

# Fuzzy Logic Control for Automatic Impedance Matching

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**Abstract**—In this work an automatic impedance matching for wireless communication systems is proposed. A Takagi-Sugeno (T-S) fuzzy logic controller is used to change adaptively the value of one capacitor in the  $\pi$  matching network. Asymptotic stability is established via a common Lyapunov function for all the subsystems of the T-S fuzzy system. Results obtained using Verilog-A and a Least Mean Square (LMS) impedance matching approach are presented.

## I. INTRODUCTION

One of the most important issues on communication systems is signal integrity. As the operation speed of the communication systems increases some effects like reflection and crosstalk, which may produce important signal losses, become crucial [1,2]. Reflection is due to impedance mismatch between the source impedance and the load impedance. In this case, signal reflections traveling through the line are present in either source to load or load to source directions. Formulation of a complete mathematical model for impedance mismatch is a very complex process, since the parameters involved depend on many factors like process variations, length variations of the interconnection lines, temperature, etc. In this sense, knowledge based algorithms represent interesting alternatives which can be explored when looking for solutions to the impedance mismatch problem using adaptive schemes. Several approaches oriented to solve the problem of impedance mismatch have been reported in recent years. Yichuang et al. [3], presented an evolutionary tuning method for automatic impedance matching in radio communication systems based on genetic algorithms. Hemminger [4] reported an algorithm based on neural networks, aimed to perform real time impedance matching over a wide range of frequencies during transmitter operation, in the driving point impedance of an antenna. Munshi et al. [5] described a scheme for adaptive impedance matching using a model reference adaptive controller. They designed and implemented the scheme using adaptive delta-sigma filters with a LMS approach. Genetic algorithms and neural networks have shown to have a good performance in adaptive matching systems; however they require huge resources, which could make them impractical when implemented on integrated circuits for applications in wireless portable devices. Sjoblom et al. [6] presented an adaptive impedance tuning unit based on switched shunt capacitor banks, however, the heuristics to detect the signal,

compare its strength, and operate switches through all states are not described. Fuzzy Logic, proposed by Zadeh [7], models the uncertainty of human thought and it offers a mathematical formalism, which attempts to emulate the scheme of human deduction. Fuzzy logic formalizes the treatment of vague knowledge and approximates reasoning through inference rules. It establishes the mechanisms to generate practical solutions to problems where traditional methods, which may require precise mathematical models, may not be suitable. Because of this, fuzzy control represents a good alternative to solve the impedance mismatch problem through on-chip adaptive mechanisms.

Stability analysis is one of the most important concepts in control theory. Tanaka and Sugeno [8] proposed a sufficient condition through the existence of a common quadratic Lyapunov function for all the subsystems, i.e. they obtained a common positive definite matrix by solving a Linear Matrix Inequality (LMI) problem. Several approaches have been reported based on the work of Tanaka and Sugeno [8], e.g. in [9] Chin-Tzong and Sy-Ming study the stability issues of the linear T-S free fuzzy systems. The stability of T-S systems with time-varying uncertainties is addressed in [10], where the authors derived a unique solution for the deterministic T-S free fuzzy system to be strongly stable. The particular case when the consequents are singletons is reported by Sugeno in [11], where he also gave stability theorems for discrete time and continuous time systems.

In this work, an adaptive impedance matching circuit based on a zero-order T-S fuzzy controller, coupled to a passive  $\pi$  circuit, is presented. The stability of the system is proved by solving a LMI problem.

## II. PROPOSED IMPEDANCE MATCHING SYSTEM

Fig. 1 depicts a block diagram of the proposed scheme for the impedance matching system. It is composed of the  $\pi$  network, the source impedance  $Z_S$ , and the load impedance  $Z_L$ , both allowing complex values in the general case. The reference model is used to generate the reference signal  $y(t)$ , which is necessary to obtain the error. The error signal is used as the input to the fuzzy controller and is given by

$$e(t) = v(t) - y(t) \quad (1)$$

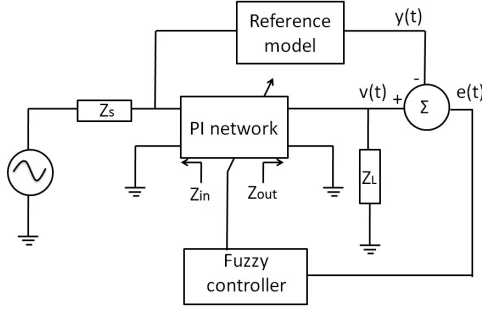


Fig. 1. Impedance matching system

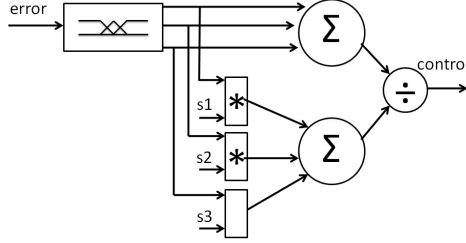


Fig. 2. General structure of the fuzzy controller

where  $e(t)$  is the error,  $v(t)$  is the current output of the system and  $y(t)$  is the desired output. The output of the fuzzy controller is used to adaptively change the value of one of the capacitors of the  $\pi$  network. The system iterates until the impedance matching condition given by (2) is fulfilled, i.e.

$$Z_L = Z_{out}^* \quad (2)$$

where  $(*)$  denotes the complex conjugate. It is clear that the fulfillment of the condition implies:  $e(t) = 0$ .

### III. FUZZY CONTROLLER

In this paper, a zero-order T-S fuzzy controller is used. Fig. 2 shows the basic structure of the proposed fuzzy controller. The system has one input corresponding to the error given by (1) and one output which is further used to adapt the value of one capacitor in the  $\pi$  network. The error signal is fuzzified using three membership functions (S, Z, and Triangular Type), uniformly distributed over the input range with an overlapping degree of two. Fig. 3 shows the form and distribution of the membership functions. The output is obtained using the

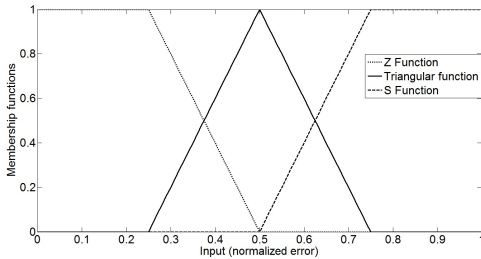


Fig. 3. Membership functions of the input signal in the fuzzy controller

defuzzification method referred to as *center of gravity for singletons* (COGS) [12], which is given by

$$u = \frac{\sum_i \mu(S_i) S_i}{\sum_i \mu(S_i)} \quad (3)$$

In this equation  $S_i$  represents the position of singleton  $i$  in the universe of discourse and  $\mu(S_i)$  represents the firing strength of rule  $i$ . The crisp output value  $u$  corresponds to the abscissa in the center of gravity of the fuzzy set, obtained from the aggregation of the values derived from the input singletons.

### IV. STABILITY

Stability is one of the most important issues in control theory. Tanaka and Sugeno [8] proposed a sufficient condition for stability through the existence of a common Lyapunov function for all the subsystems of the T-S fuzzy system. The rules of T-S fuzzy systems can be expressed as

$$\begin{aligned} \text{IF } i_1(t) \text{ is } M_1 \text{ and } \dots \text{ and } i_p(t) \text{ is } M_p \\ \text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \end{aligned} \quad (4)$$

$$i = 1, 2, \dots, r$$

where  $i_j$  is the  $j^{th}$  premise variable,  $M_j$  is the  $j^{th}$  fuzzy set,  $x(t)$  is the state vector,  $u(t)$  is the input vector,  $A_j$  and  $B_j$  are the system matrices, and  $r$  is the number of rules.

Given the pair  $(x(t), u(t))$ , the final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^r w(i(t)) \{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^r w_i(i(t))} \quad (5)$$

where

$$w_i(i(t)) = \prod_{j=1}^p M_j(i_j(t)) \quad (6)$$

for all  $t$ .

A sufficient stability condition for ensuring stability is given as follows:

**Theorem 1**[8]: The equilibrium of a linear time-invariant fuzzy system is asymptotically stable in the large if there exists a common positive definite matrix  $P$  such that

$$A_i^T P A_i - P < 0, i = 1, 2, \dots, r \quad (7)$$

i.e., a common  $P$  has to exist for all subsystems.

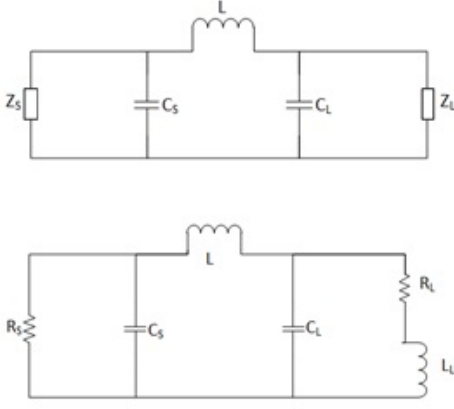


Fig. 4. a) impedance matching circuit, b) impedance matching circuit with the equivalent load impedance

In order to get the state space model, the system of Fig. 1 can be redrawn as shown in Fig. 4 a). Since the load impedance may be complex, it can be represented by

$$Z_L = R_L + jX_L \quad (8)$$

where  $R_L$  and  $X_L$  are the load resistance and reactance, respectively.

Based on the circuit of Fig. 4 b) and considering the voltage across the capacitors  $C_L$  and  $C_S$  and the current through the inductors  $L$  and  $L_L$  as state variables, the state model of the system is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_s C_s} & -\frac{1}{C_s} & 0 & 0 \\ \frac{1}{L} & 0 & -\frac{1}{L} & 0 \\ 0 & \frac{1}{C_L} & 0 & -\frac{1}{C_L} \\ 0 & 0 & \frac{1}{L_L} & -\frac{R_L}{L_L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (9)$$

Now, let the source impedance be  $50\Omega$ , the load impedance be  $25 + j43.33\Omega$ , and the frequency be  $2.4\text{GHz}$ , then the value of the elements of the  $\pi$  network can be calculated using basic circuit theory concepts:  $C_S = 3.97\text{pF}$ ,  $L = 2.43\text{nH}$ ,  $C_L = 4.05\text{pF}$ . It is clear that the value of the elements of the load impedance are  $R_L = 25\Omega$  and  $L_L = 2.87\text{nH}$ . Since the fuzzy controller has three membership functions and one input, the rules of the system can be expressed as:

$$\begin{aligned} \text{RULE1 : IF } i(t) \text{ is } M_1 \text{ THEN } \dot{x}(t) &= A_1 x(t) \\ \text{RULE2 : IF } i(t) \text{ is } M_2 \text{ THEN } \dot{x}(t) &= A_2 x(t) \\ \text{RULE3 : IF } i(t) \text{ is } M_3 \text{ THEN } \dot{x}(t) &= A_3 x(t) \end{aligned} \quad (10)$$

where  $M_1$ ,  $M_2$  and  $M_3$  are the T, S and triangular membership functions, and  $A_1$ ,  $A_2$  and  $A_3$  are given by

$$\begin{aligned} A_1 &= 1 \times 10^{11} \begin{bmatrix} 0.05 & -2.51 & 0 & 0 \\ 0.004 & 0 & -0.004 & 0 \\ 0 & 3.52 & 0 & -3.52 \\ 0 & 0 & 0.003 & -0.087 \end{bmatrix} \\ A_2 &= 1 \times 10^{11} \begin{bmatrix} 0.05 & -2.51 & 0 & 0 \\ 0.004 & 0 & -0.004 & 0 \\ 0 & 2.464 & 0 & -2.464 \\ 0 & 0 & 0.003 & -0.087 \end{bmatrix} \\ A_3 &= 1 \times 10^{11} \begin{bmatrix} 0.05 & -2.51 & 0 & 0 \\ 0.004 & 0 & -0.004 & 0 \\ 0 & 1.895 & 0 & -1.895 \\ 0 & 0 & 0.003 & -0.087 \end{bmatrix} \end{aligned} \quad (11)$$

The matrices  $A_1$ ,  $A_2$  and  $A_3$  are calculated assuming the error signal is small, medium or big. Solving the LMI problem given by (7) we obtain that the common  $P$  matrix is given by

$$P = 1 \times 10^{-12} \begin{bmatrix} 3 & -41 & -2 & 27 \\ -41 & 4497 & -54 & -1192 \\ -20 & -54 & 6 & -76 \\ 27 & -1192 & -76 & 3209 \end{bmatrix} \quad (12)$$

The positive definite matrix  $P$  satisfies the stability condition given by Theorem 1. In other words, the closed-loop fuzzy control system is asymptotically stable.

## V. SIMULATION RESULTS

In order to perform the simulations, we use the initial set of values of the elements in the matching circuit described in the previous section. The fuzzy controller iterates according to the process previously described, modifying in each step the value of the capacitor  $C_L$ , until the system reaches the matching point.

Fig. 5 shows the evolution in time of the transfer function response using different initial values of the capacitor.

As can be seen, the Fuzzy Controller adapts the impedance matching network, leading the system in every case to reach the point of  $H = -3\text{dB}$ , which corresponds to the maximum power transfer. From Fig. 5 it can be seen that the speed of convergence depends on the initial value of the capacitor. Fig. 6 depicts the system magnitude response in the frequency domain, after it has been adapted. As can be seen from Fig. 6, the system has been designed to have a resonant frequency of  $2.4\text{GHz}$ , which allows its implementation on wireless communication systems such as WI-FI [13] and Bluetooth [14]. For comparison purposes, a least mean square (LMS) algorithm was implemented, using the same impedance matching circuits. In a LMS adaptive system the criterion is to optimize temporal estimations of the error (13). This error is used to adapt the system through an iterative scheme, in a similar way to the fuzzy controller proposed in this work.

$$\rho(t) = -2\mu \int_{-\infty}^t e(u) \nabla_{\rho} e(u) du \quad (13)$$

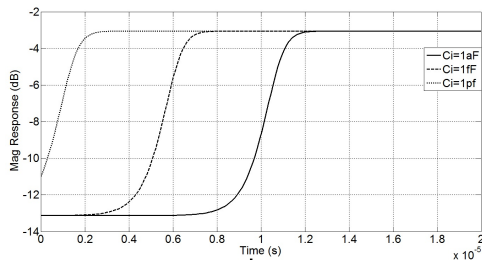


Fig. 5. Magnitud response for the PI network adapted by the fuzzy controller for different initial values of the capacitor

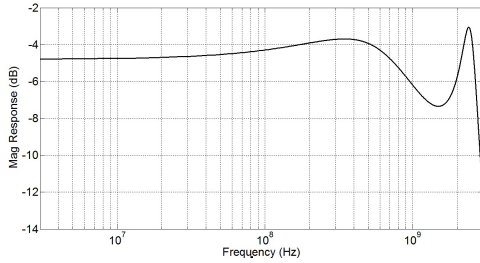


Fig. 6. Magnitud response for the PI network adapted by the fuzzy controller for different initial values of the capacitor

where  $\rho(t)$  is an impedance matching coefficient,  $\mu$  is a constant, which establishes the adaptation speed of the system,  $\nabla_{\rho} e(u)$  represents the gradient of the error related to the parameter  $\rho$ . The optimal value of the impedance matching coefficient  $\rho$  is achieved when the source impedance is equal to the load impedance.

Since the load impedance is expected to change over time, the fuzzy controller has to be able to adapt the output impedance of the  $\pi$  network to match it to the new value of the load impedance. Fig. 7 depicts the behavior of the adapted impedance when a disturbance is present.

Fig. 8 shows the results obtained in both cases: The fuzzy controller and the LMS algorithm. It can be seen that the Fuzzy Controller matches the impedance after  $8\mu s$ , compared to the LMS algorithm which requires about  $30\mu s$ , under similar conditions.

Fig. 9 shows a comparison of the normalized mean square error through the iterative process for the two cases: The pro-

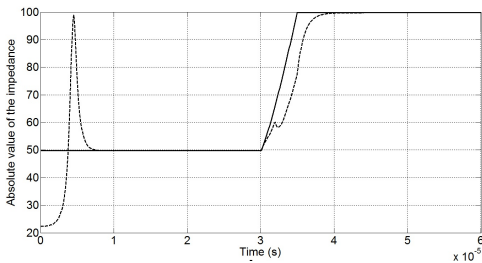


Fig. 7. Matching for on-fly changes of the load impedance

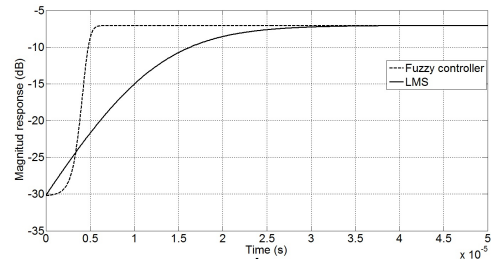


Fig. 8. Comparison between fuzzy controller and LMS

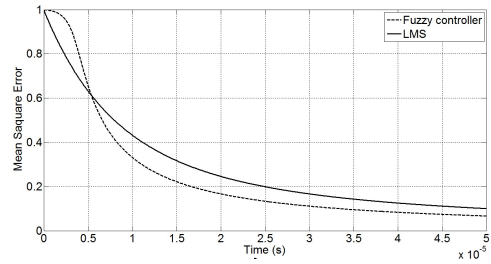


Fig. 9. Normalized mean square error

posed fuzzy controller and the LMS algorithm. The normalized mean square error is given by

$$E[e^2(n)] = E[(d(n) - y(n))^2] \quad (14)$$

where  $d(n)$  is the desired output and  $y(n)$  is the actual output.

## VI. CONCLUSION

This paper presented a novel scheme for adaptive impedance matching using a fuzzy controller with application to wireless communication systems. Results based on Verilog-A simulations indicate that the proposed fuzzy controller represents a feasible and promising scheme for the design and implementation of an on-chip adaptive impedance matching circuit. Experiments using the  $\pi$  matching network were carried out with excellent results. The results showed that the proposed fuzzy controller is, on average, six times faster than the LMS algorithm. The stability of the T-S system was proved by solving a LMI problem. The adapted system was designed to present a resonance frequency of 2.4 GHz, for applications in wireless communication systems such as WI-FI and Bluetooth. A VLSI design, simulation, manufacturing and characterization of the proposed adaptive impedance matching scheme, is currently under way in our research group.

## REFERENCES

- [1] Grant Mick, *Signal Integrity Considerations for High Speed Digital Hardware Design*, White Paper, Calyptech. Issue: 01; November 2002.
- [2] Lopez-Delgadillo E., Garcia-Andrade M.A., Diaz-Mendez J.A., Maloberti, F., *Automatic impedance control for chip-to-chip interconnections*, 15th IEEE International Conference on Electronics, Circuits and Systems, pp. 332-335, 2008.
- [3] Yichuang Sun, Wai Kit Lau, *Evolutionary Tuning Method for Automatic Impedance Matching in Communication Systems*, IEEE International Conference on Electronics, Circuits and Systems, Vol 3, pp. 73-77, 1998.

- [4] Hemminger, Thomas, *Antenna Impedance Matching With Neural Networks*, International Journal of Neural Systems (IJNS); Vol: 15; pp. 357-361, 2005.
- [5] Munshi Anees, Johns David, Sedra Adel, *Adaptive Impedance Matching*, IEEE International Symposium on Circuits and Systems, 1994. ISCAS '94., Vol. 2, pp. 69-72, 1994.
- [6] Peter Sjoblom, Henrik Sjolandl, *An adaptive impedance tuning CMOS circuit for ISM 2.4-GHz band*, IEEE Transactions on Circuits and Systems-I, Regular Papers, Vol. 52, pp. 1115-1124, June 2005.
- [7] Zadeh, L., *Fuzzy sets*; Information and Control, Vol. 8, Academic Press, pp. 338-353, 1965.
- [8] K. Tanaka and M. Sugeno, *Stability analysis and design of fuzzy control systems*, Fuzzy Sets Syst., vol. 45, pp. 135-156, 1992.
- [9] Chin-Tzong Pang and Sy-Ming Guu, *Sufficient Conditions for the Stability of Linear Takagi-Sugeno Free Fuzzy Systems*, IEEE Transactions on Fuzzy Systems, Vol. 11, No. 5, October 2003.
- [10] Chin-Tzong Pang and Yung-Yih Lur, *Stability of Takagi-Sugeno Fuzzy Systems With Time-Varying Uncertainties*, IEEE Transactions on Fuzzy Systems; Vol. 16, No. 1, February 2008.
- [11] Michio Sugeno, *On Stability of Fuzzy Systems Expressed by Fuzzy Rules with Singleton Consequents*, IEEE Transactions on Fuzzy Systems; Vol. 7, No. 2, April 1999.
- [12] Z. H. Huang and Q. Shen, *A new fuzzy interpolative reasoning method based on center of gravity*, Proc. of the International Conference on Fuzzy Systems, volume 1, pp. 25-30, 2003.
- [13] IEEE Standard for information technology Telecommunications and information exchange between systems Local and metropolitan area networks Specific requirements. Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications; IEEE Computer Society; IEEE Std 802.11 1999.
- [14] IEEE Standard for information technology Telecommunications and information exchange between systems Local and metropolitan area networks Specific requirements. Part 15.1: Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Wireless Personal Area Networks (WPANs); IEEE Computer Society; IEEE Std 802.15.1 2002.