

SYNCHRONIZED CHAOTIC SIGNALS AND SYSTEMS

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Abstract Recent work using chaotic signals to drive nonlinear systems shows that chaotic dynamics is rich in new application possibilities. Among these are stable system design and synchronization.

New Driving Signals

Driven systems are easily visualized as dynamical systems which have as one of their input parameters a dynamical variable from another, often autonomous, dynamical system. We often refer to the source of the driving signal as the *drive* system and to the driven system as the *response* system. This can be viewed as the drive sending a signal to the response which then alters its behavior according to the signal. Typically, when driven systems are studied or engineered the driving signals come from constant forces or sine wave forcing. The use of signals from a chaotic system to drive a nonlinear system offers a new type of driving signal.

In our approach [1,2,3,4] two major themes stand out. One is the idea of stability as generalized to chaotic systems. Another is the use of a constructive approach to building useful, chaotically driven systems. We cut apart, duplicate, and paste together nonlinear dynamical systems. Many things can be done with some guidance from what is now known in nonlinear dynamics.

We first examine stability.

Stability of Chaotically Driven Systems

Consider a general n -dimensional, nonlinear response system, $w = h(w, v)$, where the $w = dw/dt$, the driving signal v is supplied by a chaotic system and w and h are n -dimensional vector functions. The question of stability arises when we ask: given a trajectory $w(t)$ generated by this system for a particular drive v , when is

$w(t)$ immune to small differences in initial conditions, i.e. when is the final trajectory unique, in some sense? Fig. 1 shows this schematically.

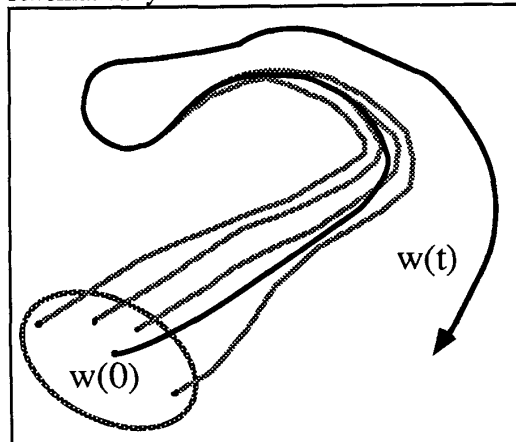


Fig. 1. Nearby trajectories converging to $w(t)$.

To appreciate how this applies to chaotically driven responses, consider two responses w and w' started at slightly different points in phase space ($w'(0) = w(0) + \Delta w(0)$). We want to know the conditions under which $w'(t) \rightarrow w(t)$ as $t \rightarrow \infty$. Assume that $\Delta w(0)$ is small and subtract the equations of motion for w' and w . This gives an equation of motion for Δw : $\Delta w = D_w h(w, v) + \phi(w, v)$, where $D_w h$ is the Jacobian of h with respect to w and ϕ represents the remainder of the terms. The question of stability is now when does $\Delta w \rightarrow 0$?

Typically, a linear stability analysis invoked in this case employs the fundamental theorem of linear stability [5]. If we can show that $\Delta w \rightarrow 0$ we have the result that, despite a chaotic drive, the w system will always settle into the same

trajectory and the points on that trajectory will eventually be at the same place at the same time for any initial condition in an open set around $w(0)$. Note that this does not mean that the trajectory will be simple, only that the w system will fall into the same trajectory (pattern) given the same drive, despite differences in starting points.

To do this stability analysis for a chaotic drive (or any system with chaotic motion) we need to calculate the Lyapunov exponents [6] of the w system. These exponents λ measure the rate ($\sim e^{\lambda t}$) at which nearby points converge or diverge in a dynamical system. They will, in general depend on the drive v . When the spectrum of exponents is all negative, then we have stability of the w system with respect to the drive $v(t)$. This gives us a new tool for building stable, chaotically driven systems.

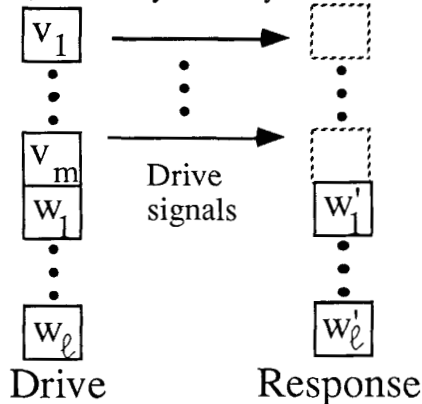


Fig. 2. Schematic of Building a driven subsystem.

Synchronization of Chaotically Driven Systems

It is impossible to synchronize two isolated chaotic systems. So, we consider not two isolated systems, but one whole compound system made from an original chaotic system and a duplicate of a *subsystem* of the original [1,2]. If the subsystem is a stable subsystem in the above terms, we have synchronizable systems by driving the duplicate with signals from the original system that are absent in the duplicate. This is shown schematically in Fig. 2.

The concept of stable subsystems is rather new and non-trivial in nonlinear systems. Note that the Lyapunov exponent spectrum for a subsystem of a nonlinear system is generally not a subset of the Lyapunov spectrum for the

whole system. Finding stable subsystems is therefore also a non-trivial problem. Testing the (conditional) Lyapunov exponents of the subsystems is the only way known for now.

The equations of motion for these synchronized systems are easily derived and various results about them appear in Refs. [1,2]. In order to test all these ideas we performed numerical tests using this scheme with Lorenz, Rössler, and hysteretic Newcomb systems. The integration was done with a Runge-Kutta 4-5 scheme with adaptable step size.

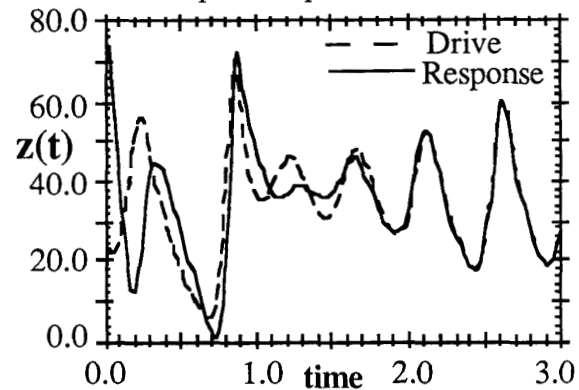


Fig. 3. Convergence of the z -component of the response to the drive z component.

For the Lorenz system in the chaotic regime ($\sigma=10$, $r=24.06$ to > 200.0 , and $b=8/3$)

$$\begin{aligned} \dot{x} &= -\sigma(x-y) \\ \dot{y} &= r x - y - xz \\ \dot{z} &= xy - bz \end{aligned} \quad \begin{array}{l} \text{stable} \\ \text{subsystem} \end{array}$$

calculations of the conditional Lyapunov exponents for two-dimensional subsystems show that we can construct a stable subsystem by choosing either $w'=(x,z)$ or $w'=(y,z)$ [2]. Fig. 3 shows the convergence of a time series for the x driven (y,z) response. Fig. 4 shows the same results for the trajectory of the (y,z) subsystem.

We tested the robustness of this synchronization against parameter variation by changing the parameter of the response and drives by various amounts. This is logical, since if we are to build such devices we will always have parameter differences. Our results show that parameter variations will degrade the synchronization, but small differences on the order of 10% in most systems will not eliminate it. We show that there is a mathematical reason for this [1].

Results for the Rössler chaotic system show that in the chaotic regime usually only one subsystem is stable (the (x,z) subsystem) and occasionally, for certain parameter regimes, none of the Rössler subsystems are stable.

To test the practical application of these concepts we built an electronic circuit based on the hysteretic circuit of Newcomb [7]. Details of this circuit are published elsewhere [3].

Lorenz (yz) Drive

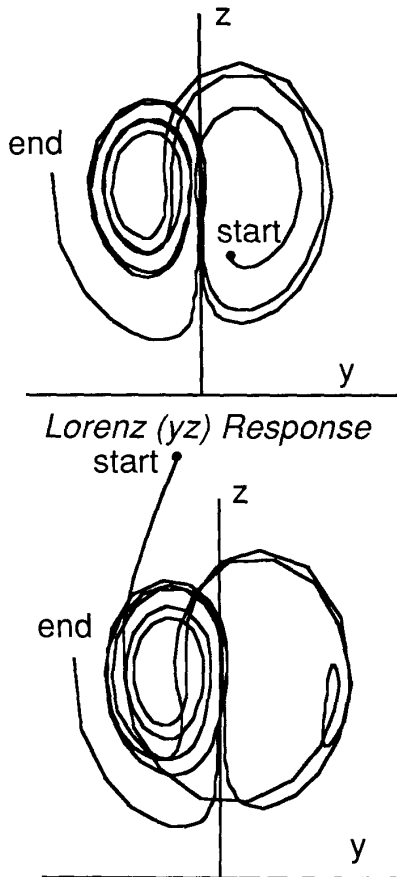


Fig. 4. Trajectories of the drive and response (yz) subsystems during convergence.

This circuit had a stable subsystem consisting mostly of the unstable oscillator subcircuit. We constructed our response from a circuit with this subcircuit in it. An easy way to demonstrate synchronization in a pair of circuits is to display voltages on the oscilloscope from the same points the the stable subsystems, using one as the x and the other as

the y scope drives. This directly shows that synchronized chaotic devices can be built and that the synchronization is robust (Fig.5).

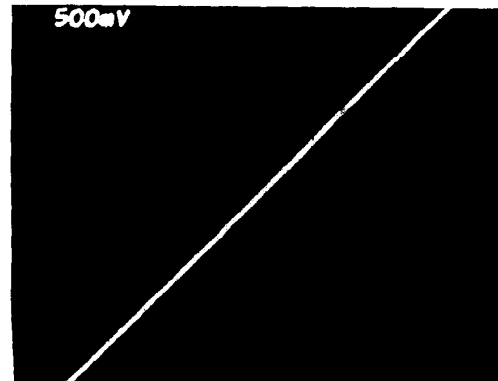


Fig. 5. voltages in drive vs. response.

Applications to Signal Processing

These results are recent and surely must be developed further to define the concepts and methods necessary for engineering such systems in actual applications. However, we speculate here, rather generally, on a few applications.

The synchronization of two chaotic systems raises the question: Could chaotic signals be used to hide signals or encrypt information? One could envision a sender "adding" chaos from a stable subsystem in some way to his signal and sending the sum to a receiver. The receiver has a duplicate of the stable subsystem of the sender's equipment. At the same time the sender may send another chaotic signal to cause (drive) the receiver's equipment to synchronize with the sender's subsystem. The receiver now has the means to "strip" away the chaos and reveal the information.

This is obviously oversimplified. Many questions need to be answered to guarantee the security of the method. Can information signals be buried in a chaotic signal in an undetectable way? Research by Marteau and Abarbanel [8] suggest that simple addition of signals may be a flawed approach, but results of testing so far are equivocal. Embedding techniques [9] show that certain topological information about an entire system can be extracted from one of its signals. Can enough information about the stable subsystem's signal be extracted from the driving signal to allow an interceptor to strip away the chaos and reveal the hidden information? Can this all be done with one signal? How do the usual

signal processing concepts (e.g. Shannon entropy and rates) apply to chaotic carriers?

If we "step back" from the synchronization application and think again just about stability, we see that we have found several systems and circuits that do indeed show a stable, predictable behavior to input signals. This leads to the possible use of nonlinear systems as signal identifiers and converters (filters). If we can design systems which behave in certain ways (patterns) for specific chaotic or complex signals, we have a system which can identify that signal.

We take a simple look at this by driving a stable subsystem. This subsystem is the y-z Lorenz subsystem we examined above and x is the drive. We examined several cases of replacing x with drives from other systems. Here we show the results from two other cases.

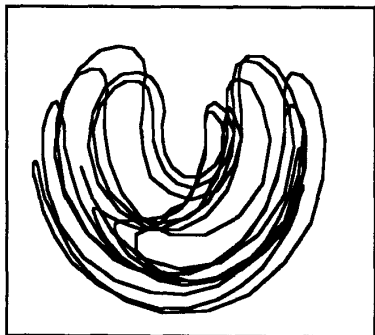


Fig. 6. Response of Lorenz y-z to Rössler chaos.

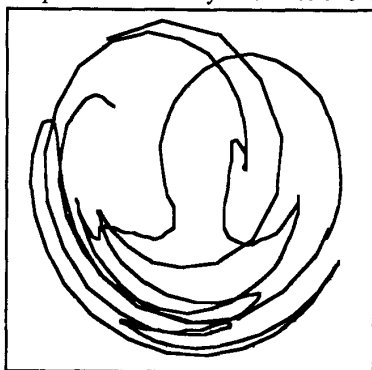


Fig. 7. Response of Lorenz y-z to spectrally scaled Rössler chaos.

Fig. 4 shows the y-z pattern (trajectory) that obtains when x comes from the Lorenz system. Fig. 6 shows the pattern that emerges when x comes from the x-component of the Rössler chaotic system. This is distinctly different from the pattern caused by the chaotic Lorenz signal. The Rössler signal does have a distinct spectrum

from the Lorenz, so to compensate for this we scaled the Rössler power spectra to the Lorenz power spectra. This gives a more complex chaotic drive and produces the pattern in Fig. 7. Note that despite the spectra similarity, the y-z stable system produces a distinctly different pattern from the Lorenz. Other tests with other chaotic drive signals show similar results in that each signal has a distinct effect on the stable y-z response.

Can we now learn how to design specially tuned stable systems to be robust signal identifiers? We are presently exploring this possibility.

Conclusions

Once one begins to use chaotic driving, deficiencies in the theory immediately show up. For one thing, until now, almost no one has considered chaotic drives. There is much to read about periodic driving, but almost nothing about chaotic driving.

The mathematical questions that come up are deeply related to other concepts in nonlinear dynamics. Some of these questions relating to signal processing were asked above. There are many others.

Certainly, we believe that the research presented here is merely scratching the surface of applications using chaotic driving. One can now design all sorts of interconnected systems made from autonomous systems or subsystems. The number of permutations are impossible for any one researcher to investigate and the number of different nonlinear behaviors must surely be likewise. We feel that in this field these new systems and behaviors will find uses alongside of the standard approaches of linear systems.

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