Real-Time Optimal Scheduling of Smart Power Distribution Systems Using Integrated Receding Horizon Control and Convex Conic Programming

Seyedmahdi Moghadasi, Student Member IEEE, Sukumar Kamalasadan, Member IEEE

Power, Energy and Intelligent Systems Laboratory

Department of Electrical and Computer Engineering

University of North Carolina at Charlotte, Charlotte, NC

smoghada@uncc.edu, skamalas@uncc.edu

Abstract—In this paper, we propose a convex Optimal Power Flow (OPF) formulation integrated within Receding Horizon Control (RHC) method using Second Order Conic Programming (SOCP). The main advantages of the proposed method are a) global optimal scheduling with faster computation time b) dynamic equation models with real-time control c) integration of uncertain resources and measurements. The effectiveness of this method is evaluated using a 32 bus power distribution test system, considering various network constraints that include market interaction, energy storage dynamics and uncertain model of wind generation. The efficiency of the proposed method compared to RHC AC Optimal Power Flow (RHC-ACOPF) architecture is also evaluated. The results show that the proposed architecture outperforms the RHC-ACOPF in computation time and guarantees the global optimal solution. Also the proposed method provides effective usage of energy storage system as the RHC integration allows including dynamic modeling of energy storage within the optimization algorithm.

Index Terms—Convex Optimization, Second Order Conic Programming (SOCP), Semidefinite Programming (SDP), Receding Horizon Control (RHC), Real-time Optimization, Electricity Market, Radial Distribution System.

I. NOMENCLATURE

Indices

i, j, h	nodes of radial systems
k, T	time step and horizon time

Variables and Parameters

$T(N, E)$	radial network
S_{ij}, P_{ij}, Q_{ij}	Power from bus i to j
R_{ij}, X_{ij}	line resistance and reactance
B_i, G_i	shunt admittance
I_{ij}	current from bus i to j
λ_{ij}	relaxed current from bus i to j
v_{ij}	voltage from bus i to j
ω_{ij}	relaxed voltage from bus i to j
$B_{\perp}$	reduced incidence matrix
$\beta_{\perp}$	phase angle difference over edges
θ_*	voltage recovered angle
$P_{wg,i}^k, P_{dg,i}^k$	wind and DG generation
$P_{cl,i}^k$	contracted load curtailment
$P_{m,i}^k, P_{nd,i}^k$	power exchange
$c_{WG,i}^k, c_{DG,i}^k$	wind and DG operation cost
$c_{CL,i}^k$	contracted load cost

$c_{M,i}^k, c_{ND,i}^k$
 $E_{s,i}^k$
 $P_{ch,i}^k, P_{dich,i}^k$
 $\eta_{ch,i}, \eta_{dich,i}$

wholesale and neighboring price
 energy storage at time step k
 charging and discharging rates
 charging and discharging efficiency

ShortUsage

ACOPF
 SOCP-OPF
 RHC-ACOPF
 RHC-SOCP-OPF

Non-convex OPF
 Convex Conic OPF
 ACOPF integrated in RHC
 Convex conic OPF integrated in RHC

II. INTRODUCTION

FAST optimal scheduling of power grid is now an important requirement for economically integrating and operating electric power network which increasingly incorporate inherent stochastic players such as renewable sources, plug-in electric vehicles (PEV), price responding demands, and electricity market interactions [1]. Optimal power flow (OPF), evolved in 1962 [2], formulated the basic problem of finding a local optimum operating point for a power system given that both the demand and supply are deterministic. From then on, numerous studies have proposed optimization algorithms to solve OPF problem more efficiently. All these algorithms carry with them various well known limitations referred to as the non-exact relaxation and locally optimum solution [3]- [6]. These limitations are mainly resulting from non-convexity of OPF equations [7]. In recent years, significant contributions are observed in convexification of OPF in radial networks. In 2006, ref [8] proposed the convexification of branch flow OPF for radial system using second order conic programming (SOCP) relaxation. The branch flow OPF [9]- [10] for optimal capacitor placement in distribution systems, exploited the voltage and current angle relaxation; however, this model was non-convex as a result of quadratic equality constraints. Several continuous convex relaxation of this model were then propounded using second order cone approximation [11]- [12]. In important studies, [13]- [15] observed and proved the exactness of angle and conic relaxation in convexification, providing an exact optimal solution for OPF problem in radial networks. A distributed optimal power flow using semi-

definite programming [16] also evaluates the exactness of SDP relaxation of bus injection OPF model in distribution system. A significant proof for zero duality gap presented in [7] shows that strong duality condition exists for the proposed dual OPF using bus injection model and SDP.

Distribution systems gradually transform from a passive network to an active and smart network as a result of evolving rapid distributed energy resources (DER), storage facilities alongside with renewable supplies and electricity markets, which increase the degree of freedom on OPF problem and necessitate dealing with uncertainties through a real-time optimization [17]. Receding horizon control (RHC) [18] is a promising method to schedule the control variables over a time horizon divided by small time steps which allows for real-time implementation. In our earlier work, we proposed and examined the framework of a real-time ACOPF integrated in RHC [19] to re-forecast the future market price and incorporate this updates in our dynamic OPF, considering power and physical network constraints. Ref. [20] proposed the iterative method of prox-average message passing to decentralize the optimization over devices and suggest RHC as an extension to the approach. Recently [21] examined RHC-ACOPF to find better DGs set-point in distribution system. A parallel multi-stage dispatch using RHC propounded by [22] evaluated the control feasibility of network layers for bulk power system. Other studies have investigated the RHC in scheduling of deferrable electric loads without power constraints [23], and scenario-based RHC for optimal power dispatch [24]. Nevertheless, there are two major drawbacks of using RHC with nonlinear ACOPF: computational burden at each time step and also the locally optimum solution.

The studies presented so far focus on either the OPF convexification or RHC method independently. In this paper, we extend our proposed real-time framework RHC-ACOPF [19] by incorporating a convex conic OPF within the RHC (RHC-SOCP-OPF). This method results in finding the optimal schedule, and at the same time brings significant saving in computation time due to efficient convex optimization algorithms compared with a non-convex optimization in ACOPF. The proposed method could also include dynamic device models in the minimization problem such as energy storage and at the same time allows for including measurements and other stochastic variables. To the best of our knowledge, it is the first time that a method introduces the convex OPF integrated in RHC. The proposed method is tested on a modified IEEE distribution system, equipped with various constraints and components associated with a smart micro grid.

Rest of the paper is organized in the following order. III-A presents the RHC formulation dealing with OPF problem, and III-B discusses the convexification of OPF problem using SOCP. Section IV formulate the real-time convex OPF. Implementation of the developed method on the 32-bus distribution test system and the study results are presented in section V. The case study contains market interactions including neighboring distribution companies (DISCOs) and upstream network, uncertain wind generation, storage, distributed energy resources, and contracted loads. Study results evaluate the effectiveness of convex RHC-SOCP-OPF compared with

RHC-ACOPF. Finally, conclusion is presented in section VI.

III. SYSTEM ARCHITECTURE AND MODELING

The proposed real-time convex OPF architecture consists of two major sections, RHC as the master control and convex conic OPF as the internal slave.

A. RHC Methodology

RHC output is a set of consecutive control actions, created by minimizing the objective function over a horizon window. The process has the capability of dealing with constrained dynamic optimization problems which includes consideration of uncertain inputs or parameters. In dynamic systems, state update is captured by (1) whereby the dynamic matrix $\hat{A}_{(k|t)}$ and input matrix $\hat{B}_{(k|t)}$ is determined based on the real-time information available at the current step. The control actions $\hat{u}_{(k|t)}$ are the solution of the optimization problem (2) [18].

$$\hat{A}_{(k+1|t)} = \hat{A}_{(k|t)}\hat{x}_{(k|t)} + \hat{B}_{(k|t)}\hat{u}_{(k|t)} \quad (1)$$

$$\text{Min} \sum_{k=t}^{t+T} \hat{J}_{(k|t)}(\hat{x}_{(k|t)}, \hat{u}_{(k|t)}) \quad (2)$$

$$\text{Sub. } \hat{A}_{(k+1|t)} = \hat{A}_{(k|t)}\hat{x}_{(k|t)} + \hat{B}_{(k|t)}\hat{u}_{(k|t)} \\ (\hat{x}_{(k|t)}, \hat{u}_{(k|t)}) \in \hat{C}_{(k|t)} \quad k = t, \dots, T$$

The control procedure comprises of the following steps:

- 1) The constrained objective function $\hat{J}_{(k|t)}$ is defined over the time horizon $[t : t + T - 1]$. The system parameters and constraints $\hat{C}_{(k|t)}$ are included based on the real-time data available until time step t .
- 2) The optimization problem (2) is solved, and the system control variables is determined over the horizon window $[t : t + T - 1]$. Then the system states $\hat{x}_{(k|t)}$ and control actions $\hat{u}_{(k|t)}$ is passed on to the system.
- 3) Feedback of system variables and external signals such as market price and new weather forecast are passed on back to the controller to update the system states and these inputs are re-forecasted over the next horizon window $[t + 1 : t + T]$.
- 4) The system moves into the next time step $t + 1$, and $[t + 1 : t + T]$ becomes the new horizon window. The preceding steps are repeated over the new window.

The RHC method continues until the time step reach the horizon T . Handing the time horizon is a significant feature in the proposed approach which brings in the ability to include uncertain parameters and real-time system feedback. Thus this method creates the platform to handle the optimization incorporating dynamic components such as storage in power distribution system. Uncertainty is generally modeled using stochastic models based on random variables. We developed (3)-(5) to forecast the random variable $\hat{z}$ over the horizon window $[k : k + T - 1]$ using real-time data $z_t, z_{t-1}, \dots, z_{t-N+1}$ with the conditional expectation expression. The linear regression method is deployed to generate the forecast values. i.e. the conditional expectation of random variable at time step k ($\hat{z}_{k|t}$) is estimated as linear function of the last N real-time feedback data, $Z = [z_t, z_{t-1}, \dots, z_{t-N+1}]$, density

Fig. 1. RHC Dealing With OPF in Modern Power Grid

function expectation $E\{z\}$ and variance $E\{z_t - E\{z_t\}\}^2$. This recursive formulation deals with random variables in which the variables have a normal joint distribution.

$$\hat{z}_{(k|t)} = E(z_k | z_t, z_{t-1}, \dots, z_{t-N+1}) = \alpha + \beta^T Z \quad (3)$$

$$\alpha = E\{z_k\} - \beta^T \begin{bmatrix} E\{z_t\} \\ E\{z_{t-1}\} \\ E\{z_{t-2}\} \\ \vdots \\ E\{z_{t-N+1}\} \end{bmatrix} \quad (4)$$

$$\beta = \begin{bmatrix} \frac{E\{(z_t - E\{z_t\})(z_k - E\{z_k\})\}}{E\{z_t - E\{z_t\}\}^2} \\ \frac{E\{(z_{t-1} - E\{z_{t-1}\})(z_k - E\{z_k\})\}}{E\{z_{t-1} - E\{z_{t-1}\}\}^2} \\ \frac{E\{(z_{t-2} - E\{z_{t-2}\})(z_k - E\{z_k\})\}}{E\{z_{t-2} - E\{z_{t-2}\}\}^2} \\ \vdots \\ \frac{E\{(z_{t-N+1} - E\{z_{t-N+1}\})(z_k - E\{z_k\})\}}{E\{z_{t-N+1} - E\{z_{t-N+1}\}\}^2} \end{bmatrix} \quad (5)$$

The forecasted values $Z = [z_t, z_{t-1}, \dots, z_{t-N+1}]$ resulting from (3)-(5) is then used in RHC optimization (2) to obtain the optimal control variables $\hat{x}_{(k|t)}, \hat{u}_{(k|t)}, k \in [t, t+T-1]$ which minimizes the cost function over the next time horizon T . The process is repeated at the next time step for updates. Fig. 1 depicts the forecasting section inside the RHC structure dealing with SOCP-OPF problem. Convexification method is discussed next.

B. Convex conic OPF

The convex OPF is a critical requirement to reach an optimal solution with a lower computation time. The proposed

SOCP-OPF is formulated based on branch flow (BF) model. The branch flow model is the method of modeling power flow problem using branch variables such as line current and line power unlike the nodal variables in the bus injection method. The model is described based on a directed tree graph denoted as $T(N, E)$ representing the radial distribution system that starts with Node 0 which is connected to the upstream network. The BF model equations are demonstrated in equation set (6). Notations are given in the nomenclature.

$$S_{ij} = V_i I_{ij}^* \quad (6)$$

$$V_i - V_j = z_{ij} I_{ij}$$

$$S_j = \sum_h S_{jh} - \sum_i (S_{ij} - z_{ij} |I_{ij}|^2) + y_j^* |V_j|^2$$

$$\forall \{i, j, h, (i, j), (j, h)\} \in T(N, E)$$

The OPF model based on BF, is a non-convex optimization problem due to nonlinear equality constraints. The relaxation method consisting of two consecutive stages is exploited to convert the problem to a convex optimization problem [15]. In the first stage, the angle relaxation step consist of eliminating angle of all bus voltages and line currents yielding the angle relaxed OPF [9]. In the second stage referred as the conic relaxation, the conic equality constraints are relaxed to conic inequality constraints. These relaxations converts the ACOPF problem to the SOCP-OPF problem, making it as a convex optimization problem. This convexified SOCP-OPF is demonstrated in (7).

$$\text{Min} \sum_{i \in N_G} c_i (P_g^i) \quad (7)$$

$$P_j = \sum_h P_{jh} - \sum_i (S_{ij} - R_{ij} \lambda_{ij}) + G_j \omega_j$$

$$Q_j = \sum_h Q_{jh} - \sum_i (S_{ij} - X_{ij} \lambda_{ij}) + B_j \omega_j$$

$$\omega_j = \omega_i - 2(R_{ij} P_{ij} + X_{ij} Q_{ij}) + (R_{ij}^2 + X_{ij}^2) \lambda_{ij}$$

$$\lambda_{ij} \leq \frac{P_{ij}^2 + Q_{ij}^2}{\omega_i}$$

These relaxations are exact; i.e. the solution of the SOCP-OPF (7) gives the optimal solution to the ACOPF problem. (7) yields the magnitude of bus voltage and line current. OPF convexification algorithm is as shown in Fig. 2. The final step is the angle recovery of voltages and currents using the angle recovery algorithm. With this the reduced incidence matrix $B_{m \times n}$ and the phase angle difference $\beta_{ij} \forall (i, j)$ are calculated and the voltage angle are then recovered through (9). The necessary condition of angle recovery (8) always holds for radial networks [15];

$$B_{\perp} B_T^{-1} \beta_{\perp} - \beta_{\perp} = 0 \quad (8)$$

$$\theta_* = \text{Res.}(B_T^{-1} \beta_T) \in (-\pi, \pi) \quad (9)$$

The other variables angles are then recoverable using Ohm's law and power injection equation.

IV. PROBLEM FORMULATION OF RHC-SOCP-OPF

We developed the RHC-SOCP-OPF formulation to find the global optimum to the OPF problem in radial distribution systems in real-time. The whole structure of proposed

Fig. 2. OPF Convexification: Two Step Relaxation

Fig. 3. Optimal Scheduling using Convex RHC

method are depicted in Fig. 3. In order to implement an RHC optimization structure, The first step is to formulate the RHC-ACOPF problem for the system including the storage, wholesale market, neighboring DISCO, and wind generations.

This OPF is based on the BF model and formulated over the time horizon. In order to reach optimal solution and improving computation time, two relaxation steps are implemented: angle relaxation and conic relaxations; The resulting optimization problem is a convex conic optimization problem over the time horizon T . The problem formulation is described below.

A. Objective Function

The objective function takes into account the generation cost, contracted load cost and the cost for energy exchanged with upstream network and neighboring DISCO. The cost function at the time step t are extended in (10). The distribution system here is considered as a micro-grid with the capability of exchanging power with the rest of grid, and the objective function is the total operation cost of the system.

$$\sum_{k=t}^{t+T-1} \sum_{i \in N} (c_{WG,i}^k (P_{wg,i}^k) + c_{DG,i}^k (P_{dg,i}^k) + c_{CL,i}^k (P_{cl,i}^k) + c_{M,i}^k (P_{m,i}^k) + c_{ND,i}^k (P_{nd,i}^k)) \quad k \in [t : t + T - 1] \quad (10)$$

B. Constraints

The power flow equations are the physical laws constraining the path of electricity in the grid, and dictating the real-time balance between active (reactive) power generation and active (reactive) demand. Equation(11) represents the relaxed power balance constraints over the horizon window.

$$\begin{aligned} P_{wg,i}^k + P_{dg,i}^k + P_{cl,i}^k + P_{ch,i}^k - P_{dch,i}^k - P_{d,i}^k \\ = \sum_h P_{ih}^k - \sum_j (S_{ji}^k - R_{ji}^k \lambda_{ji}^k) + G_i^k \omega_i^k \\ Q_{wg,i}^k + Q_{dg,i}^k + Q_{cl,i}^k - Q_{d,i}^k \\ = \sum_h Q_{ih}^k - \sum_j (S_{ji}^k - X_{ji}^k \lambda_{ji}^k) + B_i^k \omega_i^k \end{aligned} \quad k \in [t : t + T - 1] \quad (11)$$

The voltage drop constraint over the power lines are formulated through Ohm's law and is relaxed using angle relaxation method. This constraint is shown in (12).

$$\omega_i^k = \omega_j^k - 2 (R_{ji}^k P_{ji}^k + X_{ji}^k Q_{ji}^k) + (R_{ji}^{k^2} + X_{ji}^{k^2}) \lambda_{ji}^k \quad k \in [t : t + T - 1] \quad (12)$$

The total power flow through the line is defined by the bus voltage and line current. This constraint is relaxed (13) using conic relaxation.

$$\lambda_{ij}^k \leq \frac{P_{ij}^{k^2} + Q_{ij}^{k^2}}{\omega_i^k} \quad k \in [t : t + T - 1] \quad (13)$$

Storage components impose dynamic constraints to the optimization problem. The state of energy stored in the storage at each time step is dictated by the preceding energy state, charging rate, and discharging rate. These are also known as efficiency coefficients representing the efficiency of charging and discharging process. Equation (14) represents this dynamic equation.

$$E_{s,i}^{k+1} = E_{s,i}^k + P_{ch,i}^k \eta_{ch,i} - P_{dich,i}^k / \eta_{dich,i} \quad k \in [t : t + T - 1] \quad (14)$$

The RHC structure creates the capability to update the constraints associated with distribution lines, generation, demand and storage at each time step as depicted in fig 3, despite the

conventional day-ahead scheduling which contains a constant limit on each time step. Equation (15) represents these constraints at time step k . The expected wind generation and spot market price is also updated at each time step over the next horizon window.

$$\begin{aligned}
 P_{wg.min,i}^k &\leq P_{wg,i}^k \leq P_{wg.max,i}^k \\
 P_{dg.min,i}^k &\leq P_{dg,i}^k \leq P_{dg.max,i}^k \\
 Q_{dg.min,i}^k &\leq Q_{dg,i}^k \leq Q_{dg.max,i}^k \\
 P_{nd.min,i}^k &\leq P_{nd,i}^k \leq P_{nd.max,i}^k \\
 Q_{nd.min,i}^k &\leq Q_{nd,i}^k \leq Q_{nd.max,i}^k \\
 P_{m.min,i}^k &\leq P_{m,i}^k \leq P_{m.max,i}^k \\
 Q_{m.min,i}^k &\leq Q_{m,i}^k \leq Q_{m.max,i}^k \\
 P_{cl.min,i}^k &\leq P_{cl,i}^k \leq P_{cl.max,i}^k \\
 Q_{cl.min,i}^k &\leq Q_{cl,i}^k \leq Q_{cl.max,i}^k \\
 P_{ch.min,i}^k &\leq P_{ch,i}^k \leq P_{ch.max,i}^k \\
 E_{s.min,i}^k &\leq E_{s,i}^k \leq E_{s.max,i}^k \\
 \lambda_{ij}^k &\leq \lambda_{ij.max}^k \\
 \omega_{i.min}^k &\leq \omega_i^k \leq \omega_{i.max}^k \\
 S_{ij}^k &\leq S_{ij.max}^k \\
 k &\in [t : t + T - 1]
 \end{aligned} \tag{15}$$

The cost function may include other objectives, such as conservation voltage reduction (CVR) and line losses however, in this study, since the distribution system is operated as a micro-grid, the objective function contains only the total generation and operation cost.

C. Real-Time Implementation Challenges and Methodology

Three main challenges related to real-time implementation of OPF are a) the need for fast convergence, b) computational speed with accurate results and c) ability to include sensor and estimated data from the field. In the proposed methodology, due the dynamic convex OPF implementation within RHC, the approach is computationally fast and activated at each time step there by feasible to include data at each time step. The results are accurate due to the proposed convexification approach. We report the simulation time based on Intel 2.4 GHz Core i5 CPU. The optimization platform is GAMS and SOCP solver is used to implement the formulation.

V. CASE STUDIES AND SIMULATION RESULTS

The 32-bus radial distribution system [25] is considered here to examine the convex RHC method. The system is 12.66 kV, and contains 33 buses (bus 0 is the contact point to upstream network) and 32 lines. The system is modified to incorporate the market transaction, wind generation, and storage (see Fig. 4). This radial network is connected to the upstream network and neighboring DISCO through tie lines from bus 0 and bus 30 respectively. Two wind farms are located at bus 13 and 16. One storage station is connected to bus 20. Contracted loads are considered at bus 7, 13, 24, and 31. Other system data are given in Appendix A and [25]. In this study, the

TABLE I
TOTAL COST FUNCTION [\$] AT DIFFERENT TIME STEP

Step (k)	1	8	16	24
RHC-SOCP	81604	82643	84237	84721
RHC-ACOPF	82440	83477	85075	85562

wind generation is considered as a stochastic variable; i.e. the expectation of wind generation at time step k of horizon window $[t, \dots, k, \dots, t + T - 1]$ is changing by moving one step forward, t to $t + 1$. The actual and forecasted wind data from the electric reliability council of Texas (ERCOT) is used for the real time wind estimation at each time step. The time horizon T is considered 24h, and each time step is considered 1h. However, since the optimization problem is solved in the order of seconds, it is viable to reduce the time step to less than a minute. Four case studies are considered to test the effectiveness of developed RHC-SOCP-OPF method.

- A: Day ahead scheduling using ACOPF.
- B: Day ahead scheduling using SOCP-OPF.
- C: Real-time scheduling using RHC-ACOPF.
- D: Real-time scheduling using RHC-SOCP-OPF.

Fig. 5 depicts the active power generation at bus 16 using convex methods A to D. It can be notified that the day ahead scheduling (method A and B) over estimated and under estimated wind generation close to 1 MW at different time steps, compared to RHC methods (method C and D). It is noticeable that both methods C and D presented same optimal scheduling at bus 16 due to low wind price causing the wind generators are dispatched as higher priority. This also allows the wind to operate at it's maximum capability. This weakness of day ahead scheduling is shown in Fig. 6 in which the optimization methods A and B under estimated the power exchange with upstream network in hours 13 to 19. According to real-time data, wind is decreased during these hours. This reduction causes the increase in power exchange with upstream network represented in the RHC methods C and D. The Fig. 7 explains the charging and discharging schedule of storage station at each time step in convex RHC method. The storage is charging during hours 1 to 10 and then discharge from 13 to 21. This output is explained with the price pattern. In this system, the spot market price is higher during the discharging hours rather than charging period.

The performance of real-time methods C and D are compared in Fig. 8. The proposed convex method (D) results in significant reduction of the cost function in all steps of RHC implementation time steps. The reduction is close \$1000 and it is achieved at all steps 1 to 24. This improvement is resulted from convex nature of proposed method D, which guarantees the global optimum solution. Table I provide the total cost function value at time steps 1,8,16,24, from which it can be seen that method D yields to 1% reduction in cost by finding the optimal schedule of the micro-grid. The other important aspect of the optimization methods is time performance. Table II illustrates the time performance of real time methods C and D at each time step. The proposed method D shows a significant reduction in simulation time, and yet converged to

Fig. 4. IEEE 32 Bus Distribution Test System

Fig. 5. Optimal Wind Schedule at Bus 16

Fig. 7. Optimal Storage P_{ch} and P_{Disch} From Convex RHC Method

Fig. 6. Power Exchange with Upstream Network

TABLE II
SIMULATION TIME COMPARISON

RHC-SOCP Average Simulation Time (Sec)	6.8333
RHC-ACOPF Average Simulation Time (Sec)	14.2751

the optimal solution in comparison with non-convex method C which yields to a local optimum solution. The average

Fig. 8. Total Cost Over The Time Horizon

simulation time decreases from 14.275 sec to 6.833 sec.

VI. CONCLUSION

We propose a real-time optimal power scheduling method for radial smart distribution system including dynamic components, market transactions, contracted loads and renewable generation. This convex RHC method takes into account the system uncertainties such as wind through re-estimating the

TABLE III
PRICE VARIATION COEFFICIENT AT DIFFERENT TIME STEPS

Step (k)	Price Coe.	Load	Step (k)	Price Coe.	Load
1	0.7	0.88	13	1.31	1.38
2	0.64	0.83	14	1.46	1.41
3	0.62	0.80	15	1.66	1.44
4	0.59	0.78	16	1.43	1.45
5	0.62	0.80	17	1.80	1.43
6	0.63	0.85	18	1.43	1.39
7	0.65	0.92	19	1.18	1.39
8	0.65	1.02	20	1.14	1.34
9	0.77	1.11	21	1.07	1.28
10	0.84	1.20	22	0.92	1.15
11	0.95	1.28	23	0.85	1.02
12	1.09	1.34	24	0.80	0.94

future inputs based on the real time data, which is repeated at each time step over the horizon window. The proposed method also address the major weakness of real-time methods based on non-convex ACOPT, referred as the local optimum solution and time inefficiency. The convex OPF is formulated within the RHC structure through two step relaxation and the second order conic programming (SOCP). The method effectiveness examined on the 32 bus test system, and the results shows the significant improvements in comparison with the current methods.

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APPENDIX A

The market price at each time step is the multiplication of market price variation coefficient and the base price. The load demand variation at different nodes are calculated with the same procedure. DISCO optimize its operation schedule based on the energy market price in upstream, neighboring DISCO, wind variation, etc. Neighboring DISCO and upstream networks are capable to exchange power with the test system based on the hourly energy price. The base market price, and neighboring price are considered \$0.09/kWh and \$0.0825/kWh consequently. The variation coefficient of energy price with respect to base price are presented in Table III. The real-time wind forecasts over the horizon window are obtained at each time step from ERCOT wind forecast data [26].

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