Incremental relaying effect on the outage probability of correlated sources Transmission

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Abstract—In this paper we investigate different relaying strategies for correlated source transmission in block Rayleigh fading channels. Outage probability close form expressions are derived for both the cases where incremental decode and forward and only simple decode and forward techniques are used in the relays. We show that when using incremental technique in the principal relay, not only we gain in terms of channel resource saving but we get also a significant outage performance improvement compared to simple decode and forward system. Another finding of this paper is that if incremental relaying technique is used in the principal relay, the system reaches a diversity order of 2 with a significant power gain compared to a two links maximum ratio combining system.

Index— Decode and forward, incremental system, Outage probability, Slepian-Wolf.

I. INTRODUCTION

Cooperative communication has been recently used as a powerful technique which can fundamentally change the abstraction of a wireless link and offer significant potential advantages for wireless communication networks [1] [2].

In general, sources can be correlated and in order to exploit this correlation to reduce the communication requirements, a commonly used technique is Distributed Source Coding (DSC). This technique consists on compressing separately highly correlated sources and decoding them jointly. From a theoretical point of view, DSC is based on Slepian-Wolf and Wyner-Ziv compression schemes [3], [4].

The correlation between the two information sources is assumed to be expressed by the bit flipping model, where the information bits transmitted from the second transmitter are the flipped version of the information bits transmitted from the first one, with a probability pe .

In [5], the authors studied the outage performance for a direct link system when two correlated sources are used based on slepian-wolf theorem. However, these studies have not been done for relaying systems. Thus, we propose in this paper a model including DSC in decode and forward (DF) system by combining the data using both maximum ratio combining (MRC) and slepian-wolf technique. To reduce the effect of error propagation, we introduce also incremental relaying technique which shows its ability to enhance the diversity order of the system.

In this paper, the main purpose is to derive the theoretical outage probability of correlated sources transmitting with the

intermediate of relays using decode and forward or incremental relaying technique. The effect of the correlation factor and of the choice of the threshold are also investigated.

The paper is organized as follows. In section II, the system model is described. In section III, we proceed to the outage probability analysis of the system. Section IV validates the analysis by simulation results. Finally, section VI summarizes and concludes the paper.

II. SYSTEM MODEL

The adopted system model in this paper is described in Figure1 and consists of two correlated sources ($S1$ and $S2$) transmitting their data ($b1$ and $b2$) to one destination (D) by the intermediate of two relays $R1$ and $R2$.

The communication is supposed to be done over channels with a slow and frequency-flat Rayleigh fading coefficients h_{XY} . Where X and Y denotes the different system nodes. We assume that all the additive white Gaussian noise (AWGN) terms in the 6 links ($S1D$, $S1R1$, $R1D$, $S2D$, $S2R2$ and $R2D$) have equal variance N_0 and all the channels coefficients are independent. All terminals are equipped with a single antenna.

To understand the effect of path-loss on the performance of the proposed techniques, we use the model (which is commonly used in the literature e.g., [2]), where $E(|h_{XY}(n)|^2) = (\frac{d_{S1D}}{d_{XY}})^\alpha$.

Where d_{XY} stands for the distance between the nodes X and Y , α is the path-loss exponent and $E(\cdot)$ denotes the statistical average operator.

We denote by $\gamma_{X,Y}$ and $\lambda_{X,Y}$ the instantaneous received SNR from the link X - Y and its inverse respectively.

For each source, the transmission is done in two time slots. Concerning $S1$, in the first time slot, it sends its signal to $R1$ and D . Then, in the second time slot, two different models are used. For the first one, denoted by "simple decode and forward" technique, the relay $R1$ directly forwards its data to D . In the second model, denoted by "incremental" technique, the relay waits for a feedback from the destination to decide if it is needed to forward the data based on the comparison of the received SNR for the link $S1D$ to a threshold γ_0 . Then, if two signals are received, they are combined using MRC technique. The same operation is done for $S2$ using time multiplexing (4 total time slots) or frequency multiplexing ($S1$ and $S2$ send on different frequencies).

Fig. 1. System model

Then, the data received from $S1$ and $S2$ are combined at the destination using slepian wolf to recuperate the original data sent by $S1$.

Since for each source, the received signals are independently combined in a first step using MRC, the system model is equivalent to the one presented by figure2 where γ_{eq1} and γ_{eq2} denotes the equivalent received SNRs for the system transmitting $b1$ and $b2$ respectively.

Fig. 2. Equivalent system model

III. OUTAGE ANALYSIS

In this section, we analyze the performance of the described systems in terms of outage probability (denoted by P_{sw}). Where an outage is defined as the event of being outside the slepian wolf admissible rate regions.

The data streams b_1 and b_2 are separately encoded and transmitted to a common receiver in different phases.

Therefore, according to the theorem of Slepian-Wolf, the admissible rate region is constituted by the parts 3, 4, 5 and 6 of the Figure (3) [6]. The errorfree transmission can be guaranteed only if the rate pair falls into this area. For instance, if b_1 is transmitted at the rate R_1 which is equal to its entropy $H(b_1)$, then b_2 can be transmitted at the rate R_2 which is less than its entropy $H(b_2)$, but must be greater than their conditional entropy $H(b_2|b_1)$. More specifically, R_1 and R_2 must meet these equations:

$$\begin{aligned} R_{b1} &\geq H(b_1|b_2) \\ R_{b2} &\geq H(b_2|b_1) \\ R_{b1} + R_{b2} &\geq H(b_1, b_2) \end{aligned}$$

With $H(b_1, b_2)$ designating the entropy of the correlated bit streams b_1 and b_2 .

We adopt in our study symmetric binary sources ($p(1) = p(0) = 0.5$). We have that $H(b_1) = H(b_2) = 1$, $H(b_1|b_2) = H(b_2|b_1) = H(pe)$, $H(b_1, b_2) = 1 + H(pe)$ with $H(pe) = -pe \log_2(pe) - (1 - pe) \log_2(1 - pe)$. The SNR threshold is equal to:

$$\gamma[H] = 2^{R_c \cdot H} - 1$$

Where R_c represents the rate taking into account the channel coding scheme and the modulation format.

Fig. 3. Slepian-wolf admissible regions

Fig 3 shows that the possible rate regions for Slepian-Wolf when two correlated sources are considered. These regions are divided into 7 parts. It is well known that the admissible rate regions for Slepian-Wolf are the parts 3, 4, 5 and 6 [7]. However, in this article and as in [5], part 7 is also included in the admissible region, because the receiver tries only to recover b_1 . Thus, only part 1 and 2 represent the inadmissible rate region. Therefore, the outage probability can be expressed as follows:

$$P_{sw} = P_1 + P_2 \quad (1)$$

where P_1 and P_2 denote respectively the probabilities that the instantaneous SNR pair γ_{eq1} and γ_{eq2} fall in the inadmissible regions 1 and 2.

Mathematically, P_1 is defined as:

$$P_1 = \int_{\gamma_{eq1}=\gamma[0]}^{\gamma[H(b_1|b_2)]} \int_{\gamma_{eq2}=\gamma[0]}^{\gamma[\infty]} f_{\gamma_{eq1} \gamma_{eq2}}(x, y) dx dy \quad (2)$$

where $f_{\gamma_{eq1} \gamma_{eq2}}(x, y)$ denotes the joint pdf of γ_{eq1} and γ_{eq2} . With the assumption that γ_{eq1} and γ_{eq2} are statistically independent, 2 becomes:

$$P_1 = \int_{\gamma[0]}^{\gamma[H(b_1|b_2)]} \int_{\gamma[0]}^{\gamma[\infty]} f_{\gamma_{eq1}}(x) f_{\gamma_{eq2}}(y) dx dy \quad (3)$$

Concerning P_2 , it can be defined as follows

$$P_2 = \int_{\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} \int_{\gamma[0]}^{\gamma[H(b_1, b_2) - H(\gamma_{eq1})]} f_{\gamma_{eq1}}(x) f_{\gamma_{eq2}}(y) dx dy \quad (4)$$

In the rest of this section, we derive approximated outage expressions for the three proposed models based on the equations (1, 3 and 4).

A. Simple DF- Simple DF

In this part, the two relays $R1$ and $R2$ are supposed to forward directly the received data stream independently from the direct links situations.

Concerning the data stream $b1$, if the relay $R1$ decoded successfully $b1$, the destination combines the data coming on the links S_1D and R_1D using MRC . Therefore, the equivalent received SNR for the first source is $\gamma_{eq1} \triangleq \gamma_{MRC1} = \gamma_{S_1,D} + \gamma_{R_1,D}$. Consequently, the pdf of γ_{eq1} becomes in this case:

$$f_{\gamma_{MRC1}}(x) = \alpha_1 (e^{-\lambda_{R_1,D}x} - e^{-\lambda_{S_1,D}x}) \quad (5)$$

where

$$\alpha_1 = \frac{\lambda_{R_1,D} \lambda_{S_1,D}}{\lambda_{R_1,D} - \lambda_{S_1,D}} \quad (6)$$

However, if $R1$ was unable to decode $b1$, it has been proven in [8],[9] that the error probability at the destination can be approximated by 0.5. This is equivalent to say that the channel was so bad that the transmission didn't give any information on $b1$. Therefore, for this case, we can approximate the equivalent SNR by 0. i.e. $\gamma_{eq1} \approx 0$.

Concerning the bit stream $b2$. By analogy to what has been done for $b1$ and according to the success or failure of $R2$ to decode $b2$, the expressions (5 and III-A) are extended to the stream $b2$ by replacing respectively γ_{MRC1} and α_1 by $\gamma_{MRC2} = \gamma_{S_2,D} + \gamma_{R_2,D}$ and $\alpha_2 = \frac{\lambda_{R_2,D} \lambda_{S_2,D}}{\lambda_{R_2,D} - \lambda_{S_2,D}}$.

To get the expression of $P1$, we have to consider the following two cases:

- 1) $H(b_1|b_2) = 0$ i.e. $H(pe) = 0$:

In this case, there will be no longer a region 1 and consequently $P1 = 0$.

- 2) $H(b_1|b_2) \neq 0$:

If the relay $R1$ decode unseccessfully the symbol, the system will be always in the zone 1, therefore

$$P1 = \alpha_1 (1 - P_{S_1R_1}) \left(\frac{1 - e^{-\lambda_{R_1,D} \gamma[H(pe)]}}{\lambda_{R_1,D}} - \frac{1 - e^{-\lambda_{S_1,D} \gamma[H(pe)]}}{\lambda_{S_1,D}} \right) + P_{S_1R_1} \quad (7)$$

where $P_{S_1R_1}$ denotes the probability of error in the relay $R1$. From [10], we get the expression of $P_{S_1R_1}$:

$$P_{S_1R_1} = a \left(1 - \sqrt{\frac{b \cdot \lambda_{S_1R_1}}{1 + b \cdot \lambda_{S_1R_1}}} \right) \quad (8)$$

where (a, b) are constants depending on the type of modulation

Concerning P_2 , we must consider first the two cases where $R2$ decoded successfully or not $b2$:

- 1) Case 1: $R2$ decoded unseccessfully $b2$

In this case, being in zone 2 is equivalent to have $H(b_1|b_2) \leq R_1 \leq H(b_1)$, therefore $P2$ (denoted in this

case by P_{2_1}) becomes:

$$P_{2_1} = \int_{\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} f_{\gamma_{eq1}}(x) dx \\ = \alpha_1 \left(\frac{e^{-\lambda_{R_1,D} \gamma[H(pe)]} - e^{-\lambda_{R_1,D} \gamma[H(b_1)]}}{\lambda_{R_1,D}} - \frac{e^{-\lambda_{S_1,D} \gamma[H(pe)]} - e^{-\lambda_{S_1,D} \gamma[H(b_1)]}}{\lambda_{S_1,D}} \right) \quad (9)$$

- 2) Case 2: If $R2$ decoded successfully the symbol

$$P_{2_2} = \alpha_2 \int_{\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} f_{\gamma_{eq1}}(x) \\ \left(\frac{1 - e^{-\lambda_{R_2,D} \gamma[H(b_1,b_2) - H(x)]}}{\lambda_{R_2,D}} - \frac{1 - e^{-\lambda_{S_2,D} \gamma[H(b_1,b_2) - H(x)]}}{\lambda_{S_2,D}} \right) dx \quad (10)$$

Moreover, $f_{\gamma_{eq1}}(x)$ may take different expressions depending on the success or failure of $R1$ to decode $b1$ and wether $H(pe) = 0$ or not:

Case 2.1 $H(pe) \neq 0$:

Case 2.1.1 $R1$ decoded unsuccessfully the symbol:

the system will not be able to be in zone 2 since $\gamma_{eq2} \approx 0 \leq \gamma[H(pe)]$ and $P_{2,1,1} = 0$

Case 2.1.2 $R1$ decoded successfully the symbol:

By using the expressions of $f_{\gamma_{MRC1}}$ in (eq. 5) and its extension for the source $s2$, P_2 becomes:

$$P_{2,1,2} = \alpha_1 \cdot \alpha_2 \cdot \left[\left(\frac{e^{-x \cdot \lambda_{R_1,D}}}{\lambda_{R_1,D}} + \frac{e^{-x \cdot \lambda_{S_1,D}}}{\lambda_{S_1,D}} \right) \left(\frac{1}{\lambda_{R_2,D}} - \frac{1}{\lambda_{S_2,D}} \right) + \frac{-e^{-\lambda_{R_2,D}} \cdot F_1(\lambda_{R_1,D}, \lambda_{R_2,D} \cdot 2^{R_c \cdot H(b_1,b_2)})}{\lambda_{R_2,D}} + \frac{e^{-\lambda_{R_2,D}} \cdot F_1(\lambda_{S_1,D}, \lambda_{R_2,D} \cdot 2^{R_c \cdot H(b_1,b_2)})}{\lambda_{R_2,D}} - \frac{-e^{-\lambda_{S_2,D}} \cdot F_1(\lambda_{R_1,D}, \lambda_{S_2,D} \cdot 2^{R_c \cdot H(b_1,b_2)})}{\lambda_{S_2,D}} + \frac{e^{-\lambda_{S_2,D}} \cdot F_1(\lambda_{S_1,D}, \lambda_{S_2,D} \cdot 2^{R_c \cdot H(b_1,b_2)})}{\lambda_{S_2,D}} \right] \gamma[H(b_1|b_2)] \\ \triangleq [P_c]_{\gamma[H(b_1)]}^{\gamma[H(b_1|b_2)]}$$

where, $F_1(c1, c2) = \int e^{-c1 \cdot x - \frac{c2}{1+x}} dx$.

Case 2.2 $H(pe) = 0$:

Case 2.2.1 $R1$ decoded unsuccessfully the symbol

Therefore, being in zone 2 is equivalent to $0 \leq R_2 \leq H(b_2)$. Hence, P_2 becomes:

$$P_{2,2,1} = \alpha_2 \left(\frac{1 - e^{-\lambda_{R_2,D} \cdot \gamma[H(b_2)]}}{\lambda_{R_2,D}} - \frac{1 - e^{-\lambda_{S_2,D} \cdot \gamma[H(b_2)]}}{\lambda_{S_2,D}} \right)$$

Case 2.2.2 $R1$ decoded successfully the symbol, Nothing changes compared to the case 2.1.2. Therefore,

$$P_{2,2,2} = P_{2,1,2}$$

Finally, by combining the different expressions of P_2 for the different cases, we conclude its genral expression:

If $H(pe) = 0$:

$$P_2 = P_{SR_2}P_{2_1} + (1 - P_{SR_2}) (P_{SR_1}P_{2_{2,2,1}} + (1 - P_{SR_1}) P_{2_{2,2,2}})$$

If $H(pe) \neq 0$:

$$P_2 = P_{SR_2}P_{2_1} + (1 - P_{SR_2}) (1 - P_{SR_1}) P_{2_{2,1,2}}$$

B. incremental DF-simple DF

To reduce the effect of error propagation caused by the use of DF technique, we assume in this section that the first relay R_1 uses incremental technique and that the second one R_2 still uses just decode and forward technique. Therefore, $f_{\gamma_{eq2}}$ still have the same expression as (5).

Concerning the data stream b_1 , we proved in the previous part that γ_{eq1} can be approximated by 0 if the relay R_1 decoded unsuccessfully the symbol, therefore, to simplify the analysis, we suppose that in such conditions, $f_{\gamma_{eq1}}(x) = \frac{1_{[0,\epsilon]}(x)}{\epsilon}$, where $\epsilon \ll 1$. Therefore, the general expression of $f_{\gamma_{eq1}}(x)$ becomes:

$$\begin{aligned} f_{\gamma_{eq1}}(x) &= P(\gamma_{S_1,D} \leq \gamma_0)(1 - P_{S_1,R_1})f_{\gamma_{MRC_1}}(x) \\ &+ P(\gamma_{S_1,D} \leq \gamma_0)P_{S_1,R_1}f_{\gamma_{eq1}}(x) \\ &+ P(\gamma_{S_1,D} > \gamma_0)f_{\gamma_{S_1,D}}(x|\gamma_{S_1,D} > \gamma_0) \\ &= 1_{[\gamma_0,\infty]}(x)\lambda_{S_1,D}e^{-\lambda_{S_1,D}x} \\ &+ \beta_1(1_{[0,\gamma_0]}(x)(e^{-\lambda_{R_1,D}x} - e^{-\lambda_{S_1,D}x}) \\ &+ 1_{[\gamma_0,\infty]}(x)\beta_2e^{-\lambda_{R_1,D}x}) + \frac{1_{[0,\epsilon]}(x) \cdot \beta_3}{\epsilon} \end{aligned}$$

where

$$\begin{aligned} \beta_1 &= \frac{(1 - P_{S,R_1})\lambda_{S_1,D}\lambda_{R_1,D}}{\lambda_{S_1,D} - \lambda_{R_1,D}} \\ \beta_2 &= 1 - e^{(\lambda_{S_1,D} - \lambda_{R_1,D}) \cdot \gamma_0} \\ \beta_3 &= P_{S_1,R}(1 - e^{\lambda_{S_1,D} \cdot \gamma_0}) \end{aligned}$$

To get the expression of P_1 , two cases must be considered:

Case 1 If $\gamma_0 \leq \gamma[H(b_1|b_2)]$

$$\begin{aligned} P_1 &= (e^{-\lambda_{S_1,D}\gamma_0} - e^{-\lambda_{S_1,D}\gamma[H(b_1|b_2)]}) + \\ &\beta_1\left(\frac{(1 - e^{-\lambda_{R_1,D}\gamma_0})}{\lambda_{R_1,D}} - \frac{(1 - e^{-\lambda_{S_1,D}\gamma[H(b_1|b_2)]})}{\lambda_{S_1,D}}\right) + \\ &\beta_2\left(\frac{e^{-\lambda_{R_1,D}\gamma_0} - e^{-\lambda_{R_1,D}\gamma[H(b_1|b_2)]}}{\lambda_{R_1,D}}\right) + \beta_3 \end{aligned} \quad (11)$$

Case 2 If $\gamma_0 > \gamma[H(b_1|b_2)]$

$$\begin{aligned} P_1 &= \beta_1\left(\frac{(1 - e^{-\lambda_{R_1,D}\gamma[H(b_1|b_2)]})}{\lambda_{R_1,D}} - \frac{(1 - e^{-\lambda_{S_1,D}\gamma[H(b_1|b_2)]})}{\lambda_{S_1,D}}\right) \\ &+ \beta_3 \end{aligned}$$

Concerning P_2 , two major cases may be considered:

Case 1 If $\gamma[H(b_1, b_2) - H(x)] = 0$

In this case $x = \gamma[H(b_1, b_2)]$, it is an impossible case since $\gamma[H(b_1, b_2)] > \gamma[H(b_1)] = 1$

Case 2 If $\gamma[H(b_1, b_2) - H(x)] \neq 0$

Case 2.1 If $H(b_1|b_2) \neq 0$ (To simplify the analysis, only this case is considered and the $case2.2H(b_1|b_2) = 0$ is not taken into account)

Case 2.1.1 If $\gamma_0 \leq \gamma[H(b_1|b_2)]$

$$\begin{aligned} P_2 &= [(1 - P_{SR_2})\alpha_2((e^{-\lambda_{S_1,D}x} - e^{-\lambda_{R_1,D}x})\frac{\beta_1 \cdot \beta_2}{\lambda_{S_1,D}}) \\ &+ (\frac{1}{\lambda_{R_2,D}} - \frac{1}{\lambda_{S_2,D}}) - \frac{\lambda_{S_1,D}}{\lambda_{R_2,D}}e^{-\lambda_{R_2,D}}F_1(\lambda_{S_1,D}, \lambda_{R_2,D}2^{R_c}H[b_1, b_2]) \\ &+ \frac{\beta_1\beta_2}{\lambda_{R_2,D}}e^{-\lambda_{R_2,D}}F_1(\lambda_{R_1,D}, \lambda_{R_2,D}2^{R_c}H[b_1, b_2]) \\ &+ (\frac{\lambda_{S_1,D}}{\lambda_{S_2,D}}e^{-\lambda_{S_2,D}}F_1(\lambda_{S_1,D}, \lambda_{S_2,D}2^{R_c}H[b_1, b_2]) \\ &+ \frac{\beta_1\beta_2}{\lambda_{S_2,D}}e^{-\lambda_{S_2,D}}F_1(\lambda_{R_1,D}, \lambda_{S_2,D}2^{R_c}H[b_1, b_2]))] \\ &+ P_{S_2,R}(e^{-\lambda_{S_1,D}x} - \frac{\beta_1\beta_2}{\lambda_{R_1,D}}e^{-\lambda_{R_1,D}x}))^{\gamma[H(b_1)]}_{\gamma[H(b_1|b_2)]} \\ &\triangleq [P_a]_{\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} \end{aligned}$$

Case 2.1.2 If $\gamma_0 > \gamma[H(b_1)]$

$$\begin{aligned} P_2 &= [((1 - P_{S,R_2})\alpha_2\beta_1((\frac{1}{\lambda_{R_1,D}}e^{-\lambda_{R_1,D}x} \\ &+ \frac{1}{\lambda_{S_1,D}}e^{-\lambda_{S_1,D}x})(\frac{1}{\lambda_{R_2,D}} - \frac{1}{\lambda_{S_2,D}}) \\ &- \frac{e^{\lambda_{R_2,D}}}{\lambda_{R_2,D}}F_1(\lambda_{R_1,D}, \lambda_{R_2,D}2^{R_c}H[b_1, b_2]) \\ &- \frac{e^{\lambda_{R_2,D}}}{\lambda_{R_2,D}}F_1(\lambda_{S_1,D}, \lambda_{R_2,D}2^{R_c}H[b_1, b_2])) \\ &+ (\frac{e^{\lambda_{S_2,D}}}{\lambda_{S_2,D}}F_1(\lambda_{R_1,D}, \lambda_{S_2,D}2^{R_c}H[b_1, b_2]) \\ &- \frac{e^{\lambda_{S_2,D}}}{\lambda_{S_2,D}}F_1(\lambda_{S_1,D}, \lambda_{S_2,D}2^{R_c}H[b_1, b_2])) \\ &+ \beta_1 \cdot P_{S_2,R}(\frac{1}{\lambda_{S_1,D}}e^{-\lambda_{S_1,D}x} \\ &- (\frac{1}{\lambda_{R_1,D}}e^{-\lambda_{R_1,D}x}))^{\gamma[H(b_1)]}_{\gamma[H(b_1|b_2)]} \\ &\triangleq [P_b]_{\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} \end{aligned}$$

Case 2.1.3 If $\gamma[H(b_1)] \geq \gamma_0 > \gamma[H(b_1|b_2)]$

$$P_2 = [P_a]_{\gamma_0}^{\gamma[H(b_1)]} + [P_b]_{\gamma[H(b_1|b_2)]}^{\gamma_0} \quad (12)$$

C. Incremental DF- Incremental DF

In this section, the transmission from R_1 and R_2 to D are supposed to be conditioned on the feedback of the destination. Since the two relays are using incremental technique, $f_{eq1}(x)$ keeps the same expression as in (11) and $f_{eq2}(x)$ is obtained by replacing respectively β_1, β_2 and β_3 by β'_1, β'_2 and β'_3 defined by:

$$\begin{aligned} \beta'_1 &= \frac{(1 - P_{S_2,R_2})\lambda_{S_2,D}\lambda_{R_2,D}}{\lambda_{S_2,D} - \lambda_{R_2,D}} \\ \beta'_2 &= 1 - e^{(\lambda_{S_2,D} - \lambda_{R_2,D}) \cdot \gamma_0} \\ \beta'_3 &= P_{S_2,R}(1 - e^{\lambda_{S_2,D} \cdot \gamma_0}) \end{aligned}$$

The expressions of $f_{\gamma_{eq1}}(x)$ and $f_{\gamma_{eq2}}(x)$ depends on γ_0 and will include a lot of cases, especially for the calculations of P_2 .

Therefore, we suppose that $\gamma_0 \geq \gamma[H(b_1)] = \gamma[H(b_2)] = 1$. This supposition is logic because too low values of γ_0 refers to a system with "simple relaying-simple relaying" which is a system that we already studied in the first part. P_1 has the same expression present in (3), and it is equals to:

$$P_1 = \beta_1 \left(\frac{1 - e^{-\lambda_{R_1,D} \gamma[H(b_1|b_2)]}}{\lambda_{R_1,D}} - \frac{1 - e^{-\lambda_{S_1,D} \gamma[H(b_1|b_2)]}}{\lambda_{S_1,D}} \right) + \beta_3 \quad (13)$$

Since we supposed that $\gamma_0 \geq \gamma[H(b_1)] = \gamma[H(b_2)] = 1$, we have that $\gamma[H(b_1, b_2) - H(x)] \leq \gamma_0$. Therefore

$$P_2 = \int_{\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} \left(\beta_1 \beta'_1 \left(\frac{1 - e^{-\lambda_{R_2,D} \gamma[H(b_1|b_2) - H(x)]}}{\lambda_{R_2,D}} - \frac{1 - e^{-\lambda_{S_2,D} \gamma[H(b_1|b_2) - H(x)]}}{\lambda_{S_2,D}} \right) \left(\frac{1 - e^{-\lambda_{R_1,D} \gamma[H(b_1|b_2) - H(x)]}}{\lambda_{R_1,D}} - \frac{1 - e^{-\lambda_{S_1,D} \gamma[H(b_1|b_2) - H(x)]}}{\lambda_{S_1,D}} \right) + \beta_1 \beta'_3 \left(\frac{1 - e^{-\lambda_{R_1,D} \gamma[H(b_1|b_2) - H(x)]}}{\lambda_{R_1,D}} - \frac{1 - e^{-\lambda_{S_1,D} \gamma[H(b_1|b_2) - H(x)]}}{\lambda_{S_1,D}} \right) \right) dx \quad (14)$$

For the simulation, this expression is evaluated using the trapezoidal numerical integration method .

IV. SIMULATION RESULTS

This section presents the simulation results which validate the analysis for the different studied scenarios where the relays use incremental relaying technique or not. Asymmetric network geometry is examined where the relay is located across the straight line connecting the source and the destination.

Fig.4 presents The slepian-wolf outage probability of a "simple DF -simple DF" system for different values of pe . We can remark that for low SNR, this system outperforms MRC technique with two branches. It is seen also that the smaller pe is, the better results we get in terms of outage probability. In fact, having small values of pe means that the two sources are sending approximately the same data which give more chance for the destination to decode b_1 . However, for high SNR, this system no longer outperforms MRC and gives the same outage for the different values of pe . In fact, this is a result of the error propagation incremental transmission technique in the relay R_1 based on the threshold γ_0 .

Figure 5 shows the effect of the forwarding threshold γ_0 on the outage probability of the system. Very high γ_0 means that the destination never asks from the relay R_1 to forward its data which will be equivalent to have a direct link transmission for S_1 . However for very low γ_0 , R_1 will always forward its data. Therefore, the system gives better outage performances for low γ_0 compared to high γ_0 which can Remarque by Figure 5. However, for a particular value of γ_0 , the system gives lower outage probability than a system where the relay always forward his data. Therefore, we conclude that the choice of γ_0 can control when R_1 forwards useful information or just

Fig. 4. The outage probability of a "simple relaying -simple relaying" system for different values of pe

Fig. 5. effect of γ_0 on the outage probability of an "incremental relaying -simple relaying" system for different values of pe and for $Eb/N_0 = 15dB$

cause error propagation. This best value of γ_0 is variable and depends on pe . However, particularly when $\frac{Eb}{N_0} = 15dB$, the variation of the optimal threshold isn't important (range from 0dB to 2dB). Therefore, in Figure 6, we fixed γ_0 to be 2dB and plotted the outage probability in function of the received SNR for different values of pe .

Fig. 6. The outage probability of a "incremental relaying-simple relaying" system for $\gamma_0 = 2dB$ for different values of p_e

Compared to Figure 4, we can see in 6 that adding the incremental transmission technique to $R1$, allowed the system to outperform MRC system not only for low SNR (as for simple DF- simple DF) but also for high SNR. Figure 6 shows also that the new system has the same diversity order as an MRC system (2) which wasn't the case when $R1$ always forwards its data.

In figure 7, we added the incremental technique also to the relay $R2$, no significant changes can be seen compared to a system when only $R1$ uses the incremental technique. Therefore we conclude that incremental technique is only benefic if used in the relay $R1$ since it is carrying the data that we aim to recover $b1$.

V. CONCLUSIONS

In this work, the outage probability of a correlated source transmission system based on the Slepian-Wolf theorem has been analyzed for simple DF-simple DF, incremental DF-simple DF and incremental DF-incremental DF models. The bit-flipping model was used to express the correlation model between the two sources, and the close form expressions of the outage probability were theoretically calculated assuming that the receiver aims to decode $b1$. We showed that simple DF-simple DF system, outperforms MRC with 2 links only for low SNR. However, when incremental relaying incremental relaying technique is used in the principal relay, beside allowing channel resource saving the outage improvement remains also for high SNR with a significant power gain compared to MRC. We showed also that the choice of the threshold is very important for the incremental transmission performance.

Fig. 7. The outage probability of an "incremental DF-incremental DF" system for $\gamma_0 = 2dB$ for different values of p_e

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