EFFECTS OF SIGNAL AND NOISE SPECTRAL SLOPES ON TIME DELAY ESTIMATES IN PASSIVE LOCALIZATION

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ABSTRACT

In passive localization, target bearing and range are determined by signal arrival-time differences at two pairs of locations. Signal and noise spectral slopes influence time delay estimates (TDE's), which, in turn, affect the bearing and range accuracy. The variance of the estimate is greater if the signal spectrum falls off faster than the noise spectrum. Analysis shows that the variance is inversely proportional to time-bandwidth product, signal-to-noise ratio (SNR) squared, and band center frequency squared for flat signal and noise spectra. The effects of mismatch of signal and noise spectral slopes on standard deviation of TDE's are discussed.

INTRODUCTION

Sonar systems not only detect targets but also determine their locations and speeds. Locating a target involves determining its range and bearing. Broadband signals, as received at a multiplicity of widely-spaced points along the length of a sonar platform or a towed array, provide the range. The difference in arrival time of the target's signal at two locations, A and B, as shown in figure 1, determines the target's bearing, θ . Target range R is determined by measuring the time differential between arrival differences measured at two pairs of points, AB and BC, as shown in figure 1.

In a passive localization system, the signal received at spatially separated points consists of signals generated by a target and noise. We assume that the target-generated signal and noise are not correlated and are stationary random processes.

The time difference utilized in bearing estimation is obtained by crosscorrelating the received signals at two spatially separated points and measuring the time displacement of the correlogram peak (figure 2). The range is estimated by measuring the difference in time displacement of two correlogram peaks. The received signals are corrupted with noise, causing an inaccuracy in measurement of correlation peaks that affects the target bearing and range estimation. The unknown

time difference, or delay, is estimated as shown in figure 2.

The accuracy of the time delay estimate (TDE) can be enhanced (1) by placing an appropriate filter at the input of the correlator (figure 2). In other words, by providing an appropriate shaping filter at the correlator input, the variance of the time delay can be minimized.

The variance of the TDE is a function of signal-to-noise ratio (SNR), integration time, bandwidth and center frequency, signal and noise, and signal and noise spectral slopes (2-4). In this paper, we investigate the effects of signal and noise spectral slopes on the variance or standard deviation of the TDE, both with and without a shaping filter for the case of low SNR. Also, we investigate the effects of mismatch in signal and noise spectral slopes.

TIME DELAY ESTIMATE

Hahn (1) has shown that the variance σ_D^2 of a TDE that is assumed to be unbiased is

$$\sigma_D^2 = \frac{2\pi}{T} \frac{\int_{-\infty}^{\infty} \omega^2 |H(\omega)|^4 [N_1(\omega)N_2(\omega) + S(\omega)\{N_1(\omega) + N_2(\omega)\}] d\omega}{\left[\int_{-\infty}^{\infty} \omega^2 |H(\omega)|^2 S(\omega) d\omega \right]^2}, \quad (1)$$

where $S(\omega)$, $N_1(\omega)$, and $N_2(\omega)$ are the power spectra of signal $s(t)$, noise $N_1(t)$, and noise $N_2(t)$, respectively; T is the observation time in seconds; $\omega = 2\pi f$; f = frequency in Hertz; and $H(\omega)$ is the transfer function of the shaping filter. The variance of the TDE is minimized when (1)

$$|H(\omega)|^2 = \frac{S(\omega)}{N_1(\omega)N_2(\omega)} \left[1 + \frac{S(\omega)}{N_1(\omega)} + \frac{S(\omega)}{N_2(\omega)} \right]^{-1}. \quad (2)$$

Substituting the value of $|H(\omega)|^2$ into equation 1 and assuming low SNR ($\ll 1$) and $N_1(\omega) = N_2(\omega) = N(\omega)$ will yield (2,3)

$$\sigma_D^2 \approx \frac{2\pi}{T} \left\{ \int_{-\infty}^{\infty} 2\omega^2 S^2(\omega)/N^2(\omega) d\omega \right\}^{-1}. \quad (3)$$

Assuming that signal and noise spectral densities are constant over the band extending from f_1 to f_2 Hz and signal spectral density is S_0 and noise density is N_0 , equation 3 may be written as (4)

$$\sigma_D^2 \approx \frac{1}{8\pi^2} \frac{1}{(S_0/N_0)^2} \frac{1}{TW} \frac{1}{f_0^2} \left(\frac{1}{1 + \frac{W^2}{12 f_0^2}} \right) \quad (4)$$

where

$$W = f_2 - f_1, \quad f_0 = f_1 + \frac{W}{2} = f_2 - \frac{W}{2}.$$

Therefore, the standard deviation of the TDE is given by

$$\sigma_D \approx \frac{1}{\sqrt{8\pi^2}} \frac{1}{\text{SNR}} \frac{1}{\sqrt{TW}} \frac{1}{f_0} \sqrt{\frac{1}{1 + \frac{W^2}{12 f_0^2}}}; \quad \text{SNR} = S_0/N_0. \quad (5)$$

If the signal and noise spectra fall off at the rate of $10p$ and $10n$ dB/decade, respectively, then equation 3 will yield

$$\sigma_D^2 \approx \frac{1}{8\pi^2 T} \frac{1}{(\text{SNR})^2} \frac{1}{f_1^{2(p-n)}} \frac{1}{f_2^{3+2(n-p)} - f_1^{3+2(n-p)}} \quad (6)$$

($p - n \neq 1.5$ and $\text{SNR} \ll 1$)

or

$$\sigma_D \approx \left(\frac{1}{8\pi^2 T} \right)^{1/2} \frac{1}{\text{SNR}} \frac{1}{f_1^{(p-n)}} \left[\frac{[3 + 2(n-p)]^{1/2}}{f_2^{3+2(n-p)} - f_1^{3+2(n-p)}} \right]^{1/2}. \quad (7)$$

Notice that if n and p equal zero, then equation 7 reduces to equation 5. Also observe that if $n = p$, then equation 7 also yields to equation 5, which shows that the standard deviation is invariant if n is equal to p .

A word of caution may be in order. Provided an appropriate shaping filter is utilized before correlation, the variance of the TDE does not change if the signal and noise spectral slopes are the same and minimum. However, if a shaping filter is not utilized, the variance of the TDE will be higher by an amount that is dependent on the signal and noise spectral slopes. Equations 5 and 7 are derived using an appropriate shaping filter.

DISCUSSION OF ANALYTICAL RESULTS

The standard deviation of the TDE is a function of SNR, bandwidth, center frequency, observation time, and signal and noise spectral slopes. Here we present analytical results that show the effects of the signal and noise spectral slopes on the standard deviation of the TDE, σ_D , assuming all parameters, such as bandwidth, center frequency, SNR, and observation time, are constants. We put special emphasis on the effects of change of

noise and signal spectral densities with frequency over the band from 2000 to 6000 Hz, center frequency $f_0 = 4000$ Hz, integration time $T = 120$ sec, and SNR in the range of -10 to -20 dB. These parameter values apply to all figures. First let us determine the influence of the signal and noise spectral slopes when both slopes are the same on σ_D as a function of SNR with and without a shaping filter. We shall also see the effects of mismatch of spectral slopes when the shaping filter is employed.

Figure 3 shows that the TDE, σ_D , as a function of SNR (S_0/N_0) in the range of -10 to -20 dB. The three curves in figure 3 are designated A, B, and C. Curve A shows σ_D as a function of SNR, and the signal and noise spectral slopes are both zero; i.e., S_0 and N_0 are constant over the band and a ratio of spectral slopes for both signal and noise are the same, for example, -10 or -20 dB/decade. When signal and noise spectral densities are constant over the band, we also obtain the same curve A if no shaping filter is used. Curves B and C show σ_D versus SNR when the S_0 and N_0 slopes are equal and -10 and -20 dB/decade, respectively, and without a shaping filter.

Curve A indicates that when signal and noise spectral densities have zero slope, no shaping filter is necessary. Similarly, curves B and C indicate that if both signal and noise spectral slopes are the same, -10 and -20 dB/decade, respectively, then a shaping filter does not improve appreciably the accuracy of the TDE. However, if no filter is used, there are losses of about 0.20 and 0.75 dB when signal and noise spectral slopes are -10 and -20 dB/decade, respectively.

Now, let us evaluate the influence of spectral slopes on the TDE, σ_D , with and without an appropriate shaping filter, if the noise spectral density is constant over the band and signal spectral slopes are varied. The effects are shown in figure 4. The four curves in the figure are designated A, B, C, and D. Curves A and B show σ_D versus SNR when the signal spectral slope is -10 dB/decade with and without a shaping filter, respectively. Curves C and D are similar to curves A and B, the only difference being that the signal spectral slope is -20 dB/decade. Comparing curve A with B and C with D shows losses of about 0.1 and 0.75 dB in effectiveness if no shaping filter is utilized when signal spectral slopes are -10 and -20 dB/decade, respectively.

Figure 5 shows the degree of mismatch of spectral slopes with a shaping filter. Curve A applies to a signal spectral slope of -20 dB/decade and noise spectral slope of 0 dB/decade, which are matched to the shaping filter; whereas, for curves B and C, noise spectral slopes are assumed to be -10 and -20 dB/decade, respectively, and the signal spectral slope is -20 dB/decade. Mismatch causes a loss of 0.8-2.4 dB, depending on the amount of mismatch.

The effects of spectral slopes on the TDE, σ_D , when the signal spectral slope is 0 dB/decade and

noise spectral slopes are varied with and without the appropriate shaping filter are illustrated in figure 6. Curves A and B show σ_D versus SNR when the noise spectral slope is -10 dB/decade with and without a shaping filter, respectively. Curves C and D are similar to curves A and B; the only difference is that the noise spectral slope is -20 dB/decade. Again there is a loss of about 0.5 to 2.0 dB for not using the appropriate filter.

Figure 7 shows the effects of mismatch of spectral slopes with a shaping filter. Curve A represents a signal spectral shape of 0 dB/decade and a noise spectral slope of -20 dB/decade matched to the shaping filter; whereas, for curves B and C, noise spectral slopes are assumed to -10 and 0 dB/decade, respectively. There is a loss of 0.25 to 2 dB, depending on the degree of mismatch.

The situation when signal and noise spectral slopes are unequal and the shaping filter is not matched is depicted in figure 8. The shaping filter is matched to a signal slope of -10 dB/decade and a noise slope of -20 dB/decade, but the signal spectral slope is -20 dB/decade and the noise spectral slope is -20, -10, and 0 dB/decade for curves A, B, and C, respectively. There is a loss of about 1.5-4 dB, depending on mismatch.

SUMMARY AND CONCLUSIONS

In this paper, we have investigated the effects of signal and noise spectral slopes and their mismatch on the variance of TDE's, where we have assumed that the estimate is unbiased. The analysis shows that the variance is minimum when the signal and noise spectral slopes are equal. When both slopes are equal, a preshaping filter, which generally enhances the accuracy of the TDE, does not provide significant improvements. In other words, a preshaping filter is not required when the rates for signal and noise spectral fall-off are equal. However, if the rates are different, and they are matched properly with a shaping filter, there is an improvement in accuracy of the TDE over that obtained without a preshaping filter. On the other hand, if the rates are different and not properly matched to the preshaping filter, the TDE may be degraded; i.e., there is a penalty for mismatching. If no knowledge is available about the signal and noise spectral slopes, or there is a possibility of changes in the spectral slopes, it may be better not to use any preshaping filter.

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Figure 1. Target's bearing and range estimation.

Figure 2. Time delay estimation using correlator.

Figure 3. Standard deviation of TDE as a function of SNR for various signal and noise spectral slopes and with and without shaping filter. Bandwidth = 4000 Hz, center frequency $f_0 = 4000$ Hz.

Figure 6. Effect of mismatch of signal and noise spectral slopes on standard deviation of TDE with and without shaping filter. Signal spectral slope is 0 dB and noise spectral slopes are 0, -10, and -20 dB/decade.

Figure 4. Effect of mismatch of signal and noise spectral slopes on standard deviation of TDE with and without a shaping filter. Noise spectral density is constant and signal spectral slopes are -10 and -29 dB/decade.

Figure 7. Effect of degree of mismatch of signal and noise spectral slopes on standard deviation of time delay estimate with shaping filter.

Figure 5. Effect of degree of mismatch of signal and noise spectral slopes on standard deviation of TDE with shaping filter.

Figure 8. Effect of mismatch of both signal and noise spectral slopes with shaping filter on standard deviation of time delay estimates.