

Nonlinear Control of Three-Phase Active Rectifiers Based L and LCL Filters

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Abstract—In this paper, a state-space modeling of three-phase voltage source type pwm rectifiers using L and LCL filters is proposed. Next, an input-output feedback linearization based control technique is developed with the aim to control the DC output voltage and the input displacement factor as well. The overall control strategy is tested using Matlab/simulink software wherein a two-level converter is modulated by comparing low-frequency sine wave modulating signals to a 10 kHz triangular wave pulse. Finally, simulation results are investigated so as to make a comparative study between the performances of two topologies in terms of unity power factor operation capability, total harmonic distortion of line currents, and DC bus voltage regulation.

Keywords—nonlinear control; L and LCL filter; pwm voltage source rectifier;

I. INTRODUCTION

Three-phase pwm Voltage Source Rectifiers (VSR) are widely used in several industrial applications, such as motor drives, battery management, and Distributed Power Generation Systems (DPGSs) due to their attractive features over conventional topologies such as their capability of providing sinusoidal line currents with near unity input displacement factor as well as a controllable dc-link voltage. [1]-[5].

This converter is usually connected to the grid through a simple L filter to reduce high frequency harmonics contents in the line current waveforms. In recent year, the LCL filter becomes however, more and more attractive as a utility interface for grid-connected VSR. Indeed, it can lead to better mitigation of switching harmonics using lower inductance which makes it suitable for higher power applications [6]-[8]. Unfortunately, control systems involving LCL filters are inherently more complicated and they need to consider many constraints, such as current ripple through inductors, total impedance of the filter, switching harmonic attenuation, resonance phenomenon and reactive power absorbed by filter capacitors [9]-[11].

In this paper, the input-output feedback linearization based control technique is used for both topologies i.e the L and LCL filters feeding the converter. [1] [2] [12]-[14]. Through simulation results we investigate the effects of each model on unity power factor operation, line current harmonics distortion and DC-link voltage.

The paper is organized as follows. In section II, the mathematical model of the VSC with L and LCL filters is presented. The input-output feedback linearization control strategy is developed in section III. The performance evaluation and simulation results are presented in section IV. Finally, conclusions are drawn in section V.

II. THE MATHEMATICAL MODEL OF THE VSR

A. Using L filter

The power circuit of L filtered VSR without neural connection is shown in Fig.1 below. The state space model reported in a d-q rotating reference frame synchronized with the grid voltage is given by (1).

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{V}_{dc} \end{bmatrix} = \begin{bmatrix} \frac{-R}{L}i_d + \omega i_q \\ \frac{-R}{L}i_q - \omega i_d \\ \frac{3}{2C_{dc}V_{dc}}(e_d i_d + e_q i_q) - \frac{V_{dc}}{C_{dc}R_{ch}} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_d - V_d \\ e_q - V_q \end{bmatrix} \quad (1)$$

$$\begin{aligned} V_d &= V_{dc} d_d \\ V_q &= V_{dc} d_q \end{aligned} \quad (2)$$

where,

- L : Inductance of the line;
- R : Resistance of the line;
- C_{dc} : Capacitance of the DC bus;
- V_{dc} : DC bus voltage;
- Ω : Angular velocity of the line voltage;
- i_d, i_q : Line currents in d-q reference frame;
- e_d, e_q : Line voltages in d-q reference frame;
- d_d, d_q : Switching functions in d-q reference frame;
- V_d, V_q : Input voltages of the rectifier in d-q reference frame;

The system (1) is nonlinear regarding to V_{dc} . The number of state variables is three whereas there are only two control inputs.

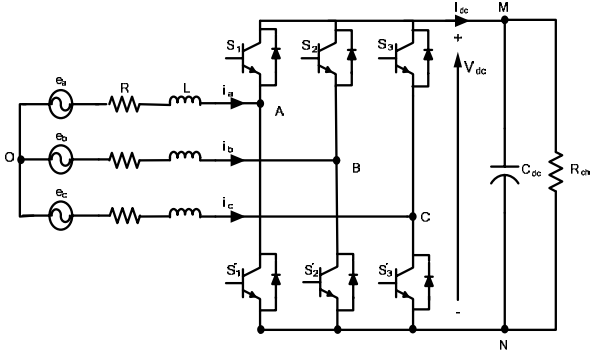


Fig. 1. Circuit diagram of a three-phase PWM rectifier with L filter

B. Using LCL

Fig.2 shows the power circuit of LCL filtered three-phase VSR. Single phase equivalent circuit of LCL filter is shown in fig.3 and its mathematical model is given with (3).

$$\begin{aligned} L_{a1} \frac{di_{a1}}{dt} &= -R_{a1}i_{a1} + e_a - V_{ca} \\ L_{a2} \frac{di_{a2}}{dt} &= -R_{a2}i_{a2} - V_a + V_{ca} \\ C \frac{dV_c}{dt} &= i_{a1} - i_{a2} \end{aligned} \quad (3)$$

Neglecting the filter capacitor C as in [3], the system can be defined using the following equation:

$$e_a = R_a i_a + L_a \frac{di_a}{dt} + V_a \quad (4)$$

where, $R_a = R_{a1} + R_{a2}$, $L_a = L_{a1} + L_{a2}$, e_a is the grid voltage, i_a is the grid current and V_a is the converter pole voltage.

Therefore, the mathematical model of LCL filtered VSR in the abc reference frame is given as follows:

$$\begin{aligned} e_a &= R_a i_{a1} + L_a \frac{di_{a1}}{dt} + V_a \\ e_b &= R_b i_{b1} + L_b \frac{di_{b1}}{dt} + V_b \\ e_c &= R_c i_{c1} + L_c \frac{di_{c1}}{dt} + V_c \\ e_a i_{a1} + e_b i_{b1} + e_c i_{c1} &= V_{dc} \left(C \frac{dV_{dc}}{dt} + \frac{V_{dc}}{R_{ch}} \right) \end{aligned} \quad (5)$$

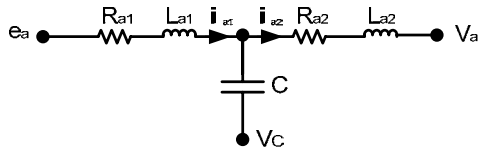


Fig. 3. Single phase equivalent circuit of LCL three-phase PWM AC-DC rectifier with LCL filter

It follows that the d-q mathematical model defined by (1) and (2), can also be applied to the LCL topology where,

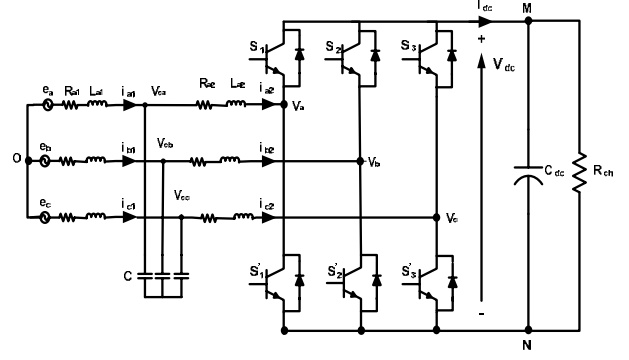


Fig. 2. Circuit diagram of a three-phase PWM rectifier with LCL filter

$$\begin{aligned} i_d &= i_{ld} \\ i_q &= i_{lq} \end{aligned} \quad (6)$$

III. NONLINEAR CONTROL FOR THREE-PHASE PWM AC-DC CONVERTER

Feedback linearization is recognized as a powerful approach to nonlinear control design. The central idea is to algebraically transform nonlinear systems dynamics into linear ones. Then, linear control techniques can be applied. Based on the input-output feedback linearization control approach already introduced in [9], the above model (1) can be rewritten in the following form

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad (7)$$

Where,

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = \begin{bmatrix} \frac{-R}{L}x_1 + \omega x_2 \\ \frac{-R}{L}x_2 - \omega x_1 \\ \frac{3}{2C_{dc}x_3}(e_d x_1 + e_q x_2) - \frac{x_3}{R_{ch}C_{dc}} \end{bmatrix} \quad (8)$$

$$g(x) = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix} \quad (9)$$

$$h(x) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} V_{dc} \\ i_d \end{bmatrix} = \begin{bmatrix} x_3 \\ x_1 \end{bmatrix} \quad (10)$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \\ V_{dc} \end{bmatrix} \quad (11)$$

$$U = \begin{bmatrix} U_d \\ U_q \end{bmatrix} = \begin{bmatrix} e_d - V_d \\ e_q - V_q \end{bmatrix} \quad (12)$$

Since the mains voltage axis d was chosen as reference in the Park transformation, then $e_d = E$ and $e_q = 0$

Thereafter, the control inputs are

$$\begin{aligned} U_d &= E - V_d \\ U_q &= -V_q \end{aligned} \quad (13)$$

The next step consists in differentiating each output many times until a control input U appears in the dynamics of the output. For the first output, it can be observed that the control input does not appear in the first derivative of y_1

$$\begin{aligned} y_1 &= x_3 \\ \dot{y}_1 &= \dot{x}_3 = f_3(x) \end{aligned} \quad (14)$$

Differentiate once again y_1 , the control input u_d appears as shown in equation (15) below

$$\ddot{y}_1 = \frac{3E}{2C} \left(\frac{f_1(x) + g_1 u_d}{x_3} - \frac{x_1 f_3(x)}{x_3^2} \right) - \frac{1}{R_{ch} C_{dc}} f_3(x) \quad (15)$$

For the second output the control input already exists in the first derivative of y_2 so we don't need to make a second derivative.

$$\begin{aligned} y_2 &= x_2 \\ \dot{y}_2 &= \dot{x}_2 = f_2(x) + g_2 u_q \end{aligned} \quad (16)$$

The above derivatives can be arranged in the following form

$$\begin{bmatrix} \ddot{y}_1 \\ \dot{y}_2 \end{bmatrix} = A(x) + E(x) \begin{bmatrix} u_d \\ u_q \end{bmatrix} \quad (17)$$

where,

$$A(x) = \begin{bmatrix} \frac{3E}{2C_{dc}} \left(\frac{f_1(x)}{x_3} - \frac{x_1 f_3(x)}{x_3^2} \right) - \frac{1}{R_{ch} C_{dc}} f_3(x) \\ f_2(x) \end{bmatrix}; E(x) = \begin{bmatrix} \frac{3E g_1}{2C_{dc} x_3} & 0 \\ 0 & g_2 \end{bmatrix} \quad (18)$$

Since $E(x)$ is nonsingular, the control law is derived as follows

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = E^{-1}(x) [A(x) + \begin{bmatrix} \ddot{y}_1 \\ \dot{y}_2 \end{bmatrix}] \quad (19)$$

where,

$$E^{-1}(x) = \begin{bmatrix} \frac{2C_{dc} x_3}{3E g_1} & 0 \\ 0 & \frac{1}{g_2} \end{bmatrix} \quad (20)$$

The new linear control inputs are

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \ddot{y}_1 \\ \dot{y}_2 \end{bmatrix} \quad (21)$$

Once the system is linearized, we can apply linear control techniques by using PI controllers. With regard to the target error dynamics given by (22), the linear control laws v_d and v_q are calculated as (24)

$$\begin{cases} \ddot{e}_1 + k_{11} \dot{e}_1 + k_{12} e_1 + k_{13} \int e_1 dt = 0 \\ \ddot{e}_2 + k_{21} e_2 + k_{22} \int e_2 dt = 0 \end{cases} \quad (22)$$

where,

$$\begin{aligned} e_i &= y_i - y_i^{ref}; (i = 1, 2) \\ y_1^{ref} &= x_3 = V_{dc} = 500V \\ y_2^{ref} &= x_2 = i_q = 0 \end{aligned} \quad (23)$$

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} -k_{11} f_3(x) - k_{12} e_1 - k_{13} \int e_1 dt \\ -k_{21} e_2 - k_{22} \int e_2 dt \end{bmatrix} \quad (24)$$

and k_{ij} are the gains allowing to impose the desired dynamics of the controlled system.

Substituting (22) and (23) into (19) yields

$$\begin{aligned} u_d &= \frac{1}{g_1} \left[\frac{2C_{dc}}{3E} x_3 v_d + f_3(x) \left(\frac{2}{3R_{ch} E} x_3 + \frac{x_1}{x_3} \right) - f_1(x) \right] \\ u_q &= \frac{1}{g_2} (v_q - f_2(x)) \end{aligned} \quad (25)$$

According to (13), the resulting voltage references to be modulated by the PWM converter are derived as follows

$$\begin{aligned} v_d^{ref} &= E - u_d \\ v_q^{ref} &= -u_q \end{aligned} \quad (26)$$

For both L and LCL filtered VSR, the same control algorithm can be used. The proposed nonlinear control block diagram of the PWM converter based L and LCL filter is depicted in Fig. 4.

IV. SIMULATION RESULTS

In order to show the effectiveness of the control technique with L and LCL filters on unity power factor operation, line currents harmonic distortions and DC bus voltage, numerical simulations are carried out using Matlab/Simulink software. Line voltages, network frequency and DC-Link parameters are taken as 220 V, 50 Hz, 60 Ω and 660 μF . Parameters of L and LCL filters are listed in Table I below.

TABLE I. VALUES OF L AND LCL FILTERS COMPONENTS

| L filter | | LCL filter | | | | |
|----------------|--------|------------------------------|----------------------|------------------------------|----------------------|---------------|
| R (Ω) | L (mH) | R _{a1} (Ω) | L _{a1} (mH) | R _{a2} (Ω) | L _{a2} (mH) | C (μF) |
| 0,5 | 3,3 | 0,2 | 0,8 | 0,3 | 2,2 | 12 |

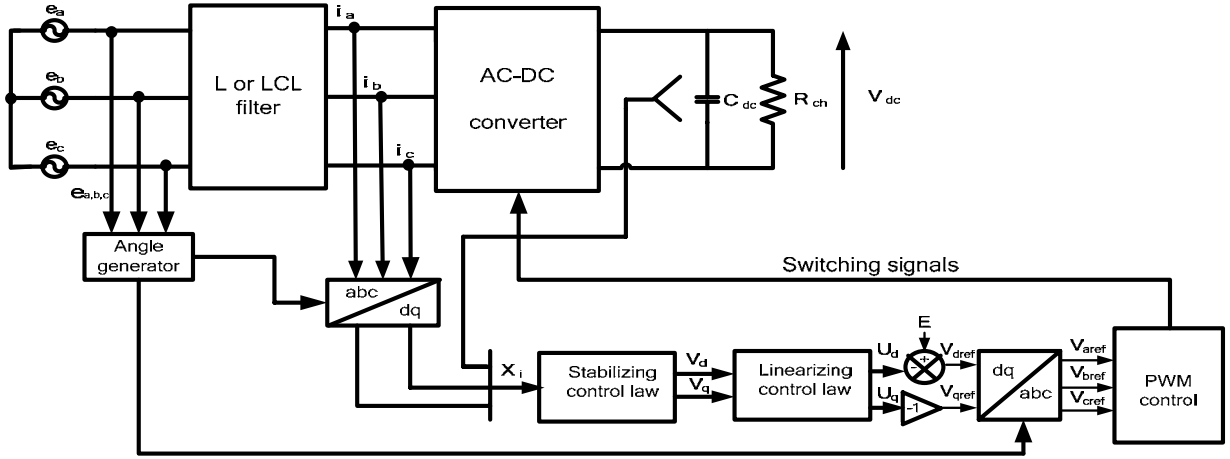


Fig. 4. Input-output control algorithm of rectifier with L or LCL filters

A. L filtered rectifier

The steady state waveforms of the mains voltage (e_a), line current (i_a), the reactive current (i_q), and the output DC bus voltage (V_{dc}) are depicted in Fig. 5. It can be seen that current and voltage are in phase. The line current is sinusoidal but it includes high order harmonic contents and its Total Harmonic Distortion value (THD) is 3,45 % which is under the standard value. Moreover, the reactive current i_q is zero which means that the converter operates at unity input power factor. On the other hand, the output DC voltage is stable and varies around the target reference. The stability of the voltage regulation loop is also verified during in transient operation by applying a step increase of the target output voltage from 500V to 600V and the settling time is shorter than 0,15s.

B. LCL filtered rectifier

The steady state waveforms of the mains voltage (e_a), line current (i_a), the reactive current (i_q), and the output DC bus voltage (V_{dc}) are depicted in Fig. 6. It can be seen that current and voltage are also in phase. The line current is sinusoidal and its THD is 2,65 % which is lower than the L filtered one. Moreover, the average value of the reactive current i_q is near zero i.e. a small DC component of the reactive current is injected into the grid. On the other hand, the output DC voltage is stable and varies around the target reference. The stability of the voltage regulation loop is also verified during in transient operation by applying a step increase of the target output voltage from 500V to 600V as shown in Fig. 6(d). The settling time is shorter than 0,15s.

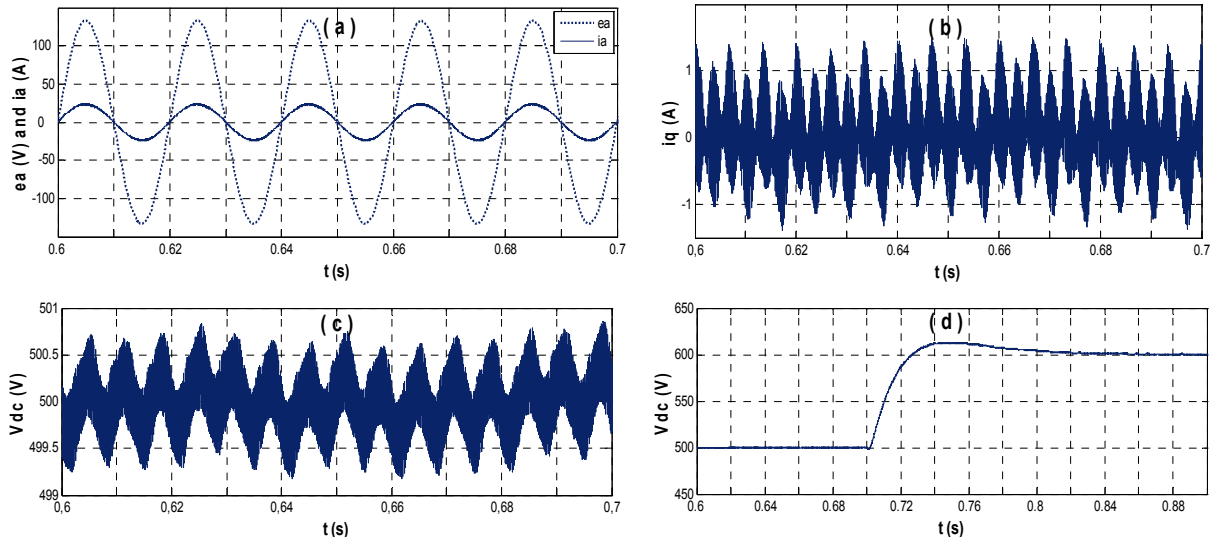


Fig. 5. L filtered rectifier simulation results -(a) Line current i_a and line voltage e_a -(b) Reactive current i_q - (c) DC bus voltage for $V_{dc}^* = 500V$ - (d) Transient response of DC bus voltage

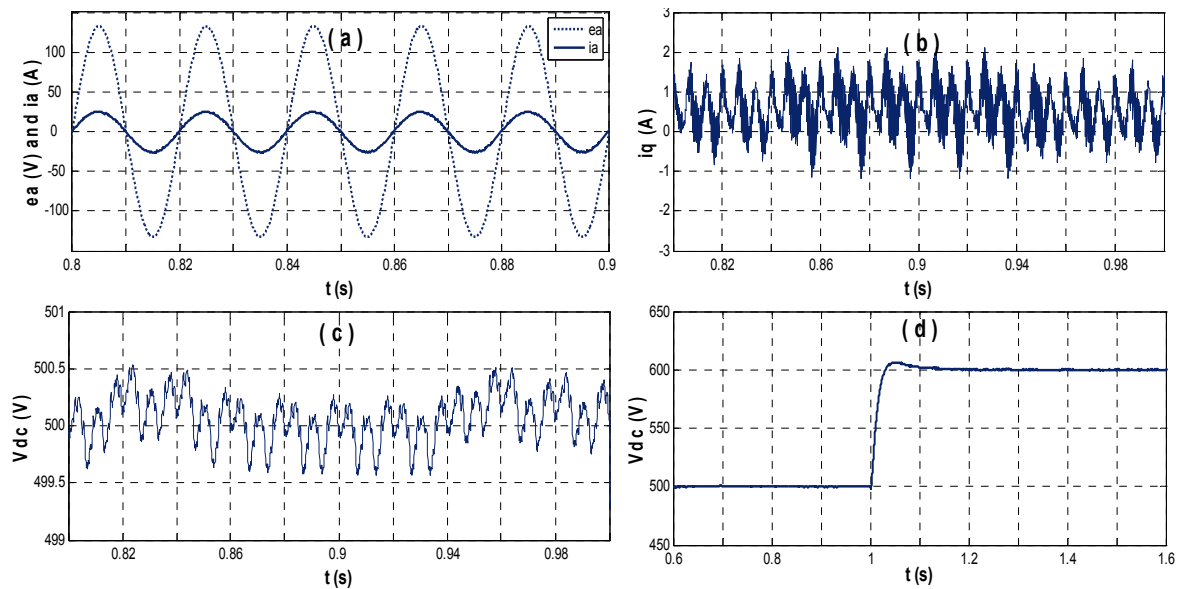


Fig. 6. LCL filtered rectifier simulation results. (a) Line current i_a and line voltage e_a - (b) Reactive current i_q -(c) DC bus voltage for $V_{dc}^* = 500V$ - (d) Transient response of DC bus voltage

V- CONCLUSION

This paper proposed a comparative study between the performances of L and LCL filters feeding three-phase PWM voltage source rectifiers under input-output feedback linearization. It was found that the LCL filter allows reducing the filter size and enhance the line current waveforms as well. However, a decrease of the input displacement factor was observed. This is due to the fact that filter capacitor is omitted in the mathematical model of the converter and thereafter it was not considered in the control algorithm. In a future work, a new algorithm will be proposed in order to achieve unity input power factor operation of the LCL topology independently of the active power demanded by the DC load.

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