A Sensor-less Speed Control of Induction Motor Based on Robust Rotor Flux Observer

Fethi Farhani¹, Abderrahmen Zaafouri², and Abdelkader Chaari³

University of Tunis

Unit C3S, Higher School of Sciences and Techniques of Tunis (ESSTT),
5 Av. Taha Hussein, BP 56, 1008 Tunis, Tunisia.

¹E-mail: fethi.farhani@issatkr.rnu.tn; ²E-mail: abderrahmen.zaafouri@isetr.rnu.tn;

³E-mail: assil.chaari@esstt.rnu.tn

Abstract—The induction motor is the principal source of the workhorse [1] in the industry. However, due to its high non linearity, a high-performance control of induction motor remains a challenge for the automation. In this paper, we present a robust solution for the observer and control of the induction machine taking into account the iron losses. The proposed technique is based on Lyapunov theory and application of the bilinear matrix inequalities framework. Thus, an adaptive mechanism is introduced to cover not only uncertainty parameters but also the rotor speed which is derived from the satisfaction of the first condition of the Lyapunov theory. The observer gain is calculated by solving the bilinear matrix inequality (BMI) to satisfy the second condition of Lyapunov. Some simulation results are given to demonstrate the robustness and performance of the proposed solution.

Index Terms—Full-Order Observer; LMI ; Induction machine; Robust Control; Vector Control.

NOMENCLATURE

v_{sd}, v_{sq}	: d and q components of stator voltage;
i_{sd}, i_{sq}	: d and q components of stator current;
$\lambda_{rd}, \lambda_{rq}$	: Rotor flux components;
L_s, L_r	: Stator and rotor self-inductances;
$l_f s, l_f r$	: Stator and rotor leakage inductances;
R_s, R_r	: Stator and rotor resistances;
R_c	: Equivalent iron core-loss;
M	: Mutual inductance;
p	: Number of pole pairs;
ω	: Rotor angular speed;
ω_e	: Angular rotating frame speed;
$\sigma = 1 - \frac{M^2}{L_r L_s}$	: Total linkage coefficient;
$\varepsilon = \frac{\sigma L_s L_r}{M}$	: —
I	: Unit matrix of appropriate dimension
J	: Skew symmetric matrix 2×2

I. INTRODUCTION

An analysis of energy consumption in industrialized countries [1] shows that the induction motors are considered the main source of mechanical energy. Because of its simplicity, reliability and efficiency, it is the type of engine most commonly employed for industrial use.

The technological evolution realized in digital calculators (DSP, μC) and power electronics permit the appearance of high-performance control technology for the induction machine. These methods are mainly based on the theory of vector control. In the literature, there are two techniques of field-oriented control, Indirect Field-Oriented Control (IFOC) [2], [3] and a Direct Field-Oriented Control (DFOC) [4], [5]. Both control strategies, led to decouple the stator current components, responsible for electromagnetic torque production and the flux. This achieves independent control of flux and electromagnetic torque.

The simplest structure of the squirrel cage motor hides some control engineering problems such as the uncertainty and inaccessibility of a number of these parameters and the state variables for the direct measurement as the rotor flux. High-performance control of induction motor requires a real-time knowledge of the evolution of its states and parameters, because these affect directly the developed torque. The scientific community developed many solutions to estimate the unmeasured variables (rotor flux and resistance) from measured quantities such as terminal stator voltage and current. For reasons of reliability and/or safety, many industrial applications require the reduction of the number of sensors such as that of the speed. To improve the robustness of the control system, many advanced solutions have been developed in the literature. A significant number of solutions for induction motor control combines the mechanical speed estimation and parameter adaptation as the rotor resistance [1], [6], the rotor time constant [7], [8], stator resistance [1], [9]. However, a few solutions [10], [11] that are interested in the monitoring of iron loss resistance. In this context, we have developed in this paper a robust rotor flux observer combined with motor parametric adaptation mechanism such as the iron loss resistance. The rotor flux observer gain as well as the adaptation mechanism is based on the Lyapunov theory. In order to optimize the DSP calculation load, we impose a constant observer gain for the entire operating range of the induction motor.

The paper is organized as follows. Section 2 presents the dynamic model of used induction motor in $\alpha - \beta$ fixed frame linked to the stator; a brief review of the theory of field

oriented control is given in Section 3. The proposed full order observer and the adaptive mechanisms of speed and parameters of three-phase squirrel cage motor are detailed in section 4. In section 5, the results are evaluated using a numerical simulation under Matlab/Simulink framework. Finally, some conclusions and perspectives are proposed.

II. MATHEMATIC MODELING OF INDUCTION MOTOR TAKING INTO ACCOUNT IRON CORE LOSS

In this section, we present the mathematical model of a three-phase squirrel cage induction motor taking into account the iron core losses. In [12] the authors emulate the iron losses by a fictive resistance (R_c) incorporated in parallel to the input of per phase equivalent circuit of the motor (Figure 1). The nominal physical parameters of the induction motor are given in Table I. Under the assumption of linear magnetic circuits, the dynamic of the induction motor is described by the following state space representation in $\alpha - \beta$ frame ($\omega_e = 0$).

$$\begin{cases} \dot{x} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} x + \begin{bmatrix} B_1 \\ 0_{2 \times 2} \end{bmatrix} v_s = A(\omega) x + B v_s \\ i_s = C x + D v_s \end{cases} \quad (1)$$

where

$$x = [i_{L\alpha} \quad i_{L\beta} \quad \lambda_{r\alpha} \quad \lambda_{r\beta}]^T; \quad i_s = [i_{s\alpha} \quad i_{s\beta}]^T;$$

$$v_s = [v_{s\alpha} \quad v_{s\beta}]^T;$$

$$A_{11} = -\left(\frac{R_r}{L_r} \frac{1-\sigma}{\sigma} + \frac{R_{sc}}{\sigma L_s}\right) I;$$

$$A_{21} = \left(\frac{M R_r}{L_r}\right) I;$$

$$A_{12} = \left(\frac{R_r}{\sigma L_r}\right) I - p\omega \left(\frac{1}{\sigma}\right) J;$$

$$A_{22} = -\left(\frac{R_r}{L_r}\right) I + p\omega J;$$

$$B_1 = \left(\frac{R_c}{R_c + R_s} \frac{1}{\sigma L_s}\right) I;$$

$$C = \left(\frac{R_c}{R_c + R_s} I \quad 0_{2 \times 2}\right);$$

$$D = \left(\frac{1}{R_c + R_s}\right) I;$$

$$R_{sc} = (R_s R_c) / (R_s + R_c);$$

The electromagnetic torque developed by the three-phase induction motor can be expressed by the rotor flux and induced rotor currents. The instantaneous electromagnetic torque in the $\alpha - \beta$ reference frame, can be calculated using the following equation

$$C_e = p(3M/2L_r)(\lambda_{r\alpha} i_{r\beta} - \lambda_{r\beta} i_{r\alpha}) \quad (2)$$

Let $C_1 = [0 \quad I]$, the rotor flux can be extracted from the state vector as follows

$$\lambda_r = C_1 x \quad (3)$$

Figure 1. Per phase equivalent circuit of induction motor in arbitrary rotating reference frame taking core loss into account iron core loss

Table I
INDUCTION MOTOR NOMINAL PARAMETERS

Symbol	Quantity	UM
R_s	Stator resistance	2.3 [Ω]
R_r	Rotor resistance	1.8 [Ω]
R_c	Equivalent iron core-loss	92 [Ω]
L_s	Stator inductance	261 [μH]
L_r	Rotor inductance	261 [μH]
M	Mutual inductance	245 [μH]
σ	Leakage factor	0.134
J	Moment of inertia	0.22 [Kg m ²]
f	Friction coefficient	0.001
V_n	Rated voltage	380 [V]
I_n	Rated current	10.4 [A]
P_n	Rated power	5.1 [KW]
p	Number of pole pairs	2

Finally, from the relationship between the developed torque, the load and the mechanical speed of the rotor, we can express the dynamic motion as follows

$$\dot{\omega} = \frac{p}{J} (C_e - C_r) \quad (4)$$

where C_r is the load torque on the shaft acting as a disturbance.

III. FIELD ORIENTED CONTROL

In the literature there exist two main techniques of field-oriented control, the DFOC and IFOC. For our application, we have adopted the DFOC. The Figure 3 illustrates the block diagram of a proposed control system. The main advantage of the FOC is to decouple control current into two components; torque component (aligned with q-axis), and flux component (aligned with d-axis). Those components are commanded by reference flux and torque, respectively. In this section, we consider the coordinate's reference frame $d - q$ locked to the rotor flux vector rotating at the stator frequency ($\omega_e = \omega_s$). The developed torque by the three-phase induction motor can be re-expressed as follows.

$$C_e = p \left(\frac{3M}{2L_r} \right) (\lambda_{rd} i_{rq} - \lambda_{rq} i_{rd}) \quad (5)$$

By aligning the d-axis of the reference frame with the rotor flux vector, the electromagnetic torque can be expressed as follows:

$$\begin{cases} \lambda_r = \lambda_{rd} \\ C_e = p \left(\frac{3M}{2L_r} \right) \lambda_{rd} i_{rq} \end{cases} \quad (6)$$

Figure 2. Adaptive State Observer of Induction Motor

The dynamic of rotor flux can be modeled by the first-order differential equation (7) in the $d-q$ frame

$$\lambda_{rd} = M i_{Ld} - \frac{L_r}{R_r} \frac{d\lambda_{rd}}{dt} \quad (7)$$

The relationship between the rotor and synchronous speed is defined as follows

$$\omega_s = \frac{MR_r}{L_r \lambda_{rd}} i_{Lq} + p\omega \quad (8)$$

IV. ROBUST ROTOR FLUX OBSERVER

In this section, we present the proposed full-order observer and its adaptive mechanisms based on the theory of Lyapunov for a three-phase squirrel cage-type induction motor.

A. Adaptive scheme for R_c and R_{sc} resistances Identification

The dynamic of the full-order observer (Figure 2) of the induction motor is given by the following equation :

$$\begin{cases} \dot{\hat{x}} = (\hat{A} + \hat{\omega} A_\omega) \hat{x} + \hat{B} v_s + H (\hat{i}_s - C \hat{x} - D v_s) \\ \hat{i}_s = C \hat{x} \end{cases} \quad (9)$$

where $A(\omega) = A + \omega A_\omega$, $\hat{A} = A + \Delta A$, $\hat{B} = B + \Delta B$, $\hat{C} = C + \Delta C$, $\hat{D} = D + \Delta D$, $\hat{\omega} = \omega + \Delta\omega$ and $A_\omega = \begin{bmatrix} 0 & -p(\frac{1}{\varepsilon})J \\ 0 & pJ \end{bmatrix}$.

The uncertain matrix's $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$ are split in two parts; the first one corresponding to nominal operation point and the second one to unknown behavior and marked $\Delta(*)$.

Let us define the estimation error as the difference between the estimated and the real states $e = x - \hat{x}$. So from this definition and based on (1) and (9) the dynamic of state estimation error is given as follows

$$\begin{aligned} \dot{e} &= (A + \Delta A + \omega A_\omega + \Delta\omega A_\omega) \hat{x} + (B v_s + \Delta B v_s) + \\ &H (\hat{i}_s - i_s) - (A + \omega A_\omega) x - B v_s \end{aligned} \quad (10)$$

By examining the influence of the variation of resistances of the three-phase induction motor, it is clear that $\Delta B \approx 0$, $\Delta C \approx 0$ and $\Delta D \approx \Delta R_c I$ since $R_s \ll R_c$,

So

$$\Delta A = A_R = \begin{bmatrix} \frac{\Delta R_{sc}}{\sigma L_s} I & 0 \\ 0 & 0 \end{bmatrix} \quad (11)$$

The dynamic of estimation error can re-expressed in the following form

$$\dot{e} = (A(\omega) + HC) e + \Delta A \hat{x} + \Delta\omega A_\omega \hat{x} + H \Delta D v_s \quad (12)$$

In order to define the observer matrix gain H and the adaptive scheme, Lyapunov theorem is utilized. Either μ_1 , μ_2 and μ_3 three positives constants, the following Lyapunov function is defined as a candidate:

$$V(e, \Delta\omega, \Delta R_{sc}, \Delta R_c) = e^T P e + \frac{\Delta\omega^2}{\mu_1} + \frac{\Delta R_c^2}{\mu_2} + \frac{\Delta R_{sc}^2}{\mu_3} \quad (13)$$

Given that the variations of parameters of the induction motor and the rotor speed to the time scale of the observation is negligible ($\dot{\omega} \approx 0$, $\dot{R}_c \approx 0$ and $\dot{R}_{sc} \approx 0$), after some calculation, the time derivative of V becomes

$$\dot{V}(e, \Delta\omega, \Delta R_c, \Delta R_{sc}) = e^T P e + V_1 + V_2 + V_3 \quad (14)$$

where

$$P = (A + \omega A_\omega + HC)^T P + P^T (A + \omega A_\omega + HC)$$

$$P = P^T \geq 0$$

$$V_1 = \Delta\omega (\hat{x}^T A_\omega^T P e + e^T P A_\omega \hat{x}) + 2 \frac{\Delta\omega}{\mu_1} \frac{d\Delta\omega}{dt}$$

$$V_2 = \Delta R_c (\hat{x}^T A_R^T P e + e^T P A_R \hat{x}) + 2 \frac{\Delta R_c}{\mu_2} \frac{d\Delta R_c}{dt}$$

$$V_3 = \Delta R_{sc} (\hat{x}^T P e + e^T P \hat{x}) + 2 \frac{\Delta R_{sc}}{\mu_3} \frac{d\Delta R_{sc}}{dt}$$

By the Lyapunov stability theory, the system is asymptotically stable if the time derivative of V is negative definite so

$$\dot{V}(e, \Delta\omega, \Delta R_c, \Delta R_{sc}) < 0 \quad (15)$$

The first condition of Lyapunov is

$$\begin{cases} \Delta\omega (\hat{x}^T A_\omega^T P e + e^T P A_\omega \hat{x}) + 2 \frac{\Delta\omega}{\mu_1} \frac{d\Delta\omega}{dt} = 0 \\ \Delta R_c (\hat{x}^T P e + e^T P \hat{x}) + 2 \frac{\Delta R_c}{\mu_3} \frac{d\Delta R_c}{dt} = 0 \\ \Delta R_{sc} (\hat{x}^T A_R^T P e + e^T P A_R \hat{x}) + 2 \frac{\Delta R_{sc}}{\mu_2} \frac{d\Delta R_{sc}}{dt} = 0 \end{cases} \quad (16)$$

The second Lyapunov condition is

$$(A(\omega) + HC)^T P + P (A(\omega) + HC) < 0 \quad (17)$$

We can rewrite the first condition of Lyapunov in the following form

Figure 6. Simulated Torque response for step varying of load torque

Figure 7. Estimated and actual rotor flux λ_α

Figure 8. Estimated and actual rotor flux λ_β

Figure 9. Tracking of R_c

Figure 10. Tracking of R_{sc}

VI. CONCLUSION

A robust full-order state observer for sensor-less three-phase induction motor drive has been developed in this paper. The proposed observer is combined with a simple mechanism to online tracking of the speed and some parameters of the induction motor. The numerical simulation is carried out to validate the proposed solution drive. Rated data of simulated induction motor are given in Table I. The observer gain matrix is obtained from solving an LMI by using LMI Toolbox of MATLAB. The simulation results show that, the convergence of the proposed robust full-order observer is guaranteed, as well as, estimated fictive resistances R_c and R_{sc} converges to actual values, and mechanical speed is controlled and estimated successfully.

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