A k -Generation Approach to Wireless Fingerprinting for Position Estimation

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Abstract—Traditional approaches to pattern based positioning have proposed schemes specifically tailored for the area of interest in which they are employed. Many, when applied to a generic search environment, may not scale computationally due to either a very large tracking area, very fine granularity in the database, or both. Here, a novel framework for fingerprinting that utilizes a k -generation structure is proposed. This significantly reduces the required number of computations while maintaining comparable accuracy to the traditionally used single-generation approach. This is the key enabler that allows generalization of our framework to any search space. We validate this approach through simulation and show that the a multi-generation approach indeed provides computational gains while simultaneously maintaining accuracy. Second, we show how a natural grouping of constituent members can improve accuracy. We use graph spectral partitioning to demonstrate this effect. The end result is a framework that can be generalized to any size search space with an arbitrary level of granularity.

Index Terms—positioning, geolocation, fingerprinting, database correlation, cell-ID, computational complexity, WiMAX, machine learning, graph theory

I. INTRODUCTION

Driven by an emerging position-aware market and requirements for emergency assistance and response, the so-called location based services (LBS) has gained significant traction over the last decade among the research community [1]. LBS has remained a challenging and difficult problem due to the increasing demand for it in urban environments. Indeed, a recent study projected that 65% of the growth in gross domestic product from 2010 to 2025 will be driven by the population in the world's 600 top cities, further substantiating the long studied urbanization movement around the globe [2]. These urban jungles and indoor settings are unique in that they are saturated with physical blockages which may lie between a transmitter and receiver creating an environment colloquially known as non-line of sight (NLOS) [3]. In this environment, electromagnetic signals must travel some non-minimal distance en route to their destination, making the time-distance mapping a challenging problem indeed. The migration of populations to urban cityscapes which are home to these severe multi-path channel environments gave rise to

pattern-based approaches to efficient LBS [4]. Pattern based approaches take advantage of this volatility in the wireless channel by taking *a priori* snapshots of signal characteristics at various physical locations. An electromagnetic observation made at an unknown location is then compared against the database of snapshots to make a position estimate. In toto, this process is referred to by the literature as scene analysis, database correlation, or, more frequently, fingerprinting.

Essentially a pattern matching problem, wireless fingerprinting has a rich body of work from which to draw in machine learning, pattern analysis, signal processing, and stochastic optimization. After its initial conception in the literature ([4], [5]) fingerprinting was improved upon by making concessions in complexity. For example, the marriage of stochastic motion models and fingerprinting has been long studied and proven useful in providing a more accurate solution [6], [7]. The literature has seen the deployment of the hidden-Markov model (HMM) and the particle filter, among others, to this end. These models bring a sense of dynamic memory to the problem and are able to use *a posteriori* probabilities to optimize the realtime solution. Additionally, recent work has shown benefits in the union of originally abandoned geometric techniques and fingerprinting [8], [9]. Here, fingerprinting is used in conjunction with traditional location techniques to improve accuracy. Finally, sparse signal recovery via convex optimization has seen an increase in momentum as a means for extracting a location estimate [10], [11].

The previously proposed approaches [6]–[11] have provided *application specific* solutions to obtaining an accurate position estimate. The increasingly complex, albeit more accurate, schemes may be justified as computational acumen continues to scale; however, the movement toward smaller mobile devices and resource sensitive applications demand an approach sensitive to these requirements yet still arbitrarily scalable. Accordingly, in this paper, we propose a novel framework, which we term k -generational, that takes these requirements into account yet still employable in a tracking area with arbitrary size and granularity. Using this method, we constitute an area of interest into different generations of fingerprint databases. This structure allows for a significant reduction

in the search space. Additionally, we propose graph spectral partitioning as an efficient means to this end.

The remainder of this paper is organized as follows. We introduce the k -generation framework in Section II. In this section the underlying theory is presented so as to highlight its scalability to any size application. In addition to the multi-generation structure, two methods of efficient organization of generation members are presented. In Section III, we validate the proposed framework through monte carlo simulation. It is shown that the k -generation approach obtains similar accuracy to the traditionally used single-generation approach when $k = \{2, 3\}$. These results are then used to suggest similar performance for $k > 3$. Further, it is shown how natural organization of generation members can improve accuracy beyond the traditional approach in some circumstances. A summary of the findings and concluding remarks are presented in Section IV.

II. K-GENERATION FINGERPRINTING MODEL

Here, we present a novel multi-generation fingerprinting scheme that results in a significant reduction in computational complexity while maintaining acceptable levels of accuracy. First, the constitution of the radio map is formalized. Next, the grouping scheme by which the radio map is tiered is presented. Finally, we show how the fingerprint construct is leveraged to effect an efficient position estimate.

A. Radio Map Constitution

First, the reference database, or radio map, is constructed. This is a critical step as this is the baseline against which the realtime positioning will depend. To this end the tracking area, Ξ , is discretized in some application appropriate manner into a series of N_t tiles. Further, throughout the tracking area there is some N_{bs} base stations (BS) distributed in some non-specific manner although it is assumed that the BS distribution is such that the entire tracking area can be adequately serviced by the network. The cell-ID fingerprint method is chosen as the desired fingerprint type for its computational simplicity relative to the alternatives; however, it should be noted that our method is easily generalized to other types of fingerprints such as RSS [7]. The radio map is constructed largely as in [7], [12] where

$$X \triangleq \{\chi_{i,j}\} \quad (1)$$

and each element is defined by

$$\chi_{i,j} = \begin{cases} 1 & \text{if BS } i \text{ is available at tile } j \\ 0 & \text{if BS } i \text{ is not available at tile } j \end{cases} \quad (2)$$

therefore $X \in \mathbb{R}^{N_{bs} \times N_t}$. Some level of noise is assumed in the channel such that there will be readings that are non-deterministic. In other words, the cell-ID fingerprint can be thought of as a thresholded version of the RSS fingerprint, which is well known to be non-deterministic. Thus, normal channel noise will effect each reading which is an independent and identically distributed random variable. It is assumed then that each reading, while binary, is taken multiple times at

Fig. 1. An example formulation of the uniform k -generation structure is presented here where $k = 3$. Each tile represents a different subgroup and each plane represents a different generation. The shaded tiles represent the fingerprint within each generation to which some notional observation matches. The arrows show how a fingerprint at one generation maps to some number of fingerprints at lower generations. The final position estimate is noted by $\hat{p}$. Note that in the example provided, $\xi_i^{(k)} \cap \xi_j^{(k)} = \emptyset, \forall i \neq j$ although this need not be the case.

each tile and then the average entry is recorded so that each database element lies on the standard probability simplex, $\chi_{i,j} \in [0, 1]$. Each fingerprint is then formally defined as each column element, X_j , of the radio map X .

B. Hierarchical Fingerprint Structure

We begin with the tracking area of some arbitrary size denoted by the continuous set Ξ , which represents the root set of the generational structure. Ξ is then discretized in an appropriate manner to some number of subareas $\xi_j^{(1)} \subset \Xi$. In this context, j represents the number of subareas discretized from Ξ and the superscript represents the generation number such that

$$\bigcup_j \xi_j^{(1)} = \Xi. \quad (3)$$

It should be noted that $\xi_j^{(k)} \cap \xi_i^{(k)}$ need not necessarily be the empty set for some $i \neq j$, although, for the simplicity of our exposition we make this assumption.

Each generation may then be decomposed into a representative generation such that

$$\bigcup_j \xi_j^{(2)} = \xi^{(1)}. \quad (4)$$

If we then set $\xi^{(0)} = \Xi$ then the scheme can be generalized to

$$\bigcup_j \xi_j^{(k)} = \xi^{(k-1)}, \quad \forall k \in 0, 1, \dots, \infty. \quad (5)$$

A representative fingerprint, $X_j^{(k)}$, is then created for each $\xi_j^{(k)}$ in the following manner

$$X_j^{(k)} = \sum_{i \in \xi_j^{(k)}} \alpha_i X_i^{(k-1)} \quad (6)$$

where α is some appropriately chosen weighting vector. Each generation will then have its own radio map $X^{(k)}$. The weighting scheme used by this work is discussed later in Section III.

This method may then be continued iteratively some number of times until the singleton group is reached, although exhaustive iteration may not be necessary.

C. Generational Structure

We propose two techniques for efficient structural organization within the fingerprint lineage. First, two and three generation approaches are taken with uniform structure. Second, a graph theoretic approach is taken to find more natural groupings of tiles. As previously mentioned, non-overlapping methods are studied such that $\xi_j^{(k)} \cap \xi_i^{(k)} = \emptyset, \forall i \neq j$.

1) *Uniform Grouping*: In this method tiles are organized in a regular way such that each generation contains members that have uniform partition. This partitioning scheme is consistent throughout all generations. We present this method in both a two and three generation format. It follows that if the generational ratios (i.e., $\xi^{(k)}/\xi^{(k+1)}$) are chosen appropriately a significant reduction in search space can be achieved. An example formulation of the uniform grouping scheme is presented in Figure 1.

2) *Graph Theoretic Grouping*: It is not hard to see that the first technique of grouping may be contrived and that more natural groupings may be realized via other methods. To this end the efficacy of graph spectral partitioning is presented as an acceptable surrogate for uniform grouping [13].

We introduce a unique application of spectral partitioning in that we apply the partitioning to the distribution of BSs and only then use that grouping to determine the correct grouping of fingerprints. This can be seen as a kind of dimensionality reduction from $\mathbb{R}^{N_{bs}} \Rightarrow \mathbb{R}^2$. This approach is intuitively satisfying if we view the database construction in the reverse as a non-linear, and sometimes non-invertible, mapping from $\mathbb{R}^2 \Rightarrow \mathbb{R}^{N_{bs}}$. Rather than attempt to perform the grouping in the hyperdimensional feature space (i.e., fingerprint space) it makes more sense to remain in the cartesian coordinate space in which our ultimate answer lies. Once we find an appropriate grouping of BSs we can then map the correlated fingerprints accordingly.

To begin the formulation of the spectral partitioning method [13], the adjacency matrix is defined as

$$A \triangleq \{a_{i,j}\} \quad (7)$$

with each element

$$a_{i,j} = \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \quad (8)$$

where each (x_i, y_i) coordinate pair is the location of BS i . We note that in this formulation each BS represents a graph node and the graph is maximally connected (i.e., mesh network architecture). Additionally, each graph edge is weighted

Fig. 2. The environmental homogeneity is presented relative to the normalized shadowing effect, both of which are controlled by the simulation parameter threshold, τ . Note that the noise contribution of the shadowing effect is the same at $\tau = 0.2$ and $\tau = 0.8$; however, the environmental homogeneity is much higher at $\tau = 0.8$.

proportional to the inverse of its euclidean distance. From, A , we can determine the degree of the j^{th} BS, defined as

$$d_{i,j} = \begin{cases} \|A_j\|_1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

The degree of each BS attempts to quantify the connectedness, or in this case, the closeness of each BS. The laplacian matrix, L , is then defined as

$$L \triangleq \{l_{i,j}\} \quad (10)$$

where

$$l_{i,j} = (d_{i,j} + a_{i,j})\delta_{i,j} - a_{i,j} \quad (11)$$

and $\delta_{i,j}$ is the Kronecker delta function.

The laplacian matrix is then factored per the standard eigen-decomposition

$$L = U\Lambda U^T \quad (12)$$

where the column-space of U is spanned by the eigenvectors of L and their associated eigenvalues are situated along the diagonal of Λ such that $\lambda_1 < \lambda_2 < \dots < \lambda_{N_{bs}}$.

The second largest eigenvector, U_2 , is also known as the Fiedler vector and its associated eigenvalue, λ_2 is used to measure a quantity called the *algebraic connectivity* of a graph [14]. Of interest to our work is using the Fiedler vector for group partitioning. Specifically, the sign of each entry $U_{i,2}$ can be used to partition a large group into two groups by minimizing what is known as the *cut-size*. For an in depth treatment on this technique the interested reader is directed to the excellent summary given in [13]. This process can be repeated iteratively on subgroups yielding a final partition of some arbitrary number of groups.

Once each BS is assigned to the j^{th} group this grouping is then used to map the associated fingerprints into appropriate

Fig. 3. The uniform community structure is presented for the 2nd generation. Each of the 16 groups is represented by a different number/color combination.

groups via a group correlation vector $\phi^{(j)}$ defined as

$$\phi_i^{(j)} = \begin{cases} 1 & \text{if BS } i \text{ is in group } j \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

The cross-correlation of each fingerprint with each $\phi^{(j)}$ is then computed and each fingerprint is assigned to the group, $\xi_j^{(k)}$, with which it is most highly correlated.

D. k-Generation Position Estimation

Once the appropriate database structure is created the position estimate can be made from a given observation. Specifically, within each generation, we seek to solve the problem

$$\begin{aligned} \min \quad & f_k(\tilde{X}^{(k)} \varphi^{(k)}, h) \\ \text{s.t.} \quad & \varphi^{(k)} = e_i \end{aligned} \quad (14)$$

where $\varphi^{(k)}$ is the position vector for generation k , h is the observation vector, e_i is a vector of the trivial orthonormal basis set where all entries are zero except for the i^{th} entry, $\tilde{X}^{(k)} \subseteq X^{(k)}$, and $f_k(\cdot)$ is an appropriate penalty function. We choose $f_k(\cdot) = |\langle \cdot, \cdot \rangle|$ the normalized dot product, as an appropriate generation function when $k > 1$. In this way, the correlation between matches is maximized and we leverage the orthogonal nature of fingerprints for $k > 1$. We choose $f_1(\cdot) = \|\cdot\|_2$ for the penalty function at the original generation as is typical in minimizing error in position. Additionally, this choice of penalty is suitable at lower levels where data is highly correlated. It follows then that a global minimum can be found for some e_i through enumeration of the possible solution space. The position estimate for generation k , $\hat{p}^{(k)}$, is then found by

$$\hat{p}^{(k)} = i. \quad (15)$$

When obtaining the solution for (14) for the largest k , all fingerprints at generation k must be used (i.e., $\tilde{X}^{(k)} = X^{(k)}$). From that point forward only the subset of fingerprints at generation $k - 1$ corresponding to the matching fingerprint as

Fig. 4. A typical grouping realized by clustering the BSs with graph spectral partitioning is presented. Each symbol represents the physical location of a BS. Each of the 15 groups is represented by a different symbol/color combination. Note that 15 groups, as opposed to 16, was realized due to the creation of a singleton group before the terminal iteration.

Fig. 5. A typical grouping of fingerprints realized by clustering the BSs with graph spectral partitioning is presented. Fingerprints are clustered in each group based on the level of influence that that group's associated BSs have on each tile. Each symbol represents the group to which a particular tile belongs (cf. Figure 4).

in (5) at generation k need be enumerated in (14). In this way the savings in computational cost can be maximized while maintaining accuracy.

III. NUMERICAL RESULTS

In this section we present results obtained by monte-carlo simulation. Our simulation environment is modeled after an actual urban WiMAX deployment covering 144 square kilometers. The tracking area is then discretized as in Section II into $N_t = 144$ one square kilometer tiles. Throughout the tracking area are placed $N_{bs} = 46$ BSs positioned as they were in the actual WiMAX deployment. The channel was modeled with noise from shadowing such that the fingerprints would be non-

deterministic and the database constructed as in Section II. The target assumes every position within the tracking area during each simulation. Observations were taken independently of database construction and 100 independent simulations were taken for each case.

The results are more illuminating when held in contrast to the operating environment which is heavily influenced by the simulation parameter termed threshold, τ . This parameter is used to adjust the shadowing effect mentioned above and shown in Figure 2. It also has the secondary effect of changing the global similarity of the environment. This effect is quantified in the *homogeneity* of the environment, or how similar the constituent fingerprints are to each other. Formally, we define the homogeneous set Ω , which contains all fingerprint satisfying

$$d(X_j, X_i) < \epsilon, i \neq j \quad (16)$$

where $d(\cdot)$ is a measure of similarity between two vectors and ϵ is some small constant. We use the ℓ_1 -norm as an appropriate measure of similarity. The homogeneity of the environment is then defined as the cardinality of the set $|\Omega|$. A more in depth treatment of the threshold parameter and its effect can be found in [12]. We will later parameterize the results with τ in order to demonstrate the effect of homogeneity, independent of the shadowing effect, on the proposed structures.

A. Realized Group Structures

The first group structure is a uniform 16×16 construct shown in Figure 3. Depicted is the 2nd generation grouping, where each generation is uniform. A 3rd generation is built upon this one as in Figure 1 such that each of the 3rd generation tiles each contain four 2nd generation tiles and are also uniformly partitioned.

The second structure evaluated is that resulting from groups derived by spectral partitioning. The original BS grouping is shown in Figure 4. Here each symbol represents the physical location of a BS and each group is represented by a different symbol/color combination. Only 15 groups were realized during the iterative partitioning as one of the groups reached singleton status before the desired number of groups was realized. The partitioning was carried out until a similar number of groups was realized as in the uniform 2nd generation so as to minimize the fundamental differences in structure. Each of the N_t fingerprints was then grouped as per Section II and the resulting organization is shown in Figure 5. Here, one can see a group structure of tiles that is aligned with the physical BS deployment. We contend this is a more natural choice of grouping from both an aesthetic and mathematical perspective.

The weighting vector used in (6) for both cases to develop the generational fingerprint databases, $X^{(k)}$, is the all ones vector scaled by the inverse of the group size (i.e., $\alpha_j = 1/|X_j^{(k-1)}|$). This has the effect of creating a representative fingerprint based on the average of the constituents. We choose this method for its relative simplicity, although more complex, and thus more accurate, means of determining the weighting vector can be conceived.

Fig. 6. Here, the positioning performance of the k -generation structure is compared with the single generation case for $\tau = 0.2$. The divergence in performance is noted at CEP 88%.

B. Complexity Reduction

Next, we turn our attention to the implications of these constructs for model complexity. First, we evaluate the uniform grouping with two generations. Enumeration of a single-generation structure requires N_t operations. With the proposed structure, the number of operations can be reduced to 25, a small fraction of the total number of operations. Additionally, if we include the 3rd generation we can further reduce the number or required operations to 17. While technically all three of the discussed approaches both scale $\mathcal{O}(N_t)$ the scaling constant in the generational approach affords large savings in computational costs. These ideas translate easily to non-uniform geometries such as that realized by spectral partitioning. The number of computations for non-uniform geometries can be computed by

$$\mathcal{N} = N_T + \mathcal{E}\{n\} \quad (17)$$

where $\mathcal{N}$ is the total number of operations, N_T represents the number of spectral partitions, and $\mathcal{E}\{n\}$ is the expected number of group constituent elements. If we assume uniform probability for the target location then, in the scenario presented, the total number of expected operations is ≈ 26 . Note that any configuration that is not uniform will yield an expected number of operations that is necessarily higher than that realized in the uniform case. We will later show how this slight capitulation of simplicity will return improvements in positioning accuracy.

C. Position Estimation

First, we examine the case of a uniform multiple generation structure. The accuracy of the schemes are presented in Figure 6 for a relatively heterogeneous environment ($\tau = 0.2$). The performance for only this value of threshold is shown as the same trends in performance are noted for varying levels of homogeneity and shadowing when a uniform structure is

employed. The first striking observation is that the scheme performs as well for both the three and two generation case.¹ The gain in computational simplicity achieved in the three generation case comes at no cost relative to the two generation case. Second, we note that all three schemes achieve the same circular error probable (CEP) up to 88%. Despite the slight decrease in performance above this range, the schemes perform roughly equivalently, making a strong case for the k -generation structure. This decrease in performance can be explained as a result of incorrect generational matching, meaning that at some generation where $k > 1$ the scheme incorrectly matched a fingerprint. As the schemes are nearly the same in the two and three generation case the mistake can likely be localized to $k = 2$.

With this in mind, we turn to the organization scheme and compare spectral partitioning and uniform structures to find a more optimal method of grouping fingerprints. The performance of these two structures relative to the single generation case is presented in Figure 7. Here, the schemes are compared in a relatively heterogeneous environment ($\tau = 0.2$). Because the environment is very diverse we do not see much gain in accuracy by using graph theory as a vehicle for grouping. Additionally, we note the same slight expected decrease in performance from the single generation approach. However, a slight increase in performance of the spectral partitioning method is noted when the error is very large, suggesting that spectral partitioning may be able to mitigate mistakes at the $k > 1$ generation by creating more natural associations.

Next, we evaluate spectral partitioning and the uniform structure against the single-generation structure in a more homogeneous environment ($\tau = 0.8$). The resulting performance is presented in Figure 8. Overall, more errors are noted across all the presented techniques as is indicative of homogeneous environments which are hostile to fingerprinting efforts. Similar performance is noted up to CEP 62%, indicating agreement between schemes at the $k = 1$ generation. As before, the divergence at CEP 62% can be attributed to unavoidable errors made at the $k > 1$ generational level. Second, the same slightly more poor accuracy is noted in the proposed schemes relative to the single-generation case for low CEP. Finally, and most striking, the notable improvement achievable by spectral partitioning above CEP 80% is observed. Above that, spectral partitioning begins to outperform even the single-generation case. This is a magnification of the effect previously noted in more heterogeneous environments ($\tau = 0.2$). Spectral partitioning can then be seen as a method of creating communal associations in the fingerprint database that are more robust against homogeneous environments than the uniform structure. This makes intuitive sense since the spectral clusters are noted to be more natural than the arbitrary boundaries drawn in the uniform case.

¹Note that we qualify "good" performance relative to the maximum accuracy possible given X . For our simulation each tile is one square kilometer. Thus, the best possible accuracy is in the vicinity of 500 meters. For increased accuracy the radio map is simply made to have more granularity.

Fig. 7. Here, the positioning performance of spectral partitioning structure and uniform partitioning is compared to the single-generation case for $\tau = 0.2$. Divergence in performance is noted at CEP 86%.

Fig. 8. Here, the positioning performance of the spectral partitioning structure and the uniform grouping structure is compared to the single generation case in a more homogeneous environment ($\tau = 0.8$). Divergence in performance from the single-generation case is noted at CEP 62%. Improvement in accuracy is noted for spectral partitioning over uniform partitioning above CEP 80%.

IV. CONCLUSION

We have presented a novel method for reducing the complexity in fingerprinting methods. First, we introduced the k -generation structure and showed how computational gains can be made by forming an appropriately layered search structure. The performance of both a two and three generation structure was shown to be congruent with each other and remarkably close to the single-generation case in simulation. Second, the group geometry was evaluated and graph spectral partitioning was presented as a possible improvement upon the previously introduced uniform partitioning. Graph spectral partitioning was shown to be a natural fit, both mathematically and aesthetically. This method was validated as superior to the uniform structure and the single-generation case for high CEP

in simulation.

The main contribution of this work is to provide a foundation from which to generalize the fingerprinting problem. The proposed scheme is the start of such an endeavor and shows that excellent performance can be achieved at a computationally tractable price for any size environment and any granularity of database. Indeed, the simulation results presented have shown that performance similar to an environment-specific solution can not only be matched, but in some circumstances outperformed.

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