A Discussion on SRAM Forward/Inverse Problem Analyses for RTN Long-Tail Distributions

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Abstract—This paper discusses, for the first time, how the statistical SRAM design analyses should be changed when: (1) the shift-amount of the *time-dependent (TD) voltage margin variations (MV)* after the screening test will become larger than that before and (2) the shapes of the MV distribution will change from the Gaussian to the complex mixtures of Gamma distributions. We discuss on the SRAM TD-MV analyses with not only the *forward problem* but also the *inverse problem*, i.e., *deconvolution* analyses. The proposed algorithm for the *deconvolution* to circumvent the issues caused by *high-pass filtering behavior* is discussed. Based on the proposed *convolution/deconvolution* design analyses, it has been shown for the first time that: (1) detecting the truncating point of the distributions of TD-MV by the screening test and (2) predicting the required the MV-shift-amount by the assisted circuit schemes to avoid the *out of specs* in the market during the life-time, etc, has become enabled based on the target specification.

Keywords—*Deconvolution, SRAM, Gamma Mixtures, Random telegraph noise, Long-tail distribution*

I. INTRODUCTION

The *guard band (GB)* design for the *static random access memory (SRAM)* will become an unprecedentedly crucial challenge because the increased *time-dependent (TD) margin variations (MV)*-caused failures can't be predicated by the ordinary Gaussian-based convolution analyses any more. This stems from the facts that: (1) TD-MV, (i.e., *unknown MV* after shipped to the market), will become much larger than the *non-TD-MV*, (i.e., *given MV* based on the measurements), resulting in the TD-MV dominating over the whole MV. This leads to increased pressure to figure out the *unknown* factors by solving the *inverse problem*, although the SRAM designers are unfamiliar with such kind of methodology until now and (2) the tail distribution of the convolution results of TD-MV and *non-TD-MV* obeys no longer Gaussian but more complex mixtures of Gamma distributions [1]-[4], as shown in Figs. 1(a) and 1(b).

The traditional SRAM statistical analyses including its convolution integral rely on the Gaussian model *given* its parameters extracted by the measured data. However, if the *non-Gaussian unknown* factors account for no longer just a fraction but a large percentage of the whole MV (shown in Fig. 1(b)), we have to solve the *non-Gaussian inverse problem* based on the pre-defined hypothesis of the *unknown* factors or final target specifications, as shown in Figs. 1(c)-1(d) and Figs.

2(b)-2(d). To make clear the challenges behind the proposed ideas in this paper, the concepts of what will be crucial in the coming process generations are shown more in detail in Fig. 2.

Fig. 1 (a) traditional and (b) new relationship between RDF, RTN, and its convolution. (c) and (d) show the *deconvolution* (inverse problem) required to detect the *unknown* factors.

Figure 2(a) shows the concept for the required convolution methods to extrapolate the whole MV distribution based on the given data of the *random dopant fluctuation (RDF)* and the

random telegraph noise (RTN). The examples for the inverse problems are shown in Figs. 2(b)-(d). Figure 2(b) recounts the following scenarios: target specs are predefined. The RTN distribution is also predefined or given, then, we want to know the required *truncating point* (TP) of the *minimum operating voltage* (V_{dmin}) for the screening test. TP should be decided based on the RDF distributions to avoid any *out of specs* after shipped to the market. This can be solved by the *deconvolution* analysis. Figure 2(c) assumes the different scenario: both of the target specs and the RTN distribution are predefined or given, then we want to know how much we need to shift the whole MV by using the *MV assisted circuit schemes* (ASSTS)[5]-[6]. Another scenario is shown in Fig. 2(d): the target specs and the truncated RDF distribution are predefined or given, then, we want to specify the device target of reducing the amplitude of RTN.

Fig. 2 (a) convolution of the both given data of RDF and RTN, (b)-(d) examples of inverse problem required to detect the *unknown* factors or establish the target specifications.

It has been well shown in the references [4]-[6] that the distributions for the V_{th} modulation (ΔV_{th}) amplitude due to RDF and RTN are obeyed to the Gaussian and Gamma distributions or more complex mixtures of them, respectively. The increasing paces of the ΔV_{th} amplitude are differently dependent on the MOSFET size scaling parameter like below the expressions of (1) and (2).

$$\Delta V_{th} \text{ (RDF)} \propto AV_t \text{ (RDF)} / \sqrt{LW} \quad (1)$$

$$\Delta V_{th} \text{ (RTN)} \propto AV_t \text{ (RTN)} / LW \quad (2)$$

where AV_t is Pelgrom coefficient and LW are MOSFET channel length and width, respectively. Assuming the LW is scaled down to 0.5 every process generation, the ΔV_{th} increasing paces of RTN is a 1.4x faster than that of RDF. In addition, since the advanced CMOS device tends to change to much less-dopant body devices like FinFET, ultra-thin body SOI, and nanowire FET, the RDF increasing trends can be slow-down, resulting in a larger difference in the increasing pace than 1.4x. According to the references [1], there will come the time soon around a 15nm scaled CMOS era.

In order to make the proposed methods more easy to use at the factory field site, we use only the Excel tool for both of the convolution and *deconvolution* including the optimization process in the *deconvolution*. The expressions used for these calculations are shown in TABLE-1. The distributions are x-directly divided into the segmentation (# of segmentation is n). The RDF, RTN, and the convolution results are referred to f , g , and h , respectively.

TABLE-1: Comparisons between the conventional and proposed *convolution/deconvolution*.

(a) Convolution $h = f * g$

$h_1 \leq f_1 * g_1$
$h_2 \leq f_1 * g_2 + f_2 * g_1$
$h_3 \leq f_1 * g_3 + f_2 * g_2 + f_3 * g_1$
$\vdots$
$h_n \leq f_1 * g_n + f_2 * g_{n-1} + f_3 * g_{n-2} + f_4 * g_{n-3} \dots + f_n * g_1$

(b) De-convolution $g = h * f^{-1}$

$g_1 \leq h_1 / f_1$
$g_2 \leq (h_2 - f_2 * g_1) / f_1$
$g_3 \leq (h_3 - f_2 * g_2 - f_3 * g_1) / f_1$
$\vdots$
$g_n \leq (h_n - f_2 * g_{n-1} - f_3 * g_{n-2} - f_4 * g_{n-3} \dots - f_n * g_1) / f_1$

(c) Proposed De-convolution

Optimization problem $g = h * f^{-1}$	Objective funct. $O(g_n)$
$g_1 : \text{Min}_{\text{search}} [h_1 - O(g_1)]$	$(f_1 : f_n) * g_1$
$g_2 : \text{Min}_{\text{search}} [(h_1 : h_2) - O(g_2)]$	$(f_1 : f_n) * (g_1 : g_2)$
$g_3 : \text{Min}_{\text{search}} [(h_1 : h_3) - O(g_3)]$	$(f_1 : f_n) * (g_1 : g_3)$
$\vdots$	$\vdots$
$g_n : \text{Min}_{\text{search}} [(h_1 : h_n) - O(g_n)]$	$(f_1 : f_n) * (g_1 : g_n)$

A. Convolution computing

The convolution can be given by the equation shown in TABLE-1(a). Since the convolution of f and g can be considered as the *forward problem*, if the mathematical

equation and the parameters at the segmentation of f_1-f_n and g_1-g_n , are given, the observation of h_1-h_n can be easily computed. This operation plays a role of *low-pass filtering*, resulting in $h=f*g$ a smoothed curve.

B. Issues facing deconvolution operations

The *deconvolution* can be basically computed based on the equation shown in TABLE-1(b). If the observation of h_1-h_n and the parameter of f_1-f_n at the segmentation are given with the mathematical equation, the *unknown* parameter of g_1-g_n can be easily computed under the limited conditions.

However, since the *deconvolution* has to be considered as the *inverse problem*, the invert operation, i.e., *high-pass filtering*, has to be executed, as shown in TABLE-1(b). This causes the issues facing the *inverse operations*: (1) *division by zero* and *Gibbs phenomenon*, resulting in a ringing curve. As can be seen in Fig. 3, the *deconvolution* of the RTN causes an *abnormal folding and ringing*, resulting in the RTN distribution significant deviation from the expected curve. This stems from the required invert operation, i.e., *high-pass filtering* to do *deconvolution*. The point of V is very sensitive to the relationship of the gradients between the f , g and the h as can be seen when comparing those of Fig. 3(a) and 3(b).

To address this issue, we try to circumvent the invert operation as much as possible and instead, solve it as the *optimization problem* to abstract the parameter RTN (g) of the objective function (partial integral of the h : $\sum(h_i \cdot h_i, g_i)$), as shown in TABLE-1(c). The unknown parameter g_1-g_n can be searched so that the difference between the given observation data of h_1-h_n and the objective function $O(g_i)$ can be minimized.

Fig. 3 The abnormal phenomenon of V-shaped folding and ringing when using the conventional deconvolution algorithm. The difference between (a) and (b) is just the x-phase shift of f .

III. DISCUSSIONS ON THE ERROR OF THE CONVOLUTION/DECONVOLUTION RESULTS

To verify the effectiveness of the proposed methodologies, the six types of RTN distributions are assumed. To normalize the ratio of the variation amplitude (ΔX) for RDF and RTN, the ΔX for RDF distribution is kept the same. It is assumed based on the references [1]-[4] that (1) the RTN distributions have several convex and concave folding points (O-P-Q-R-S) due to the variable atomistic variation factors, as shown in Figs. 4(a)-4(f). Each slope (O-P, P-Q, Q-R, R-S) is represented by the shape parameter (β) of the Gamma distribution. In addition, to make clear the RDF-truncating effect on the convolution results, the tails of RDF (f) is truncated at the $X=5$. This truncating point (TP) gives impact on the critical Vddmin and the chip yield for the screening test before shipped to the market. The interesting zone for the *fail-bit counts predictions* (FBCP) is around the *probability density function* (pdf) of 10^{-12} .

This is because the pdf of 10^{-12} corresponds to the case of 99.9% chip yield of 1Gbit-chip.

A. Convolution Results

Fig. 4 The relationships between the slope of RTN(g) and the convolution($h=f*g$), (a)-(c): different single slopes (d)-(f): different slope mixtures

As can be seen in the Figs. 4(a)-4(f), the convolution itself can be successfully done based on the equations of TABLE-1(a) even if the shape of the curves is a complex mixtures of the different sloped Gamma distributions. The results point out that the tails of the distributions of the convolution results ($h=f*g$) are no longer traditional Gaussian but strongly affected by the complex Gamma mixture tails (O-P-Q-R-S) of RTN (g).

B. New Deconvolution Results

Fig. 5 Tracing of the deconvolution $g_{deconv}=h*f^{-1}$ comparing with the original RTN(g). Relative error= $|g-g_{deconv}|/g$.

The *deconvolution* results for the RTN(g) and RDF(f) are shown in Figs. 5 and 6, respectively. The error of the *deconvolution* is defined as the relative error of the extrapolated value at each point, i.e., $|g_i - g_{deconv}|/g_i$ and $|f_i - f_{deconv}|/f_i$ for RTN and RDF, respectively. The g is the golden value for the comparison that is used when solving the forward problem of $h=g*f$. The g_{deconv} is solved by the inverse problem (*deconvolution of given h and f*). As can be seen in Fig. 5, the relative errors of RTN *deconvolution* for (a), (b), and (c) are

suppressed to small enough value of $< 10^{-6}$, 10^{-8} and 10^{-8} , respectively. The errors of RDF *deconvolution* for all of (a), (b), and (c) in Fig. 6 are suppressed to $< 10^{-8}$. It is worth noting that the truncating point of RDF (*disconnected point, i.e., jump to zero*) is also well extrapolated by the proposed *deconvolution* algorithm while circumventing the ringing caused by the *high-pass filtering*.

Fig. 6 Tracing of the deconvolution $f_{deconv}=h*g^{-1}$ comparing with the original RDF(f). Relative error= $|f-f_{deconv}|/f$.

IV. DETECTING THE TRUNCATED POINT

As explained in Fig. 2(b), the truncated point (TP) can be detected by the proposed methodology. The four types of the slope(β) of RTN(g) impacts on the convolution results ($f*g$) are shown in Figs. 7(a)-7(d). It is found that the slope of the convolution tails ($f*g$) is strongly affected by the slope of the RTN(g) when the slope of the g is steeper than RDF(f), as shown in Figs. 7(a) and 7(b). Contrary, the slope of g is shallower than that of the f , it gives much less impact on the convolution results ($f*g$), as shown in Figs. 7(c) and 7(d).

Fig. 7 RDF-truncating point dependencies of the convolution results ($f*g$). (a)-(b): short tail RTN (smaller β), (c)-(d): long tail RTN (larger β)

Fig. 8 Slope (β) and truncated width X dependencies of the amount of $f*g$ shift (ΔX).

The RTN-slope (β) dependencies of the shift amount (ΔX) of the convolution results ($f*g$) is shown in Fig. 8. It is worth noting that the shifting (ΔX) of the tails of the whole MV ($f*g$) will become harder as the RTN-slope becomes shallower, *i.e.*, larger β . It indicates that the ordinary truncating ways by the screening test can't work well any more for avoiding the *out of specs* after shipped to the market. This is because the operation of the convolution ($f*g$) does work as a *low-pass filtering*.

On the contrary, the operation of *deconvolution* does work as a *high-pass filtering*, the sensitivity to detecting the truncated point (TP) is increased as the RTN-slope becomes shallower, *i.e.*, larger β , as shown in Fig. 9. The sensitivity for $\beta=0.56$ becomes 5x larger than that for $\beta=0.03$. Since the β of RTN slope has upward trend as the device down-scaling, this operation becomes more important for the SRAM analyses.

Fig. 9 RTN-Slope (β) and $f*g$ shift (ΔX) dependencies of the amount of the truncated width X extrapolated by the deconvolution.

V. ERRORS OF DECONVOLUTION

Figure 10 shows the RTN-slope dependencies of the relative errors of the RTN-deconvolution results. The amount of the error has a slope dependency but it is suppressed within 10^{-4} to 10^{-13} .

Fig. 10 RTN-Slope (β) dependencies of the relative error of the RTN deconvolution.

The RTN-slope dependencies of the relative errors of the RDF-deconvolution results are shown in Fig. 11. The amount of the error doesn't have a slope dependency. The error is smaller than 10^{-7} .

Fig. 11 RTN-Slope (β) dependencies of the relative error of the RDF deconvolution.

VI. SUMMARY AND DISCUSSIONS

In this paper, we have discussed, for the first time, how the SRAM *forward/inverse problems* should be solved with the *convolution/deconvolution* analyses when the shift-amount of the TD-MV after the screening will become larger than that of before.

We have demonstrated that the proposed methodologies can be used for solving the SRAM *forward/inverse problems* and can work well *back-and-forth* while suppressing the errors within the orders of 10^{-4} to 10^{-13} between the objective function (whole MV) and the given parameters (either of RTN and RDF).

We have demonstrated that solving the optimization problem will be needed for getting (1) the unknown RTN, (2) the truncating points of RDF and (3) the required MV shift amount by the circuit assisted circuit schemes (ASSTS) to avoid any *out of specs* after shipped to the market. In order to circumvent the issue that the ordinary *deconvolution* can cause an abnormal folding and ringing errors,.

We have pointed out, for the first time, that the ordinary MV truncating by the screening test to avoid the *out of specs* can't work well any more in the coming process generations. Contrary, the required truncating point to avoid the *out of specs* can be detected by solving the *inverse problems* between the target MV spec. and the given RTN distributions.

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