Effect of Message Transmission Diversity on Status Age

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***Abstract*—We investigate the performance of a status monitoring system, in which a sensor sends random status updates over a network to a remote monitor. Specifically, we analyze the status age metric, which characterizes how old the information at the monitor is from the last received status update. The system on which we focus is a single queue with 2 servers (specifically, an M/M/2). In a dynamic network, different status packets may take different routes to the monitor, which allows for the possibility of packets arriving out of order. In the case of the status monitoring system, only the latest status is useful. Studying a system with 2 servers allows for the possibility of packets to arrive out-of-order while still having to queue. We present the exact approach to computing the analytical status age, and we provide an approximation that matches very closely with the simulated age. We also compare with the M/M/∞ and M/M/1, and we demonstrate the tradeoff between status age and network resource consumption.**

I. Introduction

In this paper, we revisit problem of the age of information for a real-time status monitoring system. Status monitoring systems consist of a source (e.g., a sensor) that periodically sends update messages to a monitor that is just interested in the latest status. For such systems, a recently proposed metric was defined, called the *status update age* [1], which characterizes the freshness of the information at the monitor. The status age can be loosely defined as the duration between the time of observation at the monitor (of the latest status) and the time at which that status message originated at the source.

The status age was first characterized for a system consisting of a queue with a single server [1]. It was shown that when packets are generated infrequently, the monitor receives packets infrequently, and average status age can be large, i.e., what the monitor observes, on average, is an old status. Increasing the rate of packet generation (or equivalently, system utilization) improves the age, but at some point, packets are generated too frequently, which backlogs the queue, and packets spend a long time in the queue before reaching the monitor. As a result, there is an optimal rate at which packets can be generated to minimize the average age.

The status age has also been characterized for a system in which the source is separated from the monitor by a network [2], in which packets generated at the source immediately enter the network and reach the monitor after a random exponential time. This model can be viewed as a system with infinite memoryless servers. In this system, since the packets spend a random time in the network, there is a possibility of packets arriving at the monitor out of order, so that some packets that arrive are not useful updates. It was shown that increasing the rate of packet generation will always result in a smaller average status age, but at the cost of more useless packets in the system, which equates to a waste of network resources. In [3], it was also shown that for a single server with last-come-first-served queue discipline, increasing the utilization will always reduce the average age, but in that case, older packets were dropped rather than sent to the monitor out of order.

Modeling the network as an infinite number of memoryless servers does not reflect the behavior that packets that enter the network earlier are more likely to reach the monitor first. However, a single server system does not reflect the dynamics of a network (e.g., changing routes) which allow for packets to be received out of order. In this work, we study the intermediate case of a queue with multiple servers, which balances the out-of-order reception with the in-order queueing. The combination of queueing and out-of-order reception models the effect of transmission diversity over multiple paths. In this work, we analyze the status age for a system with two servers and provide an approximation that is very close to the simulated age. We compare the performance with the the M/M/1 and M/M/∞ cases, demonstrating the tradeoff between the age and the consumption of network resources as the number of servers varies.

II. M/M/2 System Model

We study the status age for a system in which status packets enter into an M/M/2 queue, and the packets exiting both servers are observed at a monitor. The age of information is defined as $\Delta(t) = t - u(t)$, where $u(t)$ is the latest timestamp of all packets that have been received at the monitor. This timestamp is not necessarily associated with the most recently received packet, since a packet can be generated and finish service at the other server while an older packet is still being served. We call the older packet *obsolete* if it has been bypassed by a fresher packet.

As shown in Figure 1, packets arrive into the queue at times $t_0, t_1, \ldots,$, and they depart the server at times $\tau_{0,s}, \tau_{1,s}, \ldots,$ where s denotes which server the packet is served by. The interarrival time of the ith packet is given by $X_i = t_i - t_{i-1}$.

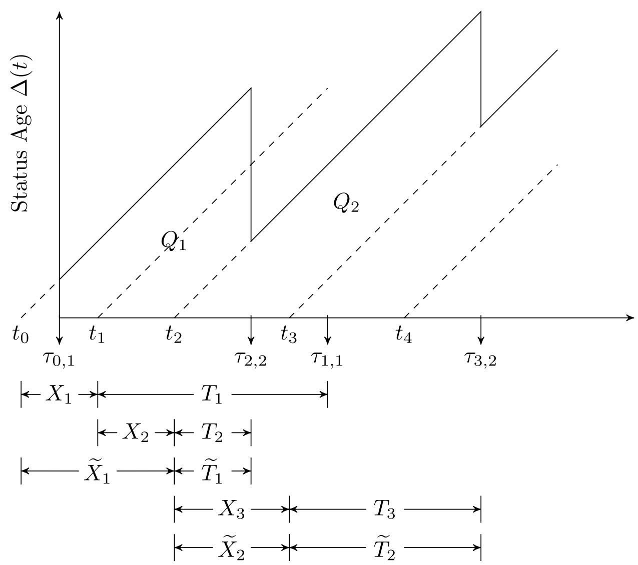

Fig. 1. Plot of status age for system with M/M/2 queue.

The X_i's are exponential i.i.d. random variables with mean $1/\lambda$. The service time of the ith packet is denoted by S_i and are also exponential i.i.d. random variable with mean $1/\mu$. The system time is given by $T_i = \tau_{i,s} - t_i$, and it is equal to the time waiting in queue W_i plus the service time S_i.

We present an example of a status age plot in Figure 1. Packet 1 arrives at time t_1, and it finds the first server empty. At time t_2, packet 2 arrives and finds the first server to be busy, so it enters into the second server. In this plot, packet 2 is shown to be served before packet 1, so the monitor recognizes packet 2 as its most recently generated packet. We refer to packet 2 as an *informative* packet and packet 1 as an *obsolete* packet, as in [2]. Note that for a two-server system, two consecutive packets cannot be made obsolete, since both packets will occupy both servers, and one of them must complete service prior to any future packets entering service. Therefore, the interarrival time of the ith informative packet, denoted by $\widetilde{X}_i$, can consist of either one or two interarrivals of regular packets (e.g., $\widetilde{X}_1 = X_1 + X_2$). We denote the system time of the ith informative packet as $\widetilde{T}_i$.

Similar to the approach for the M/M/1 [1], the average status age can be computed graphically using the areas of the informative trapezoids Q_i, which have dimensions $\widetilde{X}_i$ and $\widetilde{T}_i = \widetilde{W}_i + \widetilde{S}_i$. Using the approach in [1], the expression for the average age can be shown to be

$$\Delta = \widetilde{\lambda}(E[\widetilde{W}\widetilde{X}] + E[\widetilde{S}]E[\widetilde{X}] + E[\widetilde{X}^2]/2), \tag{1}$$

where here $\widetilde{\lambda}$ and $\widetilde{S}$ are the arrival rate and service time of informative packets. We now derive these terms that make up the status age.

III. Probability of Packet Being Informative

Our first step is to compute the probability that a packet i is informative. The event that makes a packet i informative is the event that it finishes service before the following packet: $W_i + S_i < X_{i+1} + W_{i+1} + S_{i+1}$. Direct computation of the probability of such an event requires knowledge of the joint distribution of consecutive waiting times for an M/M/2. To avoid such complication, we exploit the memoryless property of the exponential interarrivals and service times. We first consider the case where a packet entering service at time t_i sees no other packet in service. The probability of such an event is given by the steady state probability that the number of packets in the system for an M/M/2 is equal to 0 [4]:

$$\Pr[N = 0] = \frac{2\mu - \lambda}{2\mu + \lambda}$$

where N is the number of packets in the system just prior to packet i entering one of the servers. We utilize the memoryless property to compute the probability of a packet being informative, given that there is no other packet in service at t_i. Let the event that packet i is informative be $E_1(i)$.

$$\begin{aligned}\Pr[E_1(i)|N = 0] &= \Pr[S_i < X_{i+1}] \\ &+ \Pr[S_i > X_{i+1}]\Pr[S_i' > S_{i+1}] = \frac{2\mu + \lambda}{2(\lambda + \mu)}\end{aligned}$$

where S_i' indicates the residual service time, which has the same distribution as a typical service time.

We then consider the case for which a packet entering service observes a packet in the other server, but packet $i+1$ has not yet arrived in the queue. This can occur when the number of packets in the system when i arrives in the queue is equal to 1:

$$\Pr[N = 1] = \frac{\lambda}{\mu}\frac{2\mu - \lambda}{2\mu + \lambda}.$$

It can also occur if the number of packets is greater than one, but the interarrival time of packet $i+1$ is greater than the total service time of the packets in front of packet i:

$$\begin{aligned}\Pr[N = 3] &= \sum_{n=2}^{\infty} \Pr[N' = n]\Pr\left[X_{i+1} > \sum_{k=1}^{n-1} \hat{S}_k\right] \\ &= \sum_{n=2}^{\infty} 2\Big(\frac{\lambda}{2\mu}\Big)^n \frac{2\mu - \lambda}{2\mu + \lambda}\Big(\frac{2\mu}{\lambda + 2\mu}\Big)^{n-1} = \frac{\lambda^2(2\mu - \lambda)}{2\mu^2(2\mu + \lambda)}\end{aligned}$$

where N_i' is the number in the system just before packet i arrives in the queue. This turns out to be equal to the probability that 2 are in the system at a random observation time, which can be just prior to an arrival or after a departure in the case of Poisson arrivals and exponential service times. This is because for a full server ($N' >= 3$), a packet entering service coincides with a packet departing the server. The quantity $\sum_{k=1}^{n-1} \hat{S}_k$ is the total service time of some $(n-1)$ packets ahead of packet i in the system.

Next, the conditional probability that a packet is informative given $[N = 1 \text{ or } 3]$ is computed as

$$\begin{aligned}\Pr[E_1(i)|N = 1 \text{ or } 3] &= \Pr[S_i < S_{i-k}'] + \Pr[S_i > S_{i-k}'] \\ &\cdot(\Pr[X_{i+1}' < S_{i-k}'|S_i < S_{i-k}'] \cdot \Pr[S_i' < S_{i+1}] \\ &+ \Pr[X_{i+1}' > S_{i-k}'|S_i < S_{i-k}']\Pr[S_i' < X_{i+1}''] \\ &(+\Pr[S_i' > X_{i+1}'']\Pr[S_i'' < S_{i+1}])) \\ &= \frac{1}{2} + \frac{1}{2}\Big(\frac{\lambda}{2(\lambda + 2\mu)} + \frac{\mu}{\lambda + \mu}\Big)\end{aligned}$$

where X_i' (and S_i'') indicates a residual time.

For all other cases, packet $i+1$ has already arrived in the queue by the time i starts service. The probability that such a packet is informative is given by

$$\Pr[E_1(i)|N \geq 4] = \Pr[S_i < S'_{i-k}] + \Pr[S_i > S'_{i-k}]\Pr[S'_i < S_{i+1}] = \frac{3}{4}$$

Finally, we can compute the overall probability that a packet is informative as

$$\begin{aligned}\Pr[E_1(i)] &= \Pr[N=0]\Pr[E_1(i)|N=0] \\ &+ \Pr[N=1 \text{ or } 3]\Pr[E_1(i)|N=1 \text{ or } 3)] \\ &+ \Pr[N \geq 4]\Pr[E_1(i)|N \geq 4] = \frac{3\lambda^2 + 8\lambda\mu + 8\mu^2}{4(\lambda+\mu)(\lambda+2\mu)}.\end{aligned}$$

IV. Conditional Waiting Time Given Interarrival

We would like to derive the conditional waiting time for a packet given a single packet interarrival time. We first derive the conditional probability of the number in the system n just before the arrival of a packet, given its interarrival time. For $n \geq 2$,

$$\begin{aligned}p_{n|x} &= \sum_{k=n-1}^{\infty} p_n \Pr[k+1-n \text{ served}|x, \text{ both servers busy}] \\ &= 2\frac{1-\rho}{1+\rho}\rho^{n-1}e^{-2\mu(1-\rho)x}.\end{aligned}$$

The probability of packets being served during a length of time x for a G/M/m system is given in [4]. For $n < 2$, the waiting time is equal to zero. The conditional CDF of the waiting time is given by

$$\begin{aligned}F_{W_i|X_i}(w|x) &= p_{0|x} + p_{1|x} + \sum_{k=2}^{\infty}\int_0^w \frac{2\mu(2\mu z)^{k-2}}{(k-2)!}e^{-2\mu z}2 \\ &\cdot\frac{1-\rho}{1+\rho}\rho^{k-1}e^{-2\mu(1-\rho)x}\mathrm{d}z \\ &= p_{0|x} + p_{1|x} + \frac{2}{1+\rho}\rho e^{-2\mu(1-\rho)x}(1-e^{-2\mu(1-\rho)w}).\end{aligned}$$

The terms $p_{0|x}$ and $p_{1|x}$ are not functions of w since the waiting time is zero in such cases. Thus, the conditional PDF of the waiting time is given by

$$f_{W_i|X_i}(w|x) = \frac{4\mu\rho(1-\rho)}{1+\rho}e^{-2\mu(1-\rho)(x+w)} + C_1\delta(w)$$

and the conditional expected waiting time is given by

$$\begin{aligned}E[W_i|X_i = x] &= \int_0^{\infty} w f_{W_i|X_i}(w|x)\mathrm{d}w \\ &= \frac{\rho}{\mu(1+\rho)(1-\rho)}e^{-2\mu(1-\rho)x}. \qquad (2)\end{aligned}$$

Since the interarrival of informative packets may be the sum of two regular interarrivals (when the previous packet is obsolete), we need to know the conditional waiting time given two packet interarrival times. We omit the derivation here, which requires deriving the PDF of $f_{W_i|X_{i-1},X_{i-2}}(w|x_1,x_2)$. The conditional expected waiting time is given by

$$\begin{aligned}E[W_i|X_{i-1} = x_1, X_{i-2} = x_2] &= \frac{1-\rho}{2\mu(1+\rho)(1-2\rho)} \\ &\cdot((1+2\rho)e^{-\mu x_1} - 2e^{-2\mu(1-\rho)x_1})e^{-2\mu x_2} \\ &+ \frac{1}{\mu(1+\rho)(1-\rho)}e^{-2\mu(1-\rho)(x_1+x_2)}. \qquad (3)\end{aligned}$$

V. Consecutive Informative Packets

To fully characterize the interarrival of informative packets $\widetilde{X}_i$, we need to identify the cases for which an informative interarrival is equal to one vs. two regular interarrivals. We first enumerate the events that make a packet informative or not, given the number in system prior to service (N_i), and the probabilities of such events, as follows:

1) $N_i = 0$
   i is informative:
   - $C_{1a} : \Pr[S_i < X_{i+1}] = \frac{\mu}{\lambda+\mu}$
   - $C_{1b} : \Pr[S_i > X_{i+1}$ and $S'_i < S_{i+1}] = \frac{\lambda}{2(\lambda+\mu)}$

   i is not informative:
   - $C_{1c} : \Pr[S_i > X_{i+1}$ and $S'_i > S_{i+1}] = \frac{\lambda}{2(\lambda+\mu)}$
2) $N_i = 1$ or 3
   i is informative:
   - $C_{2a} : \Pr[S_i < S'_{i-k}] = \frac{1}{2}$
   - C_{2b} : $\Pr[S_i > S'_{i-k}$ and $X'_{i+1} < S'_{i-k}$ and $S'_i < S_{i+1}] = \frac{\lambda}{4(\lambda+2\mu)}$
   - C_{2c} : $\Pr[S_i > S'_{i-k}$ and $X'_{i+1} > S'_{i-k}$ and $S'_i < X''_{i+1}] = \frac{\mu^2}{(\lambda+\mu)(\lambda+2\mu)}$
   - C_{2d} : $\Pr[S_i > S'_{i-k}$ and $X'_{i+1} > S'_{i-k}$ and $S'_i > X''_{i+1}$ and $S''_i < S_{i+1}] = \frac{\lambda\mu}{2(\lambda+\mu)(\lambda+2\mu)}$

   i is not informative:
   - C_{2e} : $\Pr[S_i > S'_{i-k}$ and $X'_{i+1} < S'_{i-k}$ and $S'_i > S_{i+1}] = \frac{\lambda}{4(\lambda+2\mu)}$
   - C_{2f} : $\Pr[S_i > S'_{i-k}$ and $X'_{i+1} > S'_{i-k}$ and $S'_i > X''_{i+1}$ and $S''_i > S_{i+1}] = \frac{\lambda\mu}{2(\lambda+\mu)(\lambda+2\mu)}$
3) $N_i = 4$
   i is informative:
   - $C_{3a} : \Pr[S_i < S'_{i-k}] = \frac{1}{2}$
   - $C_{3b} : \Pr[S_i > S'_{i-k}$ and $S'_i < S_{i+1}] = \frac{1}{4}$

   i is not informative:
   - $C_{3c} : \Pr[S_i > S'_{i-k}$ and $S'_i > S_{i+1}] = \frac{1}{4}$
4) $N_i \geq 5$, cases are same as $N_i = 4$

Separating the cases in 3) and 4) is needed when computing the transition probabilities from packet i to packet $i+1$.[1] We call the overall set of cases $\mathcal{C} = \{C_{1a}, C_{1b}, \ldots, C_{4c}\}$. The informative cases are denoted as the set $\mathcal{C}_{\text{inf}} = \{C_{1a}, C_{1b}, C_{2a}, C_{2b}, C_{2c}, C_{2d}, C_{3a}, C_{3b}, C_{4a}, C_{4b}\}$, and the remaining non-informative cases are denoted as $\mathcal{C}_{\text{ninf}} = \mathcal{C}\backslash\mathcal{C}_{\text{inf}}$. For each case $C \in \mathcal{C}$, the event that there is the corresponding number in queue is given as $N(C) : N_i = n$. For example, $N(C_{4x}) : N_i \geq 5$.

[1]For systems with more than 2 servers, even more cases would be required, since more runs of packets can be rendered obsolete.

We now consider the next packet $i+1$, and determine the events and probabilities for which it finds itself with $N_{i+1}=0$, $N_{i+1}=1$, etc., given that packet i is informative under one of the conditions in $\mathcal{C}$. For example, if packet i is informative under condition 1a) from the list above, then the system is emptied before packet $i+1$ arrives, which means $N_{i+1}=0$, or condition 1) (occurs with probability 1). We denote this event as $T(C_{1a}, C_{1x})$. As another example, if packet i is informative under condition 2b), then packet $i+1$ has arrived before i or $i-k$ is served. If $X_{i+2} > S''_{i-k}$, then $N_{i+1}=3$, except that it cannot be served prior to packet i (as in 2a)), so we denote this as $T(C_{2b}, C_{2x})$, $x=b,c,d,e,f$. There is also the possibility of $N_{i+1} \geq 4$ if $X_{i+2} > S''_{i-k}$, which we denote as $T(C_{2b}, C_{3}x)$. We enumerate a few possible cases and their probabilities as follows:

- $\Pr[T(C_{1a}, C_{1x})] = 1$
- $\Pr[T(C_{1b}, C_{2x})] = 1$, $x=b,c,d,e,f$
- $\Pr[T(C_{2a}, C_{1x})] = \Pr[X_{i+1} < S_i < X_{i+1}+S_{i+1}] = \frac{\mu}{2(\lambda+\mu)}$
- $\Pr[T(C_{2a}, C_{2x})] = \Pr[X_{i+1} < S'_{i-k}, X_{i+1}+X_{i+2} > S_i] = \frac{\lambda}{2(\lambda+\mu)}(1-\frac{\lambda}{\lambda+\mu})$
- $\Pr[T(C_{2a}, C_{3x})] = \Pr[X_{i+1}+X_{i+2} < S_i] = (\frac{\lambda}{\lambda+\mu})^2$
- $\Pr[T(C_{2b}, C_{2x})] = \Pr[X'_{i+2} > S''_{i-k}] = \frac{\mu}{\lambda+\mu}$, $x = b,c,d,e,f$
- $\Pr[T(C_{2b}, C_{3x})] = \Pr[X'_{i+2} < S''_{i-k}] = \frac{\lambda}{\lambda+\mu}$

Due to spatial considerations, we omit the transition probabilities for $T(C_{2c}, C_{1x})$, $T(C_{2d}, C_{2x})$, $T(C_{2d}, C_{3x})$, $T(C_{3a}, C_{2x})$, $T(C_{3a}, C_{3x})$, $T(C_{3b}, C_{2x})$, $T(C_{3b}, C_{3x})$, $T(C_{4a}, C_{3x})$, $T(C_{4b}, C_{3x})$. For all other transitions, the probability is equal to zero.

VI. Status Age for M/M/2

Having enumerated the possible outcomes for a packet i to be informative and the next packet $i+1$ to be either informative or non-informative, we can compute the distribution of the interarrival $\widetilde{X}_{i+1}$ (and $\widetilde{X}_{i+2}$ if $i+1$ non-informative) for each combination of events for i and $i+1$. As an approximation, we let the distribution be that of a regular exponential interarrival X_i if consecutive packets are informative, or two i.i.d. exponential interarrivals if there is a non-informative packet between informative ones.

First we look at the case of consecutive informative packets. Let $\widetilde{X}_a$ be the informative interarrival given that it is equal to one regular interarrival. The values $E[\widetilde{X}_a] = 1/\lambda$ and $E[\widetilde{X}_a^2] = 2/\lambda^2$ are given from the exponential distribution. Using (2), the value $E[\widetilde{W}\widetilde{X}_a]$ can be found by iterated expectation:

$$E[\widetilde{W}\widetilde{X}_a] = \int_0^\infty x E[W_i|X_i = x] f_X(x)\mathrm{d}x = \frac{\lambda^2}{2\mu^2(2\mu+\lambda)(2\mu-\lambda)}$$

In the case where informative packets are separated by a non-informative one, we approximate the interarrival $\widetilde{X}_b$ as the sum of two exponential random variables. We compute the values $E[\widetilde{X}_b] = 2E[X] = 2/\lambda$, $E[\widetilde{X}_b^2] = E[X_1^2 + 2X_1X_2 + X_2^2] = 6/\lambda^2$. Using (3), the value $E[\widetilde{W}\widetilde{X}_b]$ can be found by iterated expectation:

$$\begin{aligned} E[\widetilde{W}\widetilde{X}_b] &= \int_0^\infty (x_1+x_2)E[W_i|X_{i-1}=x_1, X_{i-2}=x_2] \\ &\quad \cdot f_X(x_1)f_X(x_2)\mathrm{d}x_1\mathrm{d}x_2 \\ &= \frac{\lambda^2(2\mu-\lambda)}{2\mu^2(2\mu+\lambda)^2(\lambda+\mu)} + \frac{\lambda^2}{\mu^2(2\mu-\lambda)(2\mu+\lambda)}. \end{aligned}$$

The probability p_a of two consecutive packets being informative can be found by combining 1) the probability of i being informative under one of the conditions in $\mathcal{C}$, with 2) the transition probability of corresponding events in $\mathcal{T}$, and 3) the probability that $i+1$ is informative under the condition in $\mathcal{C}$:

$$\begin{aligned} p_a = \sum_{C_i \in \mathcal{C}_{\text{inf}}} \sum_{C_{i+1} \in \mathcal{C}_{\text{inf}}} &\Pr[N(C_i)]\Pr[C_i] \\ &\cdot \Pr[T(C_i, C_{i+1})]\Pr[C_{i+1}]. \end{aligned}$$

For the case where $i+1$ is not informative, we can make a similar computation to find the probability p_b except that C_{i+1} is summed over cases where it is not informative:

$$\begin{aligned} p_b = \sum_{C_i \in \mathcal{C}_{\text{inf}}} \sum_{C_{i+1} \in \mathcal{C}_{\text{ninf}}} &\Pr[N(C_i)]\Pr[C_i] \\ &\cdot \Pr[T(C_i, C_{i+1})]\Pr[C_{i+1}]. \end{aligned}$$

As another approximation, we assume that $E[\widetilde{S}]$ is the same for both consecutive and non-consecutive informative packets. We computed $E[\widetilde{S}]$ for informative packets by averaging over the conditional expectation given a packet i is informative for each category of N_i:

$$\begin{aligned} E[\widetilde{S}] &= E[S_i|S_i < X_{i+1}+S_{i+1}]\Pr[N_i = 0] \\ &\quad + (E[S_i|S_i < S'_{i-k}] + E[S_i|S'_{i-k} < S_i < X_{i+1}+S_{i+1}]) \\ &\quad \Pr[N_i = 1,3]/2 + E[S_i|S_i < S'_{i-k}+S_{i+1}]\Pr[N_i \geq 4]. \end{aligned}$$

The approximate average age is then computed as

$$\begin{aligned} \Delta \approx \lambda(&p_a(\frac{E[\widetilde{X}_a]^2}{2} + E[\widetilde{W}\widetilde{X}_a] + E[\widetilde{S}]E[\widetilde{X}_a]) \\ &+ p_b(\frac{E[\widetilde{X}_b]^2}{b} + E[\widetilde{W}\widetilde{X}_b] + E[\widetilde{S}]E[\widetilde{X}_b])). \end{aligned}$$

VII. Bounds on M/M/2 Age

A simple upper bound is to compute the average trapezoid in Figure 1 over all packets, whether or not it is informative. The average is given by

$$\begin{aligned} \Delta_{UB} &= \lambda(\frac{E[X^2]}{2} + E[WX] + E[S]E[X]) \\ &= \lambda(\frac{1}{\lambda^2} + \frac{1}{\lambda\mu} + \frac{\rho^2}{\mu(1+\rho)(1-\rho)}) \\ &= \frac{1}{\mu}(1 + \frac{1}{2\rho} + \frac{2\mu\rho^3}{(1+\rho)(1-\rho)}). \end{aligned}$$

For the lower bound, we compute the average informative packet assuming that the interarrival is the same as a typical interarrival. We first argue that an interarrival of informative packets is stochastically greater than or equal to that of a regular packet.

$$
\begin{aligned}
&\Pr[X_i > x | E_1(i)] \\
&\quad = \Pr[X_i > x | W_i + S_i < W_{i+1} + S_{i+1} + X_{i+1}] \\
&\quad = \Pr[X_i > x | \min((T_{i-1} - X_i)^+, (T_{i-2} - X_{i-1} - X_i)^+) \\
&\quad +S_i < W_{i+1} + S_{i+1} + X_{i+1}].
\end{aligned}
$$

If $W_i = 0$, that means that X_i is either greater than T_{i-1} or $T_{i-2} - X_{i-1}$, and the probability that i is informative does not otherwise depend on X_i. If $W_i > 0$, that means i is informative if X_i is greater than $T_{i-1} - W_{i+1} - S_{i+1} - X_{i+1} + S_i$ or $T_{i-2} - X_{i-1} - W_{i+1} - S_{i+1} - X_{i+1} + S_i$. Since the only constraint on X_i when it is informative is that it is greater than something, then $\Pr[X_i > x | E_1(i)] \geq \Pr[X_i > x]$. Given that i is informative, the ith interarrival $\widetilde{X}_i$ is either equal to X_i or $X_i + X_{i+1}$, so both are stochastically greater than or equal to a typical X_i. Using the notation in Section VI, our lower bound is given by

$$
\Delta_{LB} = \tilde{\lambda}(E[\widetilde{W}\widetilde{X}_a] + E[\widetilde{S}]E[\widetilde{X}_a] + E[\widetilde{X}_a^2]/2).
$$

VIII. Numerical Results

We have evaluated our approximate age and upper and lower bounds for $\mu = 0.5$ and 1 and plotted the results vs. the system utilization ρ in Figure 2. We compare the results with the simulated M/M/2 age as well as the M/M/1 age. Here $\rho = \lambda/m\mu$ for an M/M/m queue. We see that the average status age for the M/M/2 is about 1/2 that of the M/M/1 case. Also, the approximation matches the simulated value very closely.

The upper and lower bounds seem relatively tight for lower ρ, but as $\rho \to 1$, they get looser. For the upper bound, the trapezoids from Figure 1 for all packets are averaged, including the obsolete ones, which are more prevalent as $\rho \to 1$. For the lower bound, the informative interarrival are assumed to be the same as a typical interarrival. Thus, for small ρ, most packets are informative, so the interarrival of informative packets is very similar to the typical interarrival, and the bound is tighter. As $\rho \to 1$, there are more obsolete packets, meaning that informative interarrivals are more likely to consist of two typical interarrivals.

In Figure 3, we compare the status age for the M/M/2 with that of the M/M/∞ case, and as expected, the M/M/∞ has a lower age, so the M/M/2 falls in between the M/M/1 and M/M/∞. However, the lower age of M/M/∞ comes at the cost of more obsolete packets. Likewise, the M/M/2 has a lower age than the M/M/1, but the M/M/1 does not have any obsolete packets since only one packet in service at a time. For the M/M/2, the percentage of obsolete packets approaches 25% as $\lambda/2\mu$ increases, which agrees with $1 - \Pr[i \text{ is informative for a backlogged queue}] = 1 - (\Pr[C_{4a}] + \Pr[C_{4b}])$. It is also worthy to note that for the M/M/2, there is a utilization at which the age for the M/M/2 is minimum, similar to the M/M/1 and in contrast to the M/M/∞. For M/M/1, the optimal utilization is $\rho \approx 0.53$, and for the M/M/2, the optimal is found (numerically) to be $\rho \approx 0.56$,

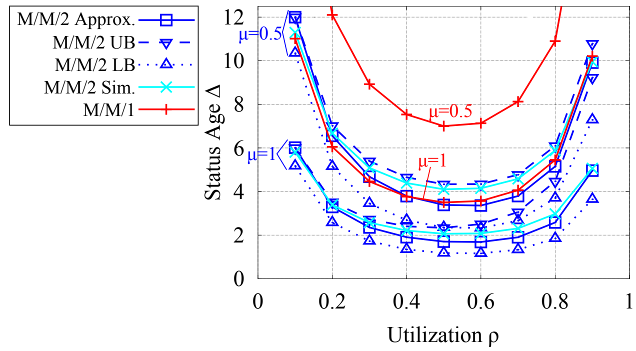

Fig. 2. Status age vs. utilization for system with M/M/2 queue, compared with M/M/1.

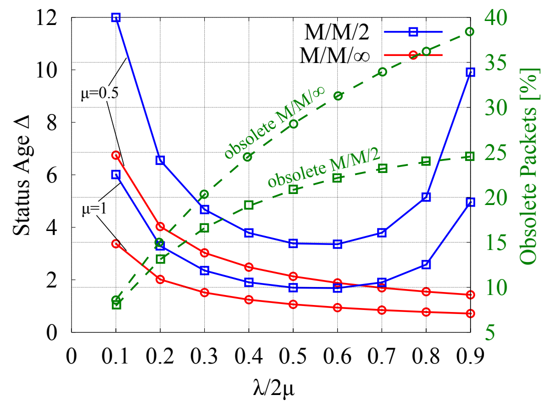

Fig. 3. Status age and % of obsolete packets vs. utilization for system with M/M/2 queue, compared with M/M/∞.

so with more servers, the utilization can be increased higher before the average age also starts to increase.

IX. Conclusion

We have looked at the status age for a monitoring system operating over a dynamic network. We model the network as an M/M/2 queue, in which packets can arrive slightly out-of-order, but with some limitations. This is in contrast to the M/M/∞ model, in which it is possible for a packet to arrive sooner than an unlimited number of future packets. While the M/M/∞ has a lower average status age due to the unlimited server capacity, it comes at the cost of wasting network resources. Future extensions include analyzing the age for a queue with m servers or multiple M/M/1 queues.

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