Placement of Distributed Energy Storage via Multidimensional Scaling and Clustering

Arthur K. Barnes

Department of Electrical Engineering
University of Arkansas
Fayetteville, AR USA
artbarnes@ieee.org

Juan Carlos Balda

Department of Electrical Engineering
University of Arkansas
Fayetteville, AR USA
jbalda@uark.edu

Abstract—Energy storage has long been proposed at the distribution level, where it can provide additional benefits via ancillary services. This work studies how to place energy storage units (ESU) on a distribution feeder in the most cost-effective manner while still meeting voltage regulation requirements. The feeder also has photovoltaic (PV) generation, and the PV ability to supply reactive power is considered. The placement of the ESU is performed via a fast heuristic, in which multidimensional scaling (MDS) is used to transform the combinatorial placement problem into a continuous-valued problem by mapping buses to points in a space. In the new space, clustering algorithms can be applied to determine the ESU locations from a set of candidate locations. The method reduces computation time by an order of magnitude, allowing for various distribution feeder configurations to be quickly compared.

Keywords—Energy storage, photovoltaic systems, renewable energy sources, optimization, clustering methods

I. INTRODUCTION

This paper focuses on ESU used principally for providing frequency regulation, but also for supplying reserve power during periods of high demand. The case addressed is a distribution system operator (DSO) that has made the purchase of a fixed amount of energy storage in terms of MWh capacity and “smart” PV inverters. The DSO seeks to determine the best solution in terms of numbers of ESU and “smart” PV units, their power ratings and placements on the system.

The “smart” PV inverter is a concept proposed by EPRI [1] that has the ability to draw or inject reactive power based on a power vs. voltage curve, allowing it to assist with voltage regulation. Because of the “smart” PV functions, it is desirable to integrate them with the ESU at the same time. When determining placement, the scenario of ESU and “smart” PV providing reserve power during peak load conditions is examined. The total load on the distribution feeder as viewed from the distribution substation can be lowered by appropriately placing the ESU and PV to flatten the voltage profile on the feeder, thereby allowing for conservation voltage reduction (CVR) to be applied [2]. CVR refers to lowering the overall voltage on the system, (usually by the substation transformer tap settings), in order to reduce overall load demand by taking advantage of the increasing power vs.

voltage curve of the system load. However, dispersing the ESU on a distribution feeder as opposed to placing them at the substation will impact their ability to provide frequency regulation services.

This placement problem is closely related to that of placing distributed generation and shunt capacitors. The existing methods for placing these devices are categorized as: analytical, numerical optimization, heuristic search, and randomized search. The most common approach is genetic algorithms (GA), a form of randomized search which comes in both pure and hybrid optimal power flow (OPF) forms [3]. In the latter, the GA only selects the ESU locations, while an inner OPF selects their required power ratings, reducing the size of the GA search space and improving convergence time.

The placement method proposed in this work is a heuristic search that takes advantage of the observation that the number of ESU to be placed is small when compared to the total number of buses in the distribution feeder. The placement problem is then converted from a combinatorial problem to a continuous-valued problem by transforming the ESU bus locations into points in a continuous space with MDS. The ESU positions are then calculated with a clustering algorithm in the new space. By contrast, it is assumed that the power ratings of the “smart” PV installations are low as is the case with rooftop installations, and many of them can be placed on the feeder.

Although hierarchical clustering algorithms exist that can operate over a pairwise distance matrix, they suffer from sensitivity to initial conditions. The MDS algorithm allows classical k-means type algorithms to be applied, which are much less sensitive [4]. Additionally, it comes at a low cost in terms of both computational and programming burden, as it can be implemented in two lines in a high-level language with a set of matrix multiplications and eigenvector decomposition.

Clustering for placement reduces computational complexity over both pure and hybrid GA approaches, because it only runs a computationally intensive OPF twice, as opposed to the 10s to 100s of OPF necessary for GA [5]. This enables a distribution feeder designer to quickly evaluate several different ESU/“smart” PV configurations in a hypothetical software application by clicking a toolbar button.

The remaining sections of this paper describe how the method operates, illustrate its application to place ESU on a distribution feeder, evaluate the benefits of the ESU, and compare its performance against an existing GA approach.

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II. CALCULATING CANDIDATE LOCATIONS AND POWER INJECTIONS USING AN OPF

The ESU inject power to increase frequency towards 60 Hz (referred to here as up-regulating) or supply power during critical peaks. They also draw power to help reduce frequency towards 60 Hz (referred to here as down-regulating) and charge during off-peak times to account for losses. Thus, while it is desirable to place ESU where they make a large impact on the voltage regulation and power drawn by the feeder during critical peak load reduction and up-regulation, this placement is a hindrance when the ESU are to down-regulate or charge.

The problem objective is to minimize the cost of power delivered from the substation bus during periods of high demand, expressed as

$$\min. \sum_{t=1}^{N_t} C_t P_{1t}, \quad (1)$$

where t is the index of the current load/ESU/PV scenario, N_t is the number of scenarios, P_{1t} is the power supplied by the substation bus (bus 1) during the current scenario, and C_t is the relative cost of electricity supplied during the current scenario.

The problem is subject to several constraints, which are discussed in the remainder of this section. It is assumed that the ratio of the total power rating to the total energy capacity of the ESU is fixed at approximately 1:1 [6]. This means that the total energy capacity in MWh of energy storage purchased is equal to the total power rating of all ESU in MVA. Thus,

$$\sum_{n=1}^N S_n^{esu} \leq S_{tot}^{esu}, \quad (2)$$

where N is the number of buses, S_n^{esu} is the rated power of the ESU at bus n , and S_{tot}^{esu} is the total rated power of all the ESU in pu. For an optimal solution, the two terms in (2) will be equal. Similarly,

$$\sum_{n=1}^N S_n^{pv} \leq S_{tot}^{pv}, \quad (3)$$

where S_n^{pv} is the rated power of the PV inverter at bus n and S_{tot}^{pv} is the total rated power of all the PV inverters in MVA.

The apparent power scaling is a vector whose length is the number of scenarios included in the problem, where each element K_t^{esu} represents the fraction of rated power that each ESU operates at during the corresponding scenario. An equivalent element K_t^{pv} applies for the PV. The power scalings for ESU and PV are not included as decision variables because these quantities are typically determined by a scheduler independent of network constraints; this assumption represents current practice [7]. Similarly, power factors for ESU and PV are also not included.

Rather than having these quantities as decision variables, they are used as inputs to the problem. The power factors are not represented directly, but instead as the angle between voltage and current. For the ESU and PV, these are represented as ϕ_{nt}^{esu} and ϕ_{nt}^{pv} , respectively. The real and reactive powers for the ESU and PV are given by

$$P_{nt}^{esu} = K_t^{esu} S_n^{esu} \cos \phi_{nt}^{esu} \quad (4)$$

$$Q_{nt}^{esu} = K_t^{esu} S_n^{esu} \sin \phi_{nt}^{esu} \quad (5)$$

$$P_{nt}^{pv} = K_t^{pv} S_n^{pv} \cos \phi_{nt}^{pv} \quad (6)$$

$$Q_{nt}^{pv} = K_t^{pv} S_n^{pv} \sin \phi_{nt}^{pv}. \quad (7)$$

The cost of installing the ESU is

$$c = \sum_{n=1}^N (c' S_n^{esu} + c_0 u_n), \quad (8)$$

where c is the total cost of placing the ESU in U.S. dollars, c' is the incremental cost of placing an ESU in dollars per pu, c_0 is the fixed installation cost of placing an ESU, and u_n is a binary variable indicating if an ESU is placed at bus n .

These quantities are not used again, except to highlight that the cost of installing each ESU at bus n consists of a fixed installation cost (c_0) plus a component that depends on the capacity of the ESU (c'). The effect of this piecewise linear cost function is that it is more cost-effective to have a smaller number of larger ESU, as less fixed installation cost is incurred. In this study, the PV does not have any such fixed cost, and there is no incentive to aggregate PV installations.

Including a nonlinear cost makes the problem considerably harder, since it now has a mixed-integer nonlinear formulation [3]. In order to circumvent this issue, the OPF formulation is only used to select candidate buses for ESU placement. The placement is solved as a separate problem, using the candidate buses and the necessary power injections as inputs to a clustering algorithm which determines the actual ESU power ratings and locations. The clustering algorithm requires that the number of ESU be selected beforehand. However, as this number is small, it is reasonable to determine its optimal value by an exhaustive iteration over a fixed range from 0 to a maximum number of ESU N_{max}^{esu} .

Using a static load model, which is discussed later in more detail, the power injections into each bus are represented as

$$P_{nt} = P_{nt}^{esu} + P_{nt}^{pv} - P_{nt}^d \quad (9)$$

$$Q_{nt} = Q_{nt}^{esu} + Q_{nt}^{pv} - Q_{nt}^d \quad (10)$$

$$P_{nt}^d = K_t^d P_n^{d0} |V_{nt}|^{\beta_p} \quad (11)$$

$$Q_{nt}^d = K_t^d Q_n^{d0} |V_{nt}|^{\beta_q}. \quad (12)$$

In the above, P_{nt} and Q_{nt} are the real and reactive powers supplied by bus n during scenario t . P_{nt}^d and Q_{nt}^d are the load real and reactive powers for bus n . P_n^{d0} and Q_n^{d0} are the rated load real and reactive power demands for bus n . K_t^d is the feeder loading during the current scenario t . V_{nt} is the voltage at bus n during scenario t . The exponents β_p and β_q represent the relationship between voltage and load power, discussed in more detail later. The power flow constraints are represented as

$$P_{nt} = (P_{nt}^{esu} + P_{nt}^{pv} - P_{nt}^d) - \sum_{m=1}^N |V_{nt}| |V_{mt}| y_{mn} \cos(\theta_{mn} - \delta_{mt} - \delta_{nt}) \quad (13)$$

$$Q_{nt} = (P_{nt}^{esu} + P_{nt}^{pv} - P_{nt}^d) - \sum_{m=1}^N |V_{nt}| |V_{mt}| y_{mn} \sin(\theta_{mn} - \delta_{mt} - \delta_{nt}) \quad (14)$$

$$S_n^{esu} \leq S_{max}^{esu} \quad (15)$$

$$V_{min} \leq |V_{nt}| \leq V_{max} \quad (16)$$

$$V_{max} = 1.05V_{base} \quad (17)$$

$$V_{min} = 0.95V_{base}. \quad (18)$$

In the above, y_{mn} is the magnitude of complex impedance matrix $\mathbf{Y}$, while θ_{mn} is the phase of the impedance between buses m and n . The variable δ_{nt} is the phase of the voltage at bus n during scenario t . V_{min} and V_{max} are the minimum and maximum allowable voltage magnitudes on the feeder respectively, while V_{base} is the base voltage.

For this OPF, this exponential load model is used

$$P_{nt}^d = K_t^d P_n^{d0} |V_{nt}|^{\beta_p}$$

$$Q_{nt}^d = K_t^d Q_n^{d0} |V_{nt}|^{\beta_q}.$$

In the above, P_{nt}^d and Q_{nt}^d are the load real and reactive power demands. The term $|V_{nt}|$ is the voltage magnitude in pu seen by the load. The terms $\beta_p = 1.38$ and $\beta_q = 3.32$ [8]. Again, P_{nt}^{d0} and Q_{nt}^{d0} are the rated real and reactive power demands of the load at a voltage of 1 pu, while K_t^d is the feeder loading during the current scenario t . In order to calculate the required amount of power injection at each candidate bus, a nonlinear constrained optimizer is used on the problem, in this case `fmincon()` from the MATLAB™ optimization toolbox.

III. SELECTION OF THE ESU LOCATIONS

In this particular problem, the key idea is that it is not desirable to place ESU at buses that are very “close” to one another, which in this context means that the impedance between them is low. To determine how to best combine the candidate locations into a smaller number of well-spaced ESU, the candidate locations are mapped into a continuous space that is easier to work with than the original discrete locations. To accomplish this, dimensionality scaling methods are employed [9], [10]. However, even with the candidate locations mapped into a continuous space, the problem is still combinatorial in nature. Fortunately, the new problem space allows for the use of clustering methods, which represent a “good” suboptimal solution to the problem [4].

A. Multidimensional Scaling

The candidate buses are mapped to points in a q -dimensional space using MDS, a dimensionality reduction technique that is commonly used in the social sciences. It works by first calculating pairwise distances between each object (in this case, a bus) in a set (in this case, the M ESU candidate locations). The objects are then mapped to arbitrarily placed points in the q -dimensional space. The classical MDS (CMDS) algorithm is employed. It maps the objects using their pairwise distances onto a q -dimensional space. It does this by applying a pair of transformation matrices in order to convert the pairwise distance matrix $\mathbf{D}$ into a positive semidefinite matrix $\mathbf{B}$ of rank q . The matrix $\mathbf{B}$ is then used to produce matrix $\mathbf{W}$, whose columns are the position vectors $\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_M$.

The method is illustrated as follows:

First, the distance matrix $\mathbf{D}$ is transformed

$$\mathbf{B} = (\mathbf{I}_M - M^{-1}\mathbf{1}_M\mathbf{1}_M^T)\mathbf{A}(\mathbf{I}_M - M^{-1}\mathbf{1}_M\mathbf{1}_M^T). \quad (19)$$

In the above expression,

$$a_{nm} = \frac{1}{2}d_{nm} \quad (20)$$

$$\mathbf{1}_M = [1 \ 1 \ \dots \ 1]^T, \quad (21)$$

where a_{nm} and d_{nm} denote the elements of in the n^{th} row and m^{th} column of the matrices $\mathbf{A}$ and $\mathbf{D}$, respectively. The term $\mathbf{I}_M$ is a $M \times M$ identity matrix. The q eigenvectors $\mathbf{v}_n$ corresponding to the nonzero eigenvalues λ_n of $\mathbf{B}$ are selected, i.e. those corresponding to

$$\mathbf{B}\mathbf{v}_n = \lambda_n\mathbf{v}_n, \quad \lambda_n \neq 0. \quad (22)$$

These are concatenated into a new matrix

$$\mathbf{W} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_q]^T. \quad (23)$$

Each column $\mathbf{w}_n$ of $\mathbf{W}$ represents a point in the new space.

The essence of the method is that if each element of $\mathbf{A}$, a_{nm} is proportional to a distance metric d_{nm} calculated between objects n and m , then the Euclidean distance between columns $\mathbf{w}_n$ and $\mathbf{w}_m$ of $\mathbf{W}$ is also d_{nm} . Thus, by the definition of an Euclidean distance,

$$d_{nm} = \|\mathbf{w}_n - \mathbf{w}_m\|_2. \quad (24)$$

It is important to realize however, that the distances, d_{nm} , do not necessarily correspond to the distances between two points in an original space, as is the case with both similarity scores on questionnaires and the impedance between two buses in a distribution feeder.

B. Clustering Algorithm

The goal of the clustering algorithm is to group together buses that are candidate locations. To perform the clustering, a number of clusters K (corresponding to the number of ESU to be placed) is selected as an input to the clustering algorithm. The requirements of the clustering algorithm are to assign each bus to a particular ESU and to select the bus that the units will be placed at. Hence, the objective of the clustering algorithm is to minimize the intra-cluster variance

$$J = \sum_{n \in \mathcal{D}_k} \|\mathbf{w}_n - \boldsymbol{\mu}_k\|^2 \quad (25)$$

where $\mathbf{w}_n$ is a point in cluster k , $\boldsymbol{\mu}_k$ is the centroid (or center of mass) of cluster k , and $\mathcal{D}_k$ is the set of points belonging to cluster k . A last issue is that the amount of ESU power injection at each bus is not identical. This could result in erroneous results if there is a bus with only a small amount of power injection and another bus with a large amount. To account for this, the k-means algorithm is used [4], but the centroid calculation is modified to use the amount of power injection at each bus as a weight. In the original k-means algorithm, the centroids are calculated as

$$\boldsymbol{\mu}_k = \frac{1}{N_k} \sum_{n \in \mathcal{D}_k} \mathbf{w}_n, \quad (26)$$

where N_k is the number of points in cluster k . By contrast, in the modified algorithm, the centroids are calculated as

$$\boldsymbol{\mu}_k = \sum_{n \in \mathcal{D}_k} \gamma_n \mathbf{w}_n, \quad (27)$$

where the weighting γ_n is equal to

$$\gamma_i = \frac{S_n^{esu}}{\sum_{n \in \mathcal{D}_k} S_n^{esu}}. \quad (28)$$

To perform placement, the ESU are allocated to the bus nearest to their corresponding centroid. The power injections of each ESU candidate are then assigned to their corresponding ESU, as illustrated in Fig. 1.

IV. CASE STUDY AND NUMERICAL RESULTS

The proposed method is tested on the radial Italian rural distribution feeder given in [11] and illustrated in Fig. 2. There are two substations at buses 1 and 2, the load profile is shown in Fig. 3, and the parameters are listed in Tables I and II.

The total power rating of all the ESU to be placed is set at $1/10^{\text{th}}$ the total feeder real power demand, while the total power rating of the “smart” PV to be placed is $1/40^{\text{th}}$ the total feeder real power demand. Based on the results in [11], the number of ESU to be placed is 2. This is presented in Table III, along with other relevant inputs to the placement problem.

Five different scenarios were analyzed in placement. In addition to the base case without ESU or PV, a peak evening condition was selected for the objective. The ESU and PV are placed to minimize power demand from the feeder under this condition. As an evening case is used, the PV injects reactive power only. In addition, three other cases are included, which verify if the ESU is able to provide frequency regulation without causing flicker issues, as well as its ability to charge and discharge without violating voltage constraints. These scenarios are presented in Tables IV and V.

Fig. 2. Configuration of the feeder case study.

Fig. 3. Feeder average load and PV profile over one day.

Fig. 1. Flowchart for the CMDS/clustering algorithm.

TABLE I. BRANCH CONNECTIVITY AND IMPEDANCES

From	To	R	X
1	10	0.00169	0.00073
10	14	0.00676	0.00291
14	15	0.00024	0.00010
15	16	0.00507	0.00218
15	17	0.00566	0.00268
10	9	0.00338	0.00146
9	8	0.00217	0.00094
8	7	0.00048	0.00021
7	6	0.00435	0.00187
6	5	0.00483	0.00208
5	4	0.00580	0.00250
4	3	0.00386	0.00166
3	2	0.00531	0.00229
6	11	0.00435	0.00187
11	12	0.00435	0.00187
12	13	0.00966	0.00416

TABLE II. LOAD POWER AT EACH BUS

Bus Number	Power (kW)	Power Factor
1	0.00	0.00
2	0.00	0.00
3	90.00	0.90
4	100.00	0.90
5	320.00	0.90
6	90.00	0.90
7	83.67	0.90
8	420.00	0.90
9	60.00	0.90
10	90.00	0.90
11	0.00	0.00
12	600.00	0.88
13	50.00	0.71
14	550.00	0.71
15	500.00	0.90
16	400.00	0.90
17	0.00	0.00

TABLE III. CASE STUDY PARAMETERS

Parameter	Expression	Value
Base Voltage	V_{base}	20 kV
Base Power	S_{base}	2 MW
Substation 2 power	P_{2t}, Q_{2t}	$\frac{K_t^d}{6} \sum_{n=1}^N P_n^{do}, \frac{1}{6} \sum_{n=1}^N Q_n^{do}$
Number of ESU	K	2
Total ESU Power	S_{tot}^{esu}	$\frac{1}{10} \sum_{n=1}^N P_n^{do}$
Total PV Power	S_{tot}^{pv}	$\frac{1}{40} \sum_{n=1}^N P_n^{do}$
Minimum Voltage	V_{min}	0.95 pu
Maximum Voltage	V_{min}	1.05 pu
Cost	C_t	1, $t = 1$ 0.1, otherwise

TABLE IV. DESCRIPTION OF EVALUATED SCENARIOS

Scenario	Load State	PV State	ESU State
0 (base)	Peak	Off	Off
1 (objective)	Peak	Inject Q	Discharge
2 (up-regulating)	Peak	Off	Discharge
3 (down-regulating)	Peak	Off	Charge
4 (up-regulating)	Off-Peak	Off	Discharge

TABLE V. LOAD, ESU AND PV PARAMETERS FOR EACH SCENARIO

Scenario	K_t^d	K_t^{esu}	ϕ_t^{esu}	K_t^{pv}	ϕ_t^{pv}
0 (base)	1.0	0.0	0	0	0.00
1 (objective)	1.0	1.0	0	1	$\pi/2$
2 (up-regulating)	1.0	1.0	0	0	0.00
3 (down-regulating)	1.0	1.0	π	0	0.00
4 (up-regulating)	0.454	1.0	0	0	0.00

Although the *average* feeder load does not reach 1.0 pu, a 1.0 pu peak load is selected as a worst case. To evaluate whether flicker is an issue, the maximum change in voltage

$$\Delta V = \max_n (|V_{n2}| - |V_{n3}|) \quad (29)$$

at any bus is measured between scenarios 2 (up-regulating) and 3 (down-regulating). Assuming a 4-s period in the regulation signal, any change lower than 1% is considered tolerable according to the worst case for the IEC-868 flicker curve [12], [13]. To evaluate if voltage regulation problems are caused during charging and discharging, scenarios 3 (peak load, up-regulating) and 4 (off-peak load, down-regulating) are studied.

To evaluate the performance improvement against competing methods, the placement problem is also performed with genetic algorithms, following the approach of the original analysis [11]. To more fairly compare against the proposed method, the chromosome encoding is changed so that the number of ESU to be placed is fixed, as illustrated in Fig. 4. Rather than encoding the ESU placement as a binary vector whose number of elements is the number of buses N , it is now encoded as an integer vector whose number of elements is the number of ESU to be placed K . Additionally, the parameters listed in Table VI are applied.

Tables VII-IX present placement results. The genetic algorithm was able to find the optimal placement after the first generation because of the small search space. However, the clustering-based placement is able to complete in under 2 s, over an order of magnitude less than a single iteration of the genetic algorithm. The results for the different scenarios indicate that the optimal placement of the ESU for reducing peak power does not cause voltage violations under other conditions. However, the ability of the ESU to provide regulation services is impeded, as the worst-case voltage variation will be noticeable.

TABLE VI. GENETIC ALGORITHM PARAMETERS

Parameter	Value
Fitness function	rank
Selection function	remainder selection
Population replacement	generational
Elite individuals	$\frac{1}{20} \min[\max(10N, 40), 100]$ (2-5 depending on sample size)
Mutation function	uniform
Mutation rate	0.01
Crossover type	scattered
Crossover fraction	0.8
Population size	$\max(N, 30)$
Convergence iterations	100
Maximum iterations	200

1	2	K	
1, ..., N	1, ..., N	...	1, ..., N

Fig. 4. Chromosome encoding for the genetic algorithm.

TABLE VII. CLUSTERING PLACEMENT RESULTS

Parameter	Value
ESU buses	13, 16
ESU power ratings (kW)	127.8, 207.6
PV total power rating (kW)	83.84
Feeder demand without PV and ESU (MW)	3.174
Feeder demand with PV and ESU (MW)	2.898
Reduction in feeder demand (kW)	611.1
Max ΔV between charge and discharge (%)	1.333
Clustering time to complete (s)	2.073

TABLE VIII. GA PLACEMENT RESULTS

Parameter	Value
ESU buses	12, 16
ESU power ratings (kW)	175.3, 160.1
PV total power rating (kW)	83.84
Feeder demand with PV and ESU (MW)	2.985
Reduction in feeder demand (kW)	614.7
Generations to convergence	1
Time to complete	35 s / iteration

TABLE IX. OPF RESULTS FOR CLUSTERING PLACEMENT

Scenario	Feeder Power	Min. Voltage	Max. Voltage
0 (base)	3.1741	0.9500	0.9976
1 (objective)	2.8984	0.9500	0.9928
2 (up-regulating)	2.8027	0.9500	0.9912
3 (down-regulating)	3.5534	0.9500	1.0046
4 (up-regulating)	1.0096	0.9500	0.9639

V. CONCLUSIONS

A placement method for distributed ESU that considers the addition of “smart” PV inverters was proposed. It demonstrated the utility of ESU in reducing peak load, while quantifying the amount of unwanted flicker introduced by performing

frequency regulation with the placed ESU. The method employed a novel heuristic involving MDS and clustering that reduced computation time compared to the genetic algorithm methods commonly applied to this class of problems. This reduced computational time makes the method appealing for use in interactive software packages such as distribution analysis applications.

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