THE EXTRACTION OF RANGE AND DOPPLER FROM FOURTH-ORDER SPECTRA

E1.9

Roger F. Dwyer
Naval Underwater Systems Center
New London, CT 06320

ABSTRACT

A new method is proposed to extract range and Doppler information from a Doppler-spread active sonar echo. The Doppler spreading is based on Van Trees' model and the new method is based on the fourth-order cumulant spectrum. Specifically, from the envelope of an amplitude modulated coded pulse train the spectrum and fourth-order cumulant spectrum are derived. It is demonstrated by analysis and simulation that due to Doppler-spreading the conventional method cannot extract range and Doppler shift frequencies of the return pulse without degradation. However, under the same conditions the new method based on the fourth-order cumulant spectrum can. It is also shown that a special case of the new method is independent of the covariance of the Doppler spreading function.

1. Introduction

The fourth-order spectrum represents a new way of extracting information from sonar data. Used in conjunction with the second-order spectrum the fourth-order spectrum reveals otherwise hidden relationships that are important for detection and classification of sonar signals. In addition, the fourth-order spectrum can extract range and Doppler information from an active sonar return under conditions which render the conventional second-order spectrum severely degraded. The first application of the fourth-order cumulant spectrum to the range-Doppler extraction problem was given in [1]. Another reason for applying the fourth-order cumulant spectrum to this problem is that, theoretically the fourth-order spectrum eliminates additive Gaussian noise from its final result [2,3]. For the pulse lengths of this paper this property was not verified. For longer pulse lengths additional averaging may improve the estimates enough to exploit this property.

The paper will concentrate on a coded pulse (CP) waveform [4], since it includes the CW waveform as a special case. It must be emphasized that the objective of the paper is not the design of efficient codes but rather to discuss the properties of the fourth-order spectrum for a particular code. The design of efficient codes can be found in [5,6, and 7] and references therein.

First, the envelope of an amplitude modulated sonar return will be extracted. Then the autocorrelation function and fourth-order cumulant function will be derived. Finally, the second-order spectrum and the fourth-order cumulant spectrum will be obtained. The narrow-band assumption is assumed in the analysis and in the simulations. This assumption implies that target Doppler will shift all echoes by the same amount [5]. The justification for this assumption is discussed by Costas [5]. Based on this assumption, the analysis and simulation results show that the fourth-order method, based on the fourth-order cumulant spectrum, can extract range and Doppler information from a modulated return. The modulation is of course unknown at the receiver.

2. Coded Pulse Transmission System.

In this section two different ways of processing the return of a transmitted CP waveform will be compared. The first method is based on the spectrum of the envelope of the CP waveform. Whereas, the new method computes the special case of the fourth-order cumulant spectrum of the envelope of the CP waveform.

Let,

$$y(t) = \sum_{n=-k}^k h(t - nT_p) \cos(\omega_n t + \theta(t)), \quad (2.1)$$

$-\infty \leq t \leq \infty$, be the real CP transmitted signal of length, $2kT_p + T$. The subpulses are defined as follows,

$$h(t - nT_p) = u(t - nT_p + \frac{T}{2}) - u(t - nT_p - \frac{T}{2}), \quad (2.2)$$

where $u()$ is the unit step function. The parameters, T and T_p , for $T \leq T_p$, represent the pulse width and pulse repetition interval, respectively.

The received signal is assumed to be of the form

$$z'(t) = a(t - \frac{T_R}{2}) \sum_{n=-k}^k h(t - nT_p - T_R) \left(\cos[(\omega_n + \omega_{nd})(t - T_R) + \theta(t) + \phi] \right), \quad (2.3)$$

where, $T_R = 2R/c$ is the two way propagation delay to the target, c is the propagation velocity, R is the range, ω_{nd} is the Doppler shift radian frequency associated with each transmitted radian frequency ω_n , $\theta(t)$ is a random phase function, and ϕ is a random phase angle uniformly distributed between 0 and 2π . The real stochastic Gaussian modulating function $a(t)$ represents a model for a time fluctuating target[4]. According to Van Tress[4], this model applies if the pulse length of the transmitted signal is longer than the reciprocal of the reflection process.

A match filter of the transmitted signal is implemented as a correlator[8] and the output is lowpass filtered. Under the bandlimited assumption of Rihaczek[9], and [10], the envelope of the received signal,

$$z(t, T_R, T'_R) = a(t - \frac{T_R}{2}) \sum_{n_1=-k}^k \sum_{n_2=-k}^k h(t - n_1 T_P - T_R) h(t - n_2 T_P - T'_R) \left(e^{j[(\omega_{n_1} + \omega_d)(t - T_R) + \theta(t - T_R) + \phi]} e^{-j[\omega_{n_2}(t - T'_R) + \theta(t - T'_R)]} \right), \quad (2.4)$$

is in general a function of T_R and T'_R . When, $|T_R - T'_R| > 2kT_P + T$, $z(t, T_R, T'_R) = 0$. Therefore, $z(t, T_R, T'_R)$, determines the range to the target. In practice an efficient code can be designed[5,6,7] to give a peak at the true range and Doppler of the target while minimizing spurious peaks at other range-Doppler locations. However, since the Doppler information is our primary concern it will be assumed in the analysis that $T_R = T'_R$, ie, the range is known. The method is demonstrated in the paper with CW waveform with a random phase function. It should also be mentioned that the processing method of this paper is quite different from the ambiguity diagram[5]. Also, here, the data is random and, therefore, averaging is included. The averaging is done in the time domain rather than the frequency domain, because a special case requires estimation in the time domain. Nikias discusses the two approaches in some detail in [11].

Therefore, since the pulses are disjoint, ie,

$$h(t - n_1 T_P) h(t - n_2 T_P) = h(t - n_1 T_P), n_1 = n_2$$

$$h(t - n_1 T_P) h(t - n_2 T_P) = 0, n_1 \neq n_2$$

the envelope of the received signal at T_R , is approximately,

$$z(t) = a(t - \frac{T_R}{2}) \sum_{n=-k}^k h(t - n T_P - T_R) e^{j[\omega_{nd}(t - T_R) + \phi]}. \quad (2.5)$$

As stated in the beginning of this section the objective is to compare the autocorrelation function of (2.5) and its corresponding spectrum with the fourth-order cumulant function of (2.5) and its corresponding spectrum.

Let,

$$\hat{R}_z(\tau) = \int_{-\infty}^{\infty} z(t) z^*(t + \tau) dt, \quad (2.6)$$

be the estimate of the autocorrelation function. The spectrum of $\hat{R}_z(\tau)$ is,

$$\hat{S}_z(\omega) = \int_{-\infty}^{\infty} \hat{R}_z(\tau) e^{-j\omega\tau} d\tau. \quad (2.7)$$

From a transformation of variables, $t = t_1$, and $t_2 = t_1 + \tau$, (2.7) can be expressed as follows,

$$\hat{S}_z(\omega) = \sum_{n_1=-k}^k \sum_{n_2=-k}^k e^{-j(\omega_{n_2} t_2 - \omega_{n_1} t_1) T_R} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(a(t_1 - \frac{T_R}{2}) a(t_2 - \frac{T_R}{2}) h(t_1 - n_1 T_P - T_R) h(t_2 - n_2 T_P - T_R) e^{j(\omega - \omega_{n_1} t_1) t_1} e^{-j(\omega - \omega_{n_2} t_2) t_2} \right) dt_1 dt_2. \quad (2.8)$$

Taking the expected value of (2.8) and defining,

$$R_a(t_2 - t_1) = E[a(t_1 - \frac{T_R}{2}) a(t_2 - \frac{T_R}{2})], \quad (2.9)$$

(2.8) reduces to,

$$E[\hat{S}_z(\omega)] = S_z(\omega) = \sum_{n_1=-k}^k \sum_{n_2=-k}^k e^{-j(\omega_{n_2} t_2 - \omega_{n_1} t_1) T_R} \int_{-\infty}^{\infty} h(t_1 - n_1 T_P - T_R) e^{j(\omega - \omega_{n_1} t_1) t_1} \int_{-\infty}^{\infty} R_a(t_2 - t_1) h(t_2 - n_2 T_P - T_R) e^{-j(\omega - \omega_{n_2} t_2) t_2} dt_2 dt_1. \quad (2.10)$$

The modulating function, $R_a(t_2 - t_1)$, can be a non-white Gaussian process. As was shown in [1], the form of this function is not needed for the evaluation of the special case of fourth-order cumulant spectrum. However, the evaluation of (2.10) and also the evaluation of the general fourth-order cumulant spectrum depend on the form of $R_a(t_2 - t_1)$. Therefore, let,

$$R_a(t_2 - t_1) = \text{Var}(a) \frac{\alpha}{2} e^{-\alpha|t_2 - t_1|}, \quad (2.11)$$

where, $\alpha > 0$.

The general result of the evaluation of (2.10) for the above modulating function has been derived by the author. However, here, (2.10) will be evaluated for the limiting form of the modulating function. It is easy to show that $R_a(t_2 - t_1)$ has the properties of a delta function as $\alpha \rightarrow \infty$. Therefore, as $\alpha \rightarrow \infty$,

$$R_a(t_2 - t_1) \rightarrow \text{Var}(a)\delta(t_2 - t_1). \quad (2.12)$$

This represents stationary white Gaussian noise.

Under the stationary Gaussian white noise assumption for the modulating function, the spectrum of (2.10) reduces to

$$S(T_R, \omega) = \text{Var}(a)T(2k + 1), \quad (2.13)$$

which is a constant. Also, any losses or gains can be incorporated into the variance of (2.13). This clearly shows the interfering effect of the modulating white noise.

Proceeding in a similar way as above, the fourth-order cumulant spectrum [12,13] at T_R is,

$$C_{4z}(\omega_1, \omega_2, \omega_3) = \sum_{n_1=-k}^k \sum_{n_2=-k}^k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(R_a(t_2 - t_1) h(t_2 - n_2 T_P - T_R) h(t_1 - n_3 T_P - T_R) e^{-j(\omega_2 + \omega_d)t_1} e^{-j(\omega_3 + \omega_d)t_2} \right) dt_2 dt_1, \quad (2.14)$$

where, the transformation, $t_1 = t$, $t_2 = t_1 + \tau_1$, $t_3 = t_1 + \tau_2$, $t_4 = t_1 + \tau_3$, was used to derive (2.14) and the expected value taken.

The general expression for (2.14) has also been derived. However, as needed here, the limiting form of (2.14) is,

$$C_{4z}(\omega_1, \omega_2, \omega_3) = \text{Var}(a)^2 T^2 \left[\frac{\sin[(\omega_2 + \omega_d + 2\omega_d)\frac{T}{2}]}{(\omega_2 + \omega_3 + 2\omega_d)\frac{T}{2}} \right]^2 \left[\frac{\sin[(\omega_2 + \omega_3 + 2\omega_d)\frac{T}{2}]}{\sin[(\omega_2 + \omega_3 + 2\omega_d)\frac{T}{2}]} \right]^2, \quad (2.15)$$

which is not a function of ω_1 . There are many solutions to (2.15), however the peaks are reduced by the $[\frac{\sin(x)}{x}]^2$ function except at, $\omega_2 + \omega_3 = -2\omega_d$, which is,

$$C_{4z}(\omega_2, -2\omega_d - \omega_3) = [T(2k + 1)]^2. \quad (2.16)$$

This is the result derived by the author in [1]. In the simulations, $T_P = T$, so there are no other solutions. As can be seen, the spectrum (2.13) is not a function of the Doppler frequency, whereas, the fourth-order cu-

mulant spectrum (2.15) gives a solution at the Doppler frequency.

3. RESULTS

In order to demonstrate the results of the paper, the received signal and the corresponding matched filter were simulated for a modulated return. Both the second-order range-Doppler map, obtained from (2.8), and the fourth-order range-Doppler map, obtained from (2.14), are compared for the zero Doppler case.

The return signal was processed by estimating the spectrum and the special case of fourth-order cumulant spectrum [1] over a finite data segment representing the pulse length. Subsequent data segments were processed similarly. Each line in the following plots represents a processed data segment equal to the pulse length. In this way both Doppler and range can be extracted by finding the peak in the range-Doppler map. It should be also mentioned again that the processing method is quite different from the ambiguity diagram. Although there are similarities. The most significant differences are that, here, the data are random, averaging is, therefore, included, and the procedure follows the definition of the fourth-order cumulant spectrum. Again, no attempt was made to pick the most efficient waveform.

Figure 1 shows the results for the CW waveform with a random phase angle. A peak can be seen at the zero Doppler location. The "hash" is due to the random phase angle spreading the energy of the waveform out over the range-Doppler map. Although it is not obvious from the figure the CW signal builds up over range until it peaks at the true range.

Figure 2 is the same as figure 1 except that modulation is included. Since the modulation is unknown, it spreads out the received signal and the Doppler location cannot be recovered by the second-order method.

Figure 3 shows the results for the CW waveform with random phase that was processed by the fourth-order method. The Doppler location can be identified and there appears to be less "hash".

Figure 4 is the same as figure 3 except that the modulation is included. Now the Doppler location is still identified. This is due to the way the data were processed.

4. Conclusions

Based on Van Tress' model, it was shown in the paper that when a long pulse, with respect to the reciprocal of the reflection process, is transmitted the return echo is amplitude modulated from Doppler spreading. Under these conditions the second-order method can be severely distorted making detection and classification impossible. However, Using an amplitude modulated CP waveform, it was shown in the paper that the

fourth-order method could extract range and Doppler information while for the same conditions the second-order method could not.

4. References

1. Roger Dwyer, "Fourth-Order Spectra of Sonar Signals," Workshop On Higher-Order Spectral Analysis, Vail, Colorado, 28-30 June 1989.
2. M. Rosenblatt, "Some Models Exhibiting Non-Gaussian Intermittency," IEEE Trans. on Information Theory, Vol. IT-33, No. 2, March 1987.
3. C. L. Nikias and M. Raghuveer, "Bispectrum Estimation: A Digital Signal Processing Framework," Proceedings of IEEE, Vol. 75, No. 7, July 1987.
4. H. Van Trees, **Detection, Estimation, and Modulation Theory: Part III, Radar-Sonar Signal Processing and Gaussian Signals in Noise**, John Wiley & Sons, Inc. 1971.
5. John P. Costas, "A Study of a Class of Detection Waveforms Having Nearly Ideal Range-Doppler Ambiguity Properties," Proceedings of the IEEE, Vol. 72, No. 8, August 1984.
6. E. L. Titlebaum, "Time-Frequency Hop Signals Part 1: Coding Based Upon the Theory of Linear Congruences," IEEE Trans. AES, Vol. AES-17, No. 4, pp 490-493, July 1981.
7. J. R. Bellegarda, "Time-Frequency Properties of Extended Quadratic Congruential Frequency Hop Signals," IEEE ICASSP-89, Conference Record, pp2669-2672, 1989.
8. D. Barton, **Modern Radar System Analysis**, Artech House, 1988. Chapter 5.
9. W. Rihaczek, "Delay-Doppler Ambiguity Function for Wideband Signals," IEEE Trans. AES, Vol. AES-3, No. 4, July 1967.
10. W. Gardner, "Rice's Representation for Cyclostationary Processes," IEEE Trans. Comm., Vol. Com-35, No. 1, January 1987.
11. C. Nikias and R. Pan, "Time Delay Estimation in Unknown Gaussian Spatially Correlated Noise," IEEE Trans. ASSP, Vol. 36, No. 11, November 1988.
12. M. Hinich, "Testing for Gaussianity and Linearity of a Stationary Time Series," Journ. Time Series Anal., Vol. 3, No. 3, pp. 169-176, 1982.
13. D. Brillinger and M. Rosenblatt, "Asymptotic Theory of Kth-Order Spectra." In: **Spectral Analysis of Time Series** (B. Harris, ed.) 1967, pp. 153-188, Wiley, New York.

FIGURE 1. Second-Order Range-Doppler Map of CW with Random Phase.

FIGURE 2. Second-Order Range-Doppler Map of Modulated CW with Random Phase.

FIGURE 3. Fourth-Order Range-Doppler Map of CW with Random Phase.

FIGURE 4. Fourth-Order Range-Doppler Map of Modulated CW with Random Phase.