Orbital Satellite Pursuit-Evasion Game-Theoretical Control

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***Abstract*—This paper develops and evaluates a trust-based sensor management game-theoretical control approach for orbital pursuit-evasion for satellite interception and collision avoidance. Using a coupled zero-sum differential pursuit-evasion (PE) game, the pursuer minimizes the satellite interception time and evader tries to maximize interception time for collision avoidance. A trust-based decentralized sensor manager performs sensor-to-target assignment and nonlinear tracking. The interception-avoidance (IA) game approach provides a worst-case solution, which is the robust lower-bound performance case. We divide our IA algorithm into two parts: first, the pursuer will rotate its orbit to the same plane of the evader, and second, the two spacecrafts will play a zero-sum PE game. A two-step setup saves energy during the PE game because rotating a pursuer orbit requires more energy than maneuvering within the orbit plane. For the PE orbital game, an optimum open loop feedback saddle-point equilibrium solution is calculated between the pursuer and evader control structures. Using the open-loop feedback rule, each player calculates their distributed control track state. Numerical simulations demonstrate the performance using the NASA General Mission Analysis Tool (GMAT) simulator.**

I. Introduction

The main issue of satellite interception and collision avoidance is to optimally determine a flight path for a satellite. Generally, the problem can be modeled as a one-sided optimization (optimal control) setup or a two-sided optimization (game) problem. In the optimal control setup, trajectories for spacecrafts are computed based on the observed states of space objects. However, it doesn't consider the intelligence of the space objects who may change their orbits intentionally to make it difficult for the spacecraft to capture or avoid the objects. In such situations, the problem is best modeled using two competing players and it becomes a differential game, which was first researched by Isaacs [1].

In our research, we have looked at game-theoretic solutions for threat prediction and situation awareness [2, 3] in a multiplayer scenario. We utilized the methods for space threat detection [4], attack avoidance [5], and orbital evasive maneuvering through sensor management [6]. Using multi-agent modeling, we developed a framework for space situation awareness (SSA) [7]. To further the analysis, we compared several tracking methods [8] including delayed measurements [9]. Coupled with tracking, efforts of information metrics [10], search allocation game strategies [11], and service oriented architectures [12], improved the fidelity of the SSA modeling developed a system-level analysis. Our companion paper [13] provides a collision alert detection to provide operators with decision support for satellite protection.

The state-of-the art in SSA, based on the available literature, does not directly address the method of game theory. The reasons could be many of which data availability, data security, or limits to publication. However, by using the NASA General Mission Analysis Tool (GMAT) tool, we are able to explore game-theoretical SSA methods using available information for a space scenario. Our companion papers referenced above [2-12] list some of the mathematical modeling theory of orbital mechanics, simulation parameters, and other results. In this paper, we focus on the PE model used to implement the solution. Along with the implementation, our research is presented to the GMAT working group for further exploration. Much of the development from orbital mechanics is available in the GMAT tools, so the rest of the paper utilizes these definitions in the development of the PE game-theoretic model applied to the SSA domain.

This paper develops and evaluates a three-dimensional pursuit-evasion orbital game approach for satellite interception and collision avoidance. We apply 1) a decentralized sensor management (sensor-target assignment) for nonlinear tracking and then 2) the pursuit-evader game for each pair. Using a coupled zero-sum differential pursuit-evasion game, the pursuer minimizes the satellite interception time and evader tries to maximize interception time for collision avoidance. For the satellite interception problem we design an algorithm for pursuer and one for collision avoidance, where the game solution controls the evader satellite. Using the detection of collision analysis, we investigate a distributed control track state using an open-loop feedback control rule with analysis in the NASA GMAT simulator [14].

We divide our IA algorithm into two parts: first, the pursuer will rotate its orbit to the same plane of the evader, and second, the two spacecrafts will play a zero-sum pursuit-evasion (PE) game. A two-step setup saves energy during the PE game because rotating a pursuer orbit requires more energy than maneuvering within the orbit plane. To rotate the pursuer orbit plane, we utilize a series of small velocity changes $\Delta v < \Delta v_{\max}$. For the PE orbital game, an optimum open loop feedback saddle-point equilibrium solution is calculated between the pursuer and evader control structures. Using the open-loop feedback control rule, each player will calculate their distributed control track state.

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To provide a representative scenario, we use the GMAT simulator to leverage existing satellite models. The GMAT simulator provides the geometric details of the Earth, satellite orbits, and can be enhanced with other operating conditions that are available in the simulator to test the effects of the game-theoretic solution to varying conditions. With the initial analysis, we can explore higher fidelity environmental effects (e.g.. Earth occlusions and weather) in future tracking analysis.

The rest of the paper is organized as follows. In section 2, we overview decentralized sensor management. Section 3 presents the three dimensional motion and system orbital states. Section 4 describes orbital PE game model and Section 5 the trust-based sensor manager. Numerical examples are simulated for the trust-based sensor allocation algorithm in cooperative space situation awareness (SSA) search problems using the GMAT interface in Section 6 and PE in section 7. Finally we draw conclusions in Section 8.

II. Sensor Management and Control

Sensor allocation is a challenging problem within the field of multi-agent systems. The sensor allocation problem involves deciding how to assign a number of targets or cells to a set of agents according to some allocation protocol. Generally, in order to make efficient allocations, we need to design mechanisms that consider both the task performers' costs for the service and the associated probability of success (POS). In our problem, the costs are the used sensor resource, and the POS is the target tracking performance. Usually, POS may be perceived differently by different agents because they typically have different means of evaluating the performance of their counterparts (other sensors in the search and tracking problem). Given this, we turn to the notion of *trust* to capture such subjective perceptions. In our approach, we develop a trust model to construct a control mechanism that motivates sensor agents to limit their greediness. Then we model the sensor allocation optimization problem with trust-in-loop *negotiation game* and solve it using a sub-game perfect equilibrium.

There are two major classes of sensor management algorithms: centralized and decentralized. In *centralized sensor management* (CSMgt) [15, 16], a central processing node(s) collects information from all other sensors, targets, and/or environments, and then assigns sensors to different targets based on the available exploited information. The advantages of a CSMgt strategy include a simple system design and less computational load in a small scale network. However, centralized approaches are not always suitable for modern sensing/signal processing systems which includes large networks of sensors and often require higher robustness (i.e. failure of the central node would cause the failure of the whole system) and are not appropriate for sensors with critically low signal-to-noise ratios or sensors operating in dangerous areas. CSMgt sensor network system designs, some communication links might be broken at unexpected times. Also, the information quality/correctness, (i.e. dependability, sustainability, and reliability of received information), would create network survivability issues and communication delays..

To overcome shortcomings of the centralized approach, we need to develop a *decentralized sensor network management* (DSMgt) approach for robust real-world applications. In DSMgt approaches [17, 18], coordination occurs locally (not globally) and there is no central node that determines globally optimal decisions. No sensor node can "broadcast" information (e.g. availability, coordination proposal, negotiation results, etc.) to all other sensor nodes. In addition, no sensor node is supposed to have global knowledge of all sensors. The advantages of decentralized approaches include a scalable, modular, and survivable (robust) sensor network system.

Sensor-to-target assignment aims to control the data acquisition process in a multi-sensor system to enhance the performance of target tracking. The problem of *sensor assignment* can be understood from the point of view of an economic supply and demand analysis. By treating targets as "customers," each target with explicit or implicit demand requirements is satisfied by supplying their needs with least amount of resources. In this paper, we propose a *negotiable game-theoretic based sensor management* (NG-SMgt) approach to deal with the requirements of a dynamic sensor management and assignment.

In [19], a game-theoretic approach to DSMgt for target tracking via sensor-based negotiation has been developed. If every agent or sensor is rational and cooperative (i.e. follow the negotiation result), the tracking performance is optimal. However, sometimes a sensor may choose a greedy tracking strategy to maximize its own performance which might degrade the team performance. So, we revised the negotiation game model by integrating a *trust model* to limit the greediness of each sensor.

Using the detection of collision analysis, we investigate a distributed control track state using an open-loop feedback control rule with analysis in the NASA GMAT simulator.

III. Spacecraft System States

In the analysis of the game-theoretic satellite control analysis, we use the following states to describe the kinematics and dynamics of the spacecrafts.

$$\dot{r} = v \sin\gamma \quad (1)$$

$$\dot{v} = \frac{T}{m}\cos\alpha\cos\beta - \frac{\mu\sin\gamma}{r^2} \quad (2)$$

$$\dot{\gamma} = \frac{v\cos\gamma}{r} + \frac{T}{m}\frac{\sin\alpha\cos\beta}{v} - \frac{\mu\cos\gamma}{r^2 v} \quad (3)$$

$$\dot{\xi} = \frac{v\cos\gamma\cos\zeta}{r\cos\phi} \quad (4)$$

$$\dot{\phi} = \frac{v\cos\gamma\sin\zeta}{r} \quad (5)$$

$$\dot{\zeta} = \frac{T}{m}\frac{\sin\beta}{v\cos\gamma} - \frac{v\cos\gamma\sin\phi\cos\zeta}{r\cos\phi} \quad (6)$$

The set of variables (r, v, γ, ζ, ξ, ϕ) defines the 3-D motions of spacecrafts. As shown in Fig. 1, r is the instantaneous radius from Earth center, v is the velocity magnitude, γ is the flight path angle. ζ is the velocity azimuth angle, ξ is the absolute longitude, and ϕ is the latitude. The control is conducted with the thrust direction, specified by the two angles α and β.

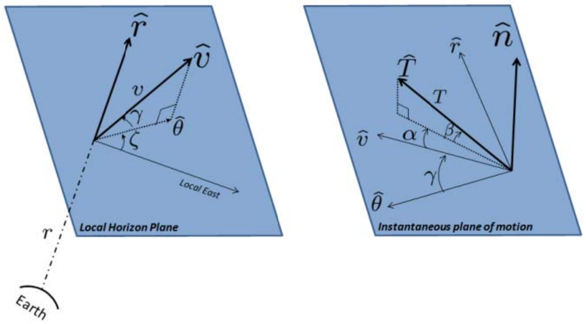

Figure 1: Spacecraft system states r, v, γ, ζ, with ξ (longitude) and ϕ (latitude) defined as the coordinate planes. α, β are the pointing angles of the thrust T and outward normal, n, from the Earth.

The transformation between the coordinate systems Earth-centered inertial (ECI) and local East-North-Up (ENU) is shown in Fig. 2 and the equations are widely available.

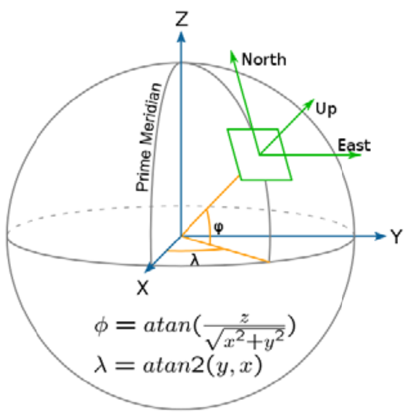

Figure 2: ECI and local ENU coordinate systems.

IV. Orbital Pursuit-Evasion Game Model

In a general zero-sum game model, each player P (purser) and E (evader) has its own system states:

$$\dot{x}_P = f_E(x_P, u_P, t) \tag{7}$$

$$\dot{x}_E = f_E(x_E, u_E, t) \tag{8}$$

The objective function in general form is

$$J = \phi(x_{P0}, x_{E0}, x_{Pf}, x_{Ef}, t_0, t_f) \tag{9}$$

The boundary condition is captured in

$$\Psi(x_{P0}, x_{E0}, x_{Pf}, x_{Ef}, t_0, t_f) = \vec{0} \tag{10}$$

In a zero-sum game, a pair of optimal strategies is a saddle-point equilibrium solution. For open-loop representation [20] of an optimal feedback strategy, we define

$$H \triangleq \lambda_P^T f_P + \lambda_E^T f_E \tag{11}$$

$$\Phi \triangleq \phi + \upsilon^T \Psi \tag{12}$$

Then

$$\dot{\lambda}_P = -\left[\frac{\partial H}{\partial x_P}\right]^T = -\left[\frac{\partial f_P}{\partial x_P}\right]^T \lambda_P \tag{13}$$

$$\dot{\lambda}_E = -\left[\frac{\partial H}{\partial x_E}\right]^T = -\left[\frac{\partial f_E}{\partial x_E}\right]^T \lambda_E \tag{14}$$

$$\lambda_P(t_f) - \frac{\partial \Phi}{\partial x_P}(t_f) = \vec{0} \quad ; \quad \lambda_E(t_f) - \frac{\partial \Phi}{\partial x_E}(t_f) = \vec{0} \tag{15}$$

The optimal control of each player is

$$u_P^* = \arg\min_{u_P} H = \arg\min_{u_P} [\lambda_P^T f_P] \tag{16}$$

$$u_E^* = \arg\min_{u_E} H = \arg\min_{u_E} [\lambda_E^T f_E] \tag{17}$$

For our satellite interception and collision avoidance problems, f_P and f_E are defined in equation (1) - (6), $J = t_f$, and

$$x_P = [r_P, v_P, \gamma_P, \xi_P, \phi_P, \zeta_P]^T = [x_1, x_2, x_3, x_4, x_5, x_6]^T \tag{18}$$

$$x_E = [r_E, v_E, \gamma_E, \xi_E, \phi_E, \zeta_E]^T = [x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T \tag{19}$$

$$u_P = [\alpha_P, \beta_P]^T = [u_1, u_2]^T \; ; \tag{20}$$

$$u_E = [\alpha_E, \beta_E]^T = [u_3, u_4]^T \; ; \tag{21}$$

$$\Psi = \begin{bmatrix} x_{1f} - x_{7f} \\ x_{4f} - x_{10f} \\ x_{5f} - x_{11f} \end{bmatrix}^T = \vec{0} \tag{22}$$

$$\lambda_P \triangleq [\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6]^T \text{, and} \tag{23}$$

$$\lambda_E \triangleq [\lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12}]^T \tag{24}$$

After we obtain the feedback control strategy, we utilize the estimated system states from our distributed tracking algorithms. Using the tracking results, such as sensor-to-target pose [21], the pursuer is able to capture the evader by modifying its control strategy to characterize the track states that afford interception.

In the game setup, we assume that a low constant thrust-to-mass ratio (*T/m*) is along the trajectory for both spacecrafts and the thrust pointing direction (α and β) is the only control option available for collision avoidance. A sufficient condition that the purser will eventually intercept the evader is $(T/m)_P > (T/m)_E$.

V. Trust-Based Sensor Management

In this section, we present a distributed trust-based negotiation game model for sensor management in multi-sensor multi-target tacking situations. It is an extension of our previous work [19]. In our negotiation framework, each negotiation agent represents a sensor. The greediness of each sensor is limited by a *trust value*, which is large when a sensor node (player in a game) stays near in the sense of sub-game perfect equilibrium and small when the team optimal result deviates from the negotiation result.

A. Subgame Perfect Equilibrium Based Negotiation Strategies

A fundamental concept in game theory is the *Nash Equilibrium* (NE) [22]. In the game, each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy (i.e., by changing unilaterally). If a player has chosen a strategy and no other player can benefit by changing his strategy while the other players keep their strategies unchanged, then the current set of strategy choices and the corresponding payoffs constitute a NE. Each strategy in a NE is a best response to all other strategies in that equilibrium. The NE may sometimes appear non-rational as a NE is not Pareto optimal. The NE may also have non-rational consequences in sequential games because players may "threaten" each other with non-rational moves.

However, in many cases all the players might improve their payoffs if they could somehow agree on strategies different from the NE, which leads the concept of *Subgame Equilibrium* (SE) [23]. SE is an attempt to choose a result from the set of Nash Equilibria and a NE structure will be maintained in each subgame. NE is a normal-form concept, which ignores the sequential structure of play in extensive-form games. As a result NE predicts some equilibria which appear problematic in the extensive form. But, SE can avoid these problems by reaching a local optimal. In a SE-based negotiation, the negotiation bargain will be finished with agreement within two time steps. Using the bargaining strategy in negotiation, we develop a *negotiable game-theoretic based sensor management* (NG-SMgt) for sensor-to-target assignments that increase target tracking efficiency.

B. Negotiation Game Model with Trust

For each sensor (represented by an agent A_i), we create a negotiation game played by the sensor and its neighboring sensors N_i = {Neighboring sensors of sensor A_i}. Negotiation is defined as the determined sensor-to-target assignments. We assume that there are communication links among the negotiation agent set $\{A_i, N_i\}$ so that each agent in the set knows the current targets of other sensors. The sensor-based negotiation game is represented by a 5-tuple G_i = < ***Agents***, ***Targets***, ***H***, ***P***, ***C*** >, where

- ***Agents*** = $\{A_i, N_i\}$; are the set of players in the game.
- ***Targets*** are the set of targets currently assigned to ***Agents***. To enable sensors be in quiescence mode, we add 0 to ***Targets***;
- ***H*** is the history sequence of the negotiation process including the offers and responses;
- ***P*** is the function to obtain to allow an agent to make an offer. *P* is defined on ***H***;
- ***C*** is the utility function set. ***C*** = $\{C_i,\ i \in$ ***Agents***$\}$.
  The utility function is based on the following equation (The gain contributed by sensor *i* to target *j* at time *k*).

$$g(i,j;k) = \min eig(M_{j,k-1} + H_{ij}^T(k)R_{ij}^{-1}(d_{ij}(k))H_{ij}(k)) - \min eig(M_{j,k-1}) \quad (25)$$

where $M_{j,\ k-1}$ is the information matrix (the inverse of the *P* matrix) of target *j* at the previous time. *Min eig* is the operator to calculate the minimal eigenvalue of a matrix.

For sensor *i*, its performance to the assigned targets $D_i(k)$ at time *k* is

$$G(i,D_i(k);k) = \begin{cases} Q_i & ,D_i(k) \text{ is empty} \\ \sum_{j\in D_i} g(i,j;k)V_j & , \text{ otherwise} \end{cases} \quad (26)$$

where V_j is the target value. The Q_i is the reward assigned to sensor *i* when it is in quiescence mode. This virtual reward will motivate the sensor to save sensing resources (e.g. power) for possible future usage. For a fixed *k*, the history (0 to *k*) are further indexed by *t* for negotiations.

For a negotiation starting at time t_0, if an agreement has been reached at time t_1, then the utility $C_i(t_0, t_1)$ for agent *i* is

$$C_i(t_0,t_1;D_i(t_0),D_i(t_1)) = \frac{Tr_i(t_0)\left[\sum_{t=t_0}^{t_1} G(i,D_i(t_0);t) + G(i,D_i(t_1);t_1)\right]}{t_1 - t_0 + 1} \quad (27)$$

where $Tr_i(t_0)$ is the trust of sensor *i* at time t_0. Here we assume that the assigned target set $\{D_i\}$ for each sensor (or agent) does not change during the negotiation process until the termination of the negotiation. We will prove later that the negotiation will be finished within one time step. The one-step approach is myopic, but the method can be extended to non-myopic approaches for larger time horizons to afford system-level threat and survivability analysis, to overcome large network communication challenges, and to sustain operation over scenarios with various mobile sensor types, hazardous environments, and intelligent targets.

Although the concept of trust has been widely used in social science literature [24, 25, 26], there is no clear consensus on the definition of trust in sensor management research area. *Trust*, by definition, implies interaction between two or more parties in either adversarial or cooperative relationships. Trust represents the reputation of an agent (sensor) and evaluates the performance as well as the cooperation (whether follow the negotiation results) of an agent in the past. We define the notion of trust to be large when it a sensor node (player in a game) stays near the negotiation choice and to be small when it goes away from it as defined below:

$$Tr_i(k) = \frac{\sum_{t=0}^{k} \max\left\{G(i,D_i^*(t);t) - \left|G(i,D_i^*(t);t) - G(i,D_i^{real}(t);t)\right|, 0\right\}}{\sum_{t=0}^{k} G(i,D_i^*(t);t)} \quad (28)$$

where $D_i^*(t)$ is the sensor *i*'s assigned target set from the team negotiation agreement of time *t*. From Eq (28), we can obtain the following trust in recursive form:

$$Tr_i(k) = Tr_i(k-1) + \frac{\max\{G_A(t) - |G_A(t) - G_R(t)|;0\} - Tr_i(k-1)\, G(i, D_i^*(k); k)}{\sum_{t=0}^{k} G_A(t)} \quad (29)$$

where $G_A(t) = G(i, D_i^*(t); t)$ is the negotiated sensor goal, and $G_R(t) = G(i, D_i^{real}(t); t)$ is the real negotiated goal.

In the NG model, we make the following assumptions:

- **Rationality**. All agents in the system are self-interested and rational. Each agent tries to maximize its own benefits in negotiations.
- **Initial Quiescence**. After the negotiation begins, agents will not make any measurements until the end of the negotiation agreement process (which will be obtained after a few time steps). Therefore the effects of initial quiescence are relatively small and repeated negations can be made for an entire game scenario

- **Sensor Capacity = 1**. Generally, the set D_i of sensor i can contain multiple targets. Here we first assume that the capacity of each sensor is 1, so that $|D_i| = 1$. Therefore each sensor will select one target from ***Targets***. $D_i = \{d_i \in$ ***Targets***$\}$. However, it is possible that one target is selected by more than one sensor. This assumption can be relaxed through utility functions.
- **Agent Negotiation Capability = 1**. It means at any time, each agent only involve one negotiation game.
- **Negotiation Game (NG) Enable Control.** A NG can be launched only after the host agent (who maintains the negotiation game) receives all acknowledge (ACK) signals from its neighboring sensors N_i. The neighboring agent will send *reschedule signal* if it is involved in another negotiation or some of its status information is updated to request it to participate in the NG. Then the host agent has to updates the game and resends the signal to enable a negotiation.

VI. SENSOR MANAGEMENT SIMULATIONS

We used the Low Earth Orbit (LEO) and Geosynchronous (GEO) satellite scenario (Fig. 3) to evaluate our negotiation game based sensor assignment algorithm. There are 4 LEO satellites, namely ARIANE 44L, OPS 0856, VANGUARD 1 an ECHO 1, as observers, and 2 GEO satellites, namely EchoStar 10 and COSMOS 2350, as space objects. The orbits are generated by the GMAT with the two-line elements (TLEs) from space-track.org.

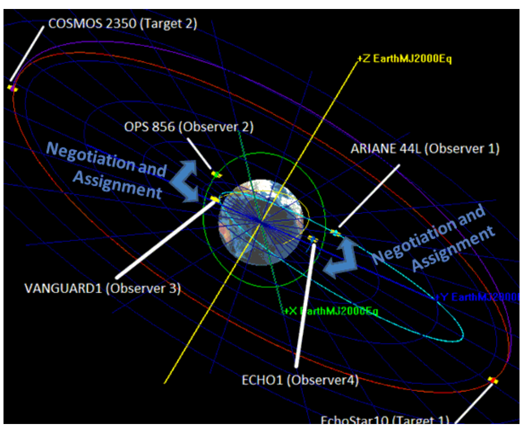

Figure 3: A space object tracking scenario with 4 LEO observers and 2 GEO targets.

After obtaining the sensor-to-target assignment (STA) based on the negotiation game, we used various trackers, such as extended Kalman filter (EKF), unscented Kalman filter (UKF), and linear minimum mean square error (LMMSE) filter to track GEO targets. We set Q_i =0 (no rewards for quiescence mode) and V_i = 1 (both GEOs have same importance). We simulated the system for 1000 time steps and each time step is 50s. The sensor assignment is shown in Fig. 4 (from the point view of targets). The results show that negotiation vary based on the capability of the sensors to track the targets.

The sensor management also includes a utility analysis and for each target, the minimal eigenvalue of the information matrix. The tracking results for object 1 are shown in Fig. 5 (which is similar to object 2). We can see that LMMSE and UKF are better than EKF in terms of reducing the satellite tracking position errors. Since the system is nonlinear, the linearization used in EKF will produce large errors. More specifically, an ellipse orbit model is not very accurate to describe the GMAT orbital propagators from the target location error (TLEs). However, in the UKF and LMMSE tracker, sigma points based sampling methods are used to approximate the nonlinear system mean and covariance.

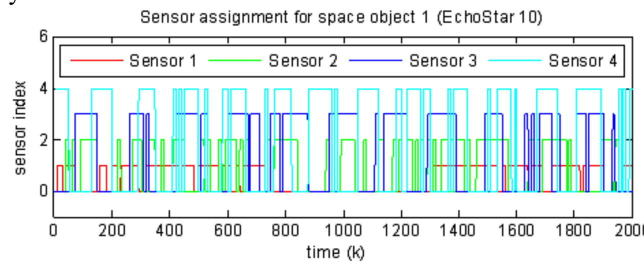

Figure 4: Sensor assignment for a space object. (Sensor i = 0 means the sensor is not assigned to a target)

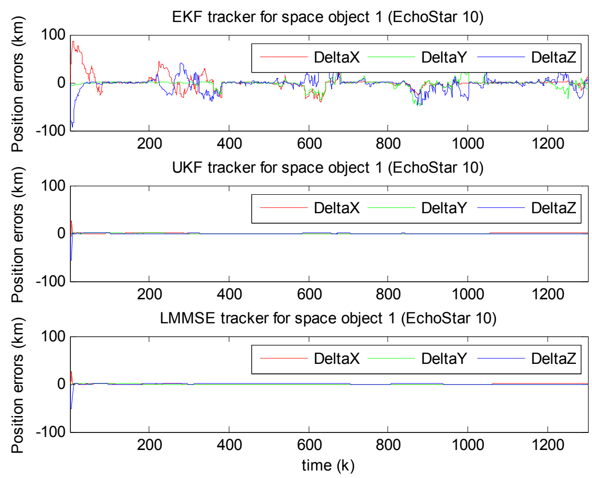

Figure 5: Tracking errors in X, Y and Z (DeltaX, DeltaY and DeltaZ) for space object 1 (Echo Star 10).

The proposed negotiation game approach for distributed sensor management has several features. First, it is a *Distributed agent system*. Each agent represents a physical sensor. Each agent hosts and maintains a negotiation game. By playing the game with its neighboring agents, the host agent generated a self-enforced target assignment among the players of the game. All the decisions are made locally. Second, *target tracking performance metrics* and *trust values* are integrated in the utility (or objective) functions of agents. We build the utilities based on the sensor gains and target values, which can be straightforwardly replaced with different metrics. Third, there is *no requirement on the number of targets and sensors*. No pseudo sensors are needed to handle the cases with more targets than sensors.

VII. PURSUIT-EVASION SIMULATIONS

To demonstrate the performance of the proposed trust-based approach, we simulate a scenario (Fig. 3) with 4 LEO observers and two GEO satellites: ECHO 1 (as Pursuer) and VANGUARD1 (as Evader). The problem is partitioned into two parts. At first, a potential pursuer (ECHO 1) satellite will rotate its orbit to place itself "behind" a high-value GEO (VANGUARD1) asset's orbital plane with a small inclination difference (e.g., 5 or 6 degrees differences). To perform rotation, we apply the minimum energy approach [27] to rotate the orbit. From the TLEs, we can calculate the initial inclination $\Delta i = 13.04$(deg). Then a zero-sum game will be played between the pursuer and the evader satellites.

After the orbit rotation, we apply the PE game interception approach and the results include the (a) position related states r, ξ (longitude) and ϕ(latitude) and (b) the velocity related states v, γ, ζ, the of the pursuer and the evader (not shown). From Fig. 6, we notice that the control angles of both satellites are approximately same, which does confirm the conclusion [28, 29] that the optimal control angles of pursuer and the evader are the same. The differences in the beginning are due to the bigger tracking errors when there are large distances. After about 160 time steps, the purser is able to intercept the evader. Given the analysis, we can effectively design control methods for collision avoidance based on the pursuer strategies.

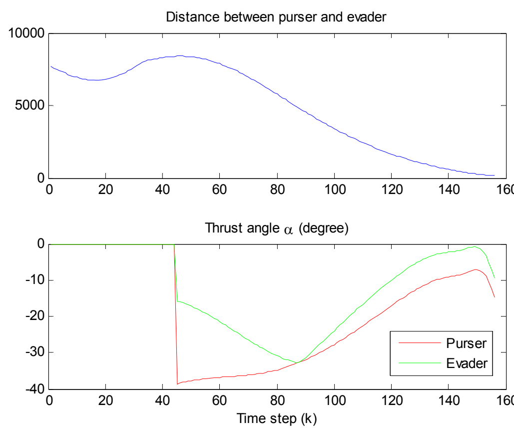

Figure 6: Pusurer and evader distance and control angles.

VIII. Conclusions

We proposed a pursuit-evasion (PE) orbital game approach for satellite interception and collision avoidance using a decentralized sensor management sensor-to-target assignment for nonlinear tracking. In the three dimensional game model, the purser minimized the satellite interception time while the evader tried to maximize it. The interception-avoidance game approach provided a worst-case solution, which is the robust lower-bound performance case. Since rotating a pursuer orbit requires more energy than maneuvering within the orbit plane, we presented a two-step setup to save energy during the PE game. First, the pursuer will rotate its orbit to the same plane of the evader, and second, the two spacecrafts will play a zero-sum PE game. We implemented the negotiation with trust orbital game approach and simulated a spacecraft interception scenario using the NASA General Mission Analysis Tool (GMAT) simulator and future efforts include multiple evaders for a [30, 31], advanced metrics, and enhanced situation awareness visualization tools.

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