantity $|\gamma_1\rho^2/h| \approx 35$, which is < 100 (see Table IV), (33) must be employed. The resulting expression for H_z is

$$H_z^{VM} \approx -\frac{9me^{-\gamma_1 h}}{2\pi\gamma_1^2\rho^5} H_z' \quad (37)$$

where

$$H_z' \approx \left(1 + \frac{25h}{2\gamma_1\rho^2} \right) - \left\{ \frac{e^{-\gamma_1(D-h)}}{9} \left[(9 + 9\gamma_1\rho + 4\gamma_1^2\rho^2 + \gamma_1^3\rho^3) - \frac{h^2}{\rho^2} (90 + 90\gamma_1\rho + 39\gamma_1^2\rho^2 + 9\gamma_1^3\rho^3 + \gamma_1^4\rho^4) \right] \right\}. \quad (38)$$

For $\rho/\delta \approx 10$, the dominant terms will be the $\gamma_1^3\rho^3$ and $\gamma_1^4\rho^4$ terms. To a first-order approximation, the $\gamma_1^3\rho^3$ terms will cancel and since $\gamma_1^4\rho^4 = -4(\rho/\delta)^4$

$$H_z' \approx 1 + \frac{25h}{2\gamma_1\rho^2} - \frac{4}{9} \left(\frac{h}{\rho} \right)^2 \left(\frac{\rho}{\delta} \right)^4 e^{-(D-h)/\delta} e^{-i(D-h)/\delta}. \quad (39)$$

This equation will be at a minimum near $(D-h)/\delta = 2\pi$, which corresponds to $\rho \approx 240$ m. The normalized VMD vertical magnetic field (H_z from (38)) is plotted in Fig. 1 versus the horizontal distance ρ . For this figure, we can

see that a rather deep null (≈ 20 -dB drop in field strength compared to the asymptotic value) occurs at a range of ≈ 240 m. This null is clearly due to the destructive interference between the direct and lateral waves.

V. CONCLUSIONS

New formulas for the electric and magnetic fields produced by the four elementary dipole antennas have been developed for the subsurface-to-subsurface propagation case. These formulas have been obtained completely from previously derived results. The main restrictions in their use are 1) the square of the index of refraction is greater than or equal to 10 and 2) the horizontal separation is greater than or equal to 3 times the sum of the depth of burial of the transmitting and receiving point sources. An additional restriction must also be applied to the lateral-wave components. The condition $|\gamma_1\rho^2/(z+h)| \geq 4c_1$ must be satisfied, where $c_1 = 3, 6, 9, 15$, or 25 , depending on the particular field-strength component. This restriction also applied to Wu and King's recently derived results [2]-[4] and to the subsurface-to-subsurface propagation equations tabulated in Kraichman [1].

These new formulas are also an improvement over Wu and King's results [3], [4]. They contain no false component that depends on the unphysical distance $z+h+\rho$. Wu and King's use of this unphysical distance leads to errors in the range where the various waves interfere, either constructively or destructively, with each other.

An analytical confirmation of the Fraser-Smith and Bubenik VMD H_z null had also been accomplished. This null is clearly due to the destructive interference between the direct and lateral waves.

The range of validity of the equations tabulated in this article can be extended down to $\rho \approx (z+h)$ for many cases if the direct and modified mirror-image terms in these equations are replaced with the equations listed in Bannister [12, appendix].

It should be noted that the two media can be inverted and the air replaced by the earth's crust (of conductivity σ_2 and dielectric constant ϵ_2). The same equations (Table I-IV) can be utilized, as long as $|n_2^2| = |\gamma_1^2/\gamma_2^2| \geq 10$ and $\rho \geq 3(z+h)$, simply by replacing $i\omega\epsilon_0$ by $\sigma_2 + i\omega\epsilon_2$.

Although the field component expressions presented in this article are valid at any frequency, they are especially useful at ELF for the air/sea and sea/seabed propagation cases.

In addition to the results presented in this article, we have also developed new formulas [18] that extend Norton's farfield equations to the quasi-nearfield range. Air-to-air, subsurface-to-air, air-to-subsurface, and surface-to-surface propagation were investigated. We have also employed image theory to derive expressions valid at any range from the source for air-to-air propagation [19].

APPENDIX

COMPARISON OF BANNISTER AND WU AND KING RESULTS

In their first article [2], Wu and King developed a new simple and very accurate formula for the radial field (E_ρ)

Fig. 1. Normalized VMD vertical magnetic field versus distance for $f = 100$ Hz, $h = 100$ m, and $\sigma_1 = 4$ S/m.

of a HED in a dissipative or dielectric half-space near its boundary with air. They examined in detail the interference patterns generated by the direct and lateral waves that originate at the dipole for three values of ϵ_1 , numerous values of σ_1 , and a wide range of frequencies. They confirmed the accuracy of the new formula by comparison with numerically evaluated Sommerfeld integral results.

In their second article [3], Wu and King developed new simple formulas for the transverse (E_ϕ) and vertical (E_z) electric-field components generated by a buried HED source. They then compared all three electric-field components with numerical integration results for the case where $\sigma_1 = 3.5$ S/m, $\epsilon_1 = 45\epsilon_0$, and $f = 600$ MHz (their Fig. 1.). The agreement between the simple-formula and numerical-integration results was excellent for the radial component when $|\gamma_1 \rho| > 2.8$ and $\rho > 2(z+h)$. However, substantial agreement between the simple-formula and numerical-integration transverse and vertical components was not achieved until $|\gamma_1 \rho| > 11.5$ and $\rho > 8(z+h)$. Furthermore, the simple formulas predicted a dip in the transverse components and no dip in the vertical component near $|\gamma_1 \rho| \approx 8$, while the numerical-integration results predicted the opposite.

In attempting to explain the major discrepancy between the HED E_ϕ and E_z simple-formula and numerical-integration results near $|\gamma_1 \rho| \approx 8$ of [3, fig. 1], Wu and King stated that very small changes in frequency significantly alter the interference pattern so that close agreement in a small range near such a region cannot be expected. This statement contradicts the E_ρ results presented in their first article (Wu and King [2]), where substantial agreement between the simple-formula and numerical-integration results was achieved in the interference region when $|n^2| \gg 1$ and $\rho \gg 5(z+h)$.

They also noted, at greater distances where the lateral wave dominates, the E_ρ , E_ϕ , and E_z expression were highly accurate. They certainly should be highly accurate at the greater distances because their lateral-wave formulas are es-

entially equivalent to Wait's results [5], which have been successfully utilized for over 20 years.

The major value of Wu and Kings' work is in determining adequate field-strength expressions in the range where the lateral and direct waves interfere (either constructively or destructively). They have succeeded with the HED E_ρ component. However, they have made some errors in their derivation of the other five HED components.

The HED E_ϕ and E_z expressions derived in this article (Table II) are in very good agreement with the numerical-integration results presented in fig. 1 of Wu and King [3] when $\rho \geq 3(z+h)$. They correctly predict a substantial dip in the E_z component (due to interference between the direct, mirror-image, and lateral waves) near $|\gamma_1 \rho| \approx 8$ and no dip in the E_ϕ component at this range (which is directly opposite to Wu and Kings' simple-formula results).

To see where Wu and King erred, we will compare their E_ϕ and E_z formulas with the expressions listed in Table II for the situation where the Sommerfeld numerical distance is small (i.e., $F \approx 1$), $\rho \geq 5(z+h)$, and $|\gamma_1 \rho| \geq 4$. For this situation, the E_ϕ and E_z expressions in Table II reduce to

$$E_\phi^{HE} \approx \frac{p \sin \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} [2(1 + \gamma_0\rho)e^{-\gamma_0\rho}e^{-\gamma_1(z+h)} - (1 + \gamma_1\rho)e^{-\gamma_1 R_1} + \frac{1}{2}(1 + \gamma_1\rho + \gamma_1^2\rho^2)(e^{-\gamma_1 R_0} - e^{-\gamma_1 R_1})] \quad (A1)$$

and

$$E_z^{HE} \approx -\frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^2} \left\{ \frac{\gamma_1}{n^2} (1 + \gamma_0\rho)e^{-\gamma_0\rho}e^{-\gamma_1(z+h)} - \frac{1}{2\rho^2} (3 + 3\gamma_1\rho + \gamma_1^2\rho^2)[(z-h)e^{-\gamma_1 R_0} + (z+h)e^{-\gamma_1 R_1}] \right\} \quad (A2)$$

When $F \approx 1$, $\rho \geq 5(z+h)$, and $|\gamma_1 \rho| \geq 4$, Wu and King's E_ϕ and E_z expressions [3] reduce to

$$E_\phi^{HE} \approx \frac{p \sin \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} [2(1 + \gamma_0\rho)e^{-\gamma_0\rho}e^{-\gamma_1(z+h)} - (2 + 2\gamma_1\rho + \gamma_1^2\rho^2)e^{-\gamma_1(z+h+\rho)} + (1 + \gamma_1\rho + \gamma_1^2\rho^2)e^{-\gamma_1 R_1} + \frac{1}{2}(1 + \gamma_1\rho + \gamma_1^2\rho^2)(e^{-\gamma_1 R_0} - e^{-\gamma_1 R_1})] \quad (A3)$$

and

$$E_z^{HE} \approx \frac{p \cos \phi}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^2} \left\{ \frac{\gamma_1}{n^2} (1 + \gamma_0\rho)e^{-\gamma_0\rho}e^{-\gamma_1(z+h)} + \frac{1}{2\rho^2} (3 + 3\gamma_1\rho + \gamma_1^2\rho^2)[(z-h)e^{-\gamma_1 R_0} + (z+h)e^{-\gamma_1 R_1}] \right\} \quad (A4)$$

A comparison of the two E_ϕ expressions ((A1) and (A3)) reveals that the difference between them is the unphysical distance $(z+h+\rho)$ term of Wu and King. If we replace $z+h+\rho$ in (A3) by the physical distance R_1 , Wu and King's formula becomes identical to (A1).

A comparison of the two E_z expressions ((A2) and (A4)) reveals that Wu and King's formula does not predict the dip in the interference region because of a sign error. Their results indicate that the sum of the direct and mirror-image waves add to the lateral wave whereas, in reality, the sum subtracts from the lateral wave.

Similar remarks also apply to the HED magnetic field formulas derived by King and Wu [4]. In particular, referring to [4, fig. 2], we see that their H_z formula does not predict the interference pattern dip near $\rho \approx 0.08$ m.

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