

# Topology-incurred Delay for Information Dissemination in Large Multi-channel Cognitive Radio Networks

Lei Sun      Wenyue Wang

Department of Electrical and Computer Engineering  
North Carolina State University, Raleigh, NC 27695

**Abstract**—Cognitive Radio (CR) networks have become an important component of the modern communication infrastructure due to their capability of improving spectrum usage efficiency by exploiting channels opportunistically. In CR networks, the network topology changes very frequently because of the temporarily available channels and dynamic transmitting parameters (e.g. transmission power and transmitting frequency), which may even result in network disconnectivity from time to time. Hence an interesting and open question is that: are there bounds on end-to-end delay between a source-destination pair with Euclidean distance  $d$  apart in such networks? These bounds are required for time-critical applications. This paper first investigates the nature of topology-incurred end-to-end delay in large multi-channel CR networks and then identifies the conditions under which the asymptotic topology-incurred delay scales linearly with the Euclidean distance ( $d$ ); that is, the conditions under which the end-to-end delay is bounded. The results in this paper are validated through extensive simulations and can advance our understanding of CR network performance.

## I. INTRODUCTION

As a promising solution to the problem of limited frequency bandwidth and inefficiency in spectrum utilization, Cognitive Radio (CR) networks have become an important component of the modern communication infrastructure in that they are expected to exploit spectrum opportunistically [1]. In CR networks, each CR node is equipped with a cognitive radio and thus can access multiple channels without interfering with the licensed nodes, which are also called primary (PR) nodes. Therefore, CR networks can facilitate the coexistence of disparate waveform technologies in the same network, which makes them suitable to a variety of application scenarios, such as military network, emergency network, cognitive mesh network and leased network [1].

Recently, there have been intensive research on understanding and optimizing performance limits of CR networks, in terms of capacity, spectrum sensing, spectrum mobility, and spectrum sharing [1]–[5]. While these research results have greatly advanced our understanding of architecture and the maximum load accommodation of CR networks, another equally important performance indicator, *packet delay*, has received less attention. In the QoS-sensitive or time-critical applications, the delay perceived by a packet is more QoS relevant than other performance metrics, such as the total network capacity. For example, when a CR network is used for emergency rescue in the aftermath of traffic accidents (e.g. vehicular networks), we

must ensure that *help* or *warning* message can be disseminated to a chosen destination within a predetermined time.

Packet delay has been studied for multihop wireless networks in pioneering work [6], [7]. A fundamental assumption for their work is the *full connectivity* of network, which is defined as the existence of a path between any pair of nodes. Denoting  $n$  as the number of nodes in a network, Gupta and Kumar [8] have provided the necessary transmission power of each node for full connectivity to be  $\Theta(\log n)$ , which is very big for large-scale networks. Therefore, considering the limited transmission power, and more importantly the opportunistically available channels of CR nodes, the full connectivity is hard and impractical to achieve for CR networks. Thus instead of full connectivity, we consider a less restrictive *percolated* network in which there exists a *giant component* consisting of most of nodes well dispersed to the entire network.

In addition, the packet delay can be categorized into the bandwidth-incurred propagation delay and topology-incurred delay. In [6], [7], the network topology remains unchanged or changes very slowly, and thus the bandwidth-incurred propagation delay, which is the transmission time spent by a packet in all the links along its transportation path, is dominant. However, in order to limit interference with primary nodes or save energy, CR nodes often dynamically adapt their transmission power and transmitting frequency to the wireless environment, resulting in frequently-changing network topology. And the network may even be disconnected from time to time. Therefore, the transmission delay incurred by topology dynamics, which is called topology-incurred delay in this paper, may be dominant. In this paper, we focus on the nature of topology-incurred delay by assuming infinite bandwidth and thus ignoring propagation delay. Specifically, we focus on the following question in this paper: *how fast can a packet be disseminated in a large multi-channel CR networks? That is, what is the topology-incurred end-to-end delay between a source-destination pair with Euclidean distance  $d$  apart in such networks?* We first derive the sufficient condition for a percolated CR network by mapping the CR network to a percolation model on discrete lattices, which has been extensively studied in [9]. Then we prove that in a percolated network, the asymptotic topology-incurred delay scales linearly with transmission distance.

This paper is organized as follows. In Section II, we introduce the models and percolation theory, and formulate the problem of topology-incurred delay. We derive the sufficient condition for percolation and analyze the transmission delay in

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Section III. In Section IV, we use simulations to validate our analysis, followed by the conclusions in Section V.

## II. PRELIMINARIES AND PROBLEM FORMULATION

In this section, we first describe network models and assumptions used in this paper. Then we introduce preliminaries of percolation theory and finally, we define the *topology-incurred delay* formally.

### A. Assumptions and Models

**CR Nodes:** we consider a large CR network consisting of  $n$  CR nodes independently and identically distributed (i.i.d) in a region  $\mathcal{A} = [0, \sqrt{\frac{n}{\lambda}}]^2$  for some constant  $\lambda$ . Let  $X_i (1 \leq i \leq n)$  denote the random locations of CR nodes that are uniformly distributed in the network. By definition [10],  $\mathcal{H}_\lambda = \{X_1, \dots, X_n\}$  is a homogeneous Poisson point process with density  $\lambda$  as  $n \rightarrow \infty$ .

In multihop wireless networks, each node is usually assumed to have a fixed transmission range. On the contrary, CR nodes may adapt their transmission range independently according to their environment, to save energy or limit interference with PR nodes. We assume that the transmission range of CR node  $v_i$  is a bounded random variable  $0 < r_i < r_{max}$ , where  $r_{max}$  is some constant. Denote  $f_r$  and  $F_r$  as the probability density function and distribution function of  $r_i$ .

**Spectrum Sensing:** because CR nodes can only opportunistically use the channels without interfering with the coexisting PR nodes, spectrum sensing is critical to CR networks in that it can prevent ongoing communications among PR nodes from interruption while selecting the “best” channel for CR nodes. Spectrum sensing does however come at some cost. A solution is to use *coordinated sensing* with global scheduling, but this requires nodes to synchronize with each other quite tightly, which can be quite complex and incur huge overhead in large networks. Letting the nodes set their sensing times in a decentralized and uncoordinated fashion reduces this complexity and overhead, but this increases the delay to transfer information. However, we will show later that even with uncoordinated sensing, there is still a rigorous bound on the delay and therefore we will adopt uncoordinated sensing in this paper. We further assume that each CR node only have a single interface and when it performs sensing, it cannot communicate with other nodes. Particularly, each CR node independently alternates between the *communicating* (active) state and *sensing* (inactive) state with periods determined by the stationary i.i.d on/off process  $W(t)$ . The  $W(t)$  includes i.i.d active periods  $Z_j, j \geq 1$  and i.i.d inactive periods  $Y_j, j \geq 1$ , where  $\mathbb{P}(0 < Z_j \leq T_{on}) = 1$  and  $\mathbb{P}(0 < Y_j \leq T_{off}) = 1$ . That is, both on and off periods are always non-zero and bounded, which is very reasonable for CR networks. Denote  $\eta_{on}$  as the stationary probability of a CR node being active.

**Channel Model:** the impact of the coexisting PR networks with different distributions, densities and traffic load on CR network performance has been studied in [5], [11]. From the CR nodes’ perspective, different PR networks lead to different channels that can be opportunistically used by CR nodes.

Therefore, instead of specifying the PR networks, we only focus on the channel model in this paper.

Specifically, we consider a sufficiently large channel set  $\mathcal{B} = \{ch_1, \dots, ch_m\}$ . Each channel  $ch_k$  is characterized by a *Bernoulli* random process  $H_{ik}$ , where  $H_{ik} = 0$  means the channel  $ch_k$  is being used by PR nodes and thus not available to the CR node  $v_i$  and  $H_{ik} = 1$  otherwise. We assume  $\{H_{i,k}\}_{i=1, \dots, n}^{k=1, \dots, m}$  are i.i.d random variables. The assumption that the channels for a particular node are independent has been widely taken in the related work [12], [13]. We further assume independent channels for different nodes by ignoring the geographical correlation among channels available for neighboring nodes. Since with uncoordinated sensing, nodes may sense channels at different time, this assumption is justified considering the time varying nature of wireless channels. A channel  $ch_k$  is called *usable* for  $v_i$  if  $ch_k$  is sensed by  $ch_k$  and  $H_{ik} = 1$ .

**Communication Links:** Unlike the PR networks, the existence of a communication link between two CR nodes depends not only the distance between them but also the channels available for CR nodes. Particularly, the link  $v_i v_j$  exists if (i)  $d \leq \min(r_i, r_j)$  and (ii) some channels *usable* for both  $v_i$  and  $v_j$ . Denote  $\mathbb{P}_u$  as the probability that any channel  $ch_k$  is *usable* for any node  $v_i$ .  $\mathbb{P}_u$  depends on the channel model and the spectrum sensing algorithm. The graph consisting of all CR nodes and links is represented by  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$ .

### B. Percolation

Recently, *percolation theory* has been used to study the connectivity of large networks [14]–[17]. Percolation theory was founded in order to model the flow of fluid in a porous medium with randomly blocked channels. The porous medium contains many tiny interstices (called sites) and neighbor interstices are connected by open paths (called bonds) with some probability. Percolation states that there exists a critical bond open probability that demarcates two different phases of the fluid distribution in the medium. If bonds are open with sufficiently high probability, the fluid can reach a giant cluster of sites. Otherwise, fluid transfusion is confined in a negligibly small space. Given the similarity between information dissemination and fluid transfusion, the analytical techniques of percolation theory can be used to characterize the information dissemination in large wireless networks.

To further explain how the percolation is related to our problem, we introduce some percolation terminology first. The percolation probability  $\mathbb{P}_\infty(\lambda)$  is the probability that an arbitrary site belongs to a cluster of infinite size. The fundamental result of percolation is the *critical phenomenon*. That is, there exists a finite, positive value  $\lambda_c$  of the site spatial density, under which the percolation probability  $\mathbb{P}_\infty(\lambda) = 0$  (*sub-critical phase*) and above which  $\mathbb{P}_\infty(\lambda) > 0$  (*super-critical phase*).  $\lambda_c$  is called *critical density* and can be formally defined as  $\lambda_c \triangleq \inf\{\lambda > 0 : \mathbb{P}_\infty(\lambda) > 0\}$ . If we assign a random variable  $r_i$  to each site  $X_i$ , where  $\{r_i\}_{i=1}^n$  are independently and identically distributed as a certain random variable  $\rho$ , and the bond  $X_i X_j$  is *open* only if  $\|X_i - X_j\| \leq (r_i + r_j)/2$ , the

resulted graph is called *Poisson boolean model* in continuum percolation [18] and denoted as  $G(\mathcal{H}_\lambda, \rho)$ .

### C. Topology-incurred Delay

Information disseminates in CR networks via rebroadcasting. When a packet is sent at time 0, denote  $\mathcal{V}(t)$  as the cluster of nodes that have received the packet by time  $t$  and  $\mathcal{A}(t) \in \mathbb{R}^2$  as *dissemination area* at  $t$ , which is defined as the total area covered by  $\mathcal{V}(t)$ . Dissemination area can be expressed as  $\mathcal{A}(t) \triangleq \bigcup_{v_i \in \mathcal{V}(t)} B(v_i, 1)$  where  $B(x, r)$  denote a ball with radius  $r$  around the point  $x \in \mathbb{R}^2$ . The topology-incurred delay between a pair of nodes  $u$  and  $v$  can be defined as follows:

**Definition 1 (Topology-incurred delay):** Given a node pair  $u$  and  $v$  and assuming no propagation delay, if  $u$  broadcasts a packet  $b$  at time  $t = 0$ , the topology-incurred delay for  $v$   $\mathcal{T}(u, v)$  is the time needed by  $v$  to receive  $b$ ; that is,  $\mathcal{T}(u, v) = \min_{t \geq 0} \{v \in \mathcal{V}(t)\}$ .

An illustration of topology-incurred delay  $\mathcal{T}(v_0, v_1)$  is shown in Fig. 1, in which a packet  $b$  is originated by node  $v_0$ .

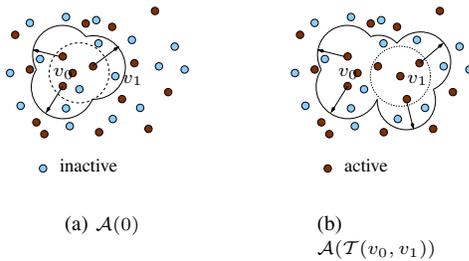


Fig. 1. Information spreading in CR networks, where  $\mathcal{T}(v_0, v_1)$  is defined as topology-incurred delay.

## III. PERCOLATION CONDITIONS AND DELAY ANALYSIS

In this section, we first study the sufficient condition for a percolated CR network, and then analyze the topology-incurred delay of such a network.

### A. Condition for Percolation

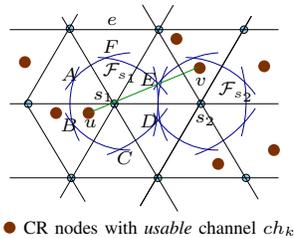


Fig. 2. The triangular lattice with the “flower”  $\mathcal{F}_{s_1} = ABCDEF$ .

Because *discrete percolation* has been well studied in [9], our approach takes following procedures. We first map  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  onto a discrete *site percolation* [9] on the triangular lattice, as shown in Fig. 2. Each site  $s$  is enclosed in an area  $\mathcal{F}_s$ , which is called a “flower” in this paper.  $\mathcal{F}_s$  is formed by the six arcs of circles, each of radius  $\|e\|$  and centered at the midpoints of the six edges incident on  $s$ . This

formation has two benefits, which are proved to be useful for reverse mapping from discrete plane to continuous plane in the following proof. First, for two adjacent sites  $s_1$  and  $s_2$ ,  $\mathcal{F}_{s_1}$  and  $\mathcal{F}_{s_2}$  are disjoint. Furthermore, for any points  $x \in \mathcal{F}_{s_1}$  and  $y \in \mathcal{F}_{s_2}$ ,  $\|x - y\| \leq 2\|e\|$ . For some fixed  $ch_k$ , a site  $s$  is declared *open* if there exist some *active* node  $v_i$  with transmission range  $r_i > 2\|e\|$  for which  $ch_k$  is *usable*, in  $\mathcal{F}_s$ . And a bond  $s_i s_j$  is declared *open* if both  $s_i$  and  $s_j$  are open. Note that by this mapping, an open bond  $s_i s_j$  indicates a link  $v_i v_j \in G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  around. Then, we investigate the conditions when infinite open paths (composed of open sites) exist in the discrete lattice, which implies giant components in  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  by a reverse mapping back to the continuous plane.

**Theorem 1:** When

$$\lambda > \min_{0 < \|e\| < \frac{r_{max}}{2}} \frac{0.843}{\mathbb{P}_u \eta_{on} \|e\|^2 (1 - F_r(2\|e\|))}, \quad (1)$$

$G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  percolates with probability 1.

**Proof:** With the mapping discussed above, each site  $s$  is open with probability  $\mathbb{P}_o = 1 - e^{-\lambda \mathbb{P}_u \eta_{on} (1 - F_r(2\|e\|))}$  by *Thinning Theorem* [18]. And from [9], we know that the discrete percolation model on the triangular lattice percolates with probability 1 when site open probability is greater than  $1/2$ . Thus the result by  $\mathbb{P}_o > 1/2$ . ■

### B. Topology-incurred Delay For A Percolated CR Network

Denote  $\lambda_{1c}$  and  $\lambda_{2c}$  as the critical densities of  $G(\mathcal{H}_\lambda, 1)$  and  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  respectively. Note that by definition,  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  is a proper subset of  $G(\mathcal{H}_\lambda, 1)$ ; that is,  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t)) \subset G(\mathcal{H}_\lambda, 1)$ . When  $\lambda > \lambda_{2c}$ , there exists a giant component  $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))) \in G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  such that a packet  $b$  originated from a node  $v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t)))$  can be disseminated to the whole network instantaneously; and when  $\lambda < \lambda_{1c}$ , there exist no giant component even in  $G(\mathcal{H}_\lambda, 1)$  and thus a packet originate from an arbitrary node can only be disseminated around a finite component.

We are more interested in case of  $\lambda_{1c} < \lambda < \lambda_{2c}$ . In this scenario, there exists a giant component  $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1)) \in G(\mathcal{H}_\lambda, 1)$  but no giant component of  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$ . Thus, a packet originated from any node cannot be disseminated to the whole network instantaneously. However, if we can show that the transmission delay  $\mathcal{T}_{i,j}$  for any link  $v_i v_j \in G(\mathcal{H}_\lambda, 1)$  is finite, a packet  $b$  originated from some node  $v_0 \in G(\mathcal{H}_\lambda, 1)$  can be disseminated to the whole network along the infinite path of  $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$  gradually. Here  $\mathcal{T}_{i,j}$  is the time needed by node  $v_j$  to receive the packet  $b$  from  $v_i$  directly after  $v_i$  has received  $b$ . We focus on the topology-incurred delay of this case.

**Lemma 2:** For each oriented link  $(v_i, v_j) \in G(\mathcal{H}_\lambda, 1)$ , let  $\mathcal{T}_{i,j}$  be a random variable associated with  $(v_i, v_j)$  denoting the time that after  $v_i$  has received the packet, the time needed by  $v_j$  to receive the packet from  $v_i$  directly. We have  $\mathbb{P}(\mathcal{T}_{i,j} < \infty) = 1$  and  $\mathbb{E}(\mathcal{T}_{i,j}) < \infty$ .

**Proof:** Without loss of generality, assume  $v_i$  receives the packet at time 0. Thus  $\mathcal{T}_{i,j} = 0$  when node  $v_j$  is *active*,

$\|v_i - v_j\| < \min\{r_i, r_j\}$  and there exist some channels *usable* for both  $v_i$  and  $v_j$ . Denote  $X = \min\{r_i, r_j\}$  and  $Y = \|v_i - v_j\|$ . It is easy to show that their distributions  $\mathbb{P}(X < x) = 1 - (1 - F_r(x))^2$  for  $x \leq 1$  and  $\mathbb{P}(Y \leq y) = \frac{\pi y^2}{\pi} = y^2$  for  $y \leq 1$ . Thus  $\mathbb{P}(Y < X) = \int_0^1 \int_0^X d\mathbb{P}(Y < y)d\mathbb{P}(X < x)$  and  $\mathbb{P}_d = \mathbb{P}(T_{i,j} = 0) = \eta_{on}\mathbb{P}(Y < X)\mathbb{P}_s$ , where  $\eta_{on}$  and  $\mathbb{P}_u$  denote the probabilities of nodes being *active* and channels *usable* for both  $v_i$  and  $v_j$ . Now consider  $\mathbb{P}(T_{i,j} > t)$ . Since the sensing frequency is bounded by  $\mathcal{T}_{off} + \mathcal{T}_{on}$  (see Section II-A), during time  $t$ , node  $v_i$  will be *active* at least  $\frac{t}{\mathcal{T}_{off} + \mathcal{T}_{on}}$  times (ignoring the fraction factor in this paper for simplicity). Once  $v_i$  turns active, the probability that the packet can be successfully delivered from  $v_i$  to  $v_j$  is at least  $\mathbb{P}_d$ . Hence,  $\mathbb{P}(T_{i,j} > t) \leq (1 - \mathbb{P}_d)^{\frac{t}{\mathcal{T}_{off} + \mathcal{T}_{on}}} < e^{-\frac{t\mathbb{P}_d}{\mathcal{T}_{off} + \mathcal{T}_{on}}}$  and  $E(T_{i,j}) = \int_0^\infty \mathbb{P}(T_{i,j} > t)dt < \infty$ . ■

With  $T_{i,j}$ , the topology-incurred delay between any nodes  $u$  and  $v$  defined in Section II-C can be coupled to the *first passage time* in the weighted graph and reexpressed as  $\mathcal{T}(u, v) \triangleq \inf_{l(u,v)} \{\sum_{(v_i, v_j) \in l(u,v)} T_{i,j}\}$ , where  $l(u, v)$  is an arbitrary path from the  $u$  to  $v$ .

We consider the information propagation in a particular direction, say  $x$ -axis without loss of generality. For any point  $\mathbf{r}_0(d) = (d, 0) \in \mathbb{R}^2$ , since the node distribution is continuous, there exists no node on  $\mathbf{r}_0(d)$  with probability 1. Thus we consider node  $\tilde{v}_0(d) \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$  nearest to  $\mathbf{r}_0(d)$ . In order to use *subadditive ergodic theorem*, we only consider discrete  $d = nx$  for integers  $n$  and some constant  $x$ . Since the discrete limit can be replaced by a continuous one by [19], the results derived here also applied for continuous  $d$  when  $d$  is sufficiently large. Let  $\tilde{\mathcal{T}}(mx, nx) \triangleq \mathcal{T}(\tilde{v}_0(mx), \tilde{v}_0(nx))$  and define the collection of indexed variables  $\{\mathcal{Z}_{m,n} \triangleq \tilde{\mathcal{T}}(mx, nx), m, n \in \mathbb{Z}^+\}$ , for some constant  $x > 0$ . Using *Liggett's subadditive ergodic theorem*, we have the following main result:

**Theorem 3:** When  $\lambda_{1c} < \lambda < \lambda_{2c}$  and for any  $v, u \in G(\mathcal{H}_\lambda, 1)$ , there exist some constant  $0 < \xi < \infty$  such that

$$\mathbb{P}\left(\lim_{\|u-v\| \rightarrow \infty} \frac{\mathcal{T}(u, v)}{\|u - v\|} = \xi\right) = 1. \quad (2)$$

**Theorem 4:** [ [20], Liggett's subadditive ergodic theorem] Let  $\{\mathcal{Z}_{m,n}\}$  be a collection of random variables indexed by integers satisfying  $0 \leq m < n$ . Suppose  $\{\mathcal{Z}_{m,n}\}$  has the following properties: (i)  $\mathcal{Z}_{0,n} \leq \mathcal{Z}_{0,m} + \mathcal{Z}_{m,n}$ , (ii) For each  $n$ ,  $\mathbb{E}(|\mathcal{Z}_{0,n}|) < \infty$  and  $\mathbb{E}(\mathcal{Z}_{0,n}) \geq cn$  for some constant  $c > -\infty$ . (iii) The distribution of  $\{\mathcal{Z}_{m,m+k:k \geq 1}\}$  does not depend on  $m$ . (iv) For each  $k \geq 1$ ,  $\{\mathcal{Z}_{nk,(n+1)k} : n \geq 0\}$  is a stationary sequence. Then: (a)  $\xi = \lim_{n \rightarrow \infty} \mathbb{E}(\mathcal{Z}_{0,n})/n = \inf_{n \geq 1} E(\mathcal{Z}_{0,n})/n$ . (b)  $\mathcal{Z} = \lim_{n \rightarrow \infty} \mathcal{Z}_{0,n}/n$  exists a.s. (c)  $\mathbb{E}(\mathcal{Z}) = \xi$ . Furthermore, if  $k \geq 1$ ,  $\{\mathcal{Z}_{nk,(n+1)k} : n \geq 0\}$  are ergodic, then (d)  $\mathcal{Z} = \xi$  a.s.

To prove Theorem 3, we first show that  $\{\mathcal{Z}_{m,n} : m, n \in \mathbb{Z}^+\}$  satisfy all the conditions of Theorem 4. By definition,  $\mathcal{Z}_{0,n}$  is the first passage time from the origin  $v_0$  to  $\tilde{v}_0(nx)$ , which is clearly at most the first passage time from  $v_0$  to  $\tilde{v}_0(mx)$   $\mathcal{Z}_{0,m}$  plus  $\tilde{v}_0(mx)$  to  $\tilde{v}_0(nx)$   $\mathcal{Z}_{m,n}$ . Condition (i) is thus verified.

Next, we show that  $\mathbb{E}(\mathcal{Z}_{m,n})$  is bounded and nonnegative for any  $m, n$ . As the first passage time cannot be negative, we have  $\mathbb{E}(\mathcal{Z}_{m,n}) \geq 0$ . To show  $\mathbb{E}(\mathcal{Z}_{m,n})$  is finite, note that by Lemma 2, the average transmission delay at each hop  $\mathbb{E}(T_{i,j}) < \infty$ , and thus we only need to find a path from  $\tilde{v}_0(mx)$  to  $\tilde{v}_0(nx)$  in  $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$  with finite hops.

**Theorem 5:** Let  $H_{m,n}$  be the number of hops in the shortest path between  $\tilde{v}_0(mx)$  and  $\tilde{v}_0(nx)$ . Then  $\mathbb{E}(H_{m,n})$  is finite.

**Proof:** See Proposition 4 in [15]. ■

Since  $\mathcal{Z}_{m,n}$  is defined in a stationary way, conditions (iii) and (iv) are clearly verified. Next, we show that  $\mathcal{Z}_{m,n}$  is also ergodic, which is implied by *mixing* (see [19]).

**Lemma 6:** The sequence  $\{\mathcal{Z}_{n,n+1}, n \geq 0\}$  is ergodic.

**Proof:** See Lemma 3 of [15]. ■

Now we have seen that  $\mathcal{Z}_{m,n}$  satisfies all the conditions of Theorem 4 and thus we have  $\lim_{n \rightarrow \infty} \mathcal{Z}_{0,n}/n = \xi = \inf_{n \geq 1} E(\mathcal{Z}_{0,n})/n$ . Next, we need to show that  $0 < \xi < \infty$ . We have shown that  $\forall m, n$ ,  $\mathbb{E}(\mathcal{Z}_{m,n}) < \infty$ . Thus  $\xi = \inf_{n \geq 1} \mathbb{E}(\mathcal{Z}_{m,n})/n \leq \mathbb{E}(\mathcal{Z}_{0,1}) < \infty$ . Note that  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  is a subgraph of the *dynamic site percolation* model  $G(\mathcal{H}_\lambda, 1, W(t))$  defined in [15]. Thus  $\mathcal{Z}_{m,n}$  is lower bounded by the first passage time from  $\tilde{v}_0(mx)$  to  $\tilde{v}_0(nx)$  in  $G(\mathcal{H}_\lambda, 1, W(t))$ , which has been proved positive in [15]. Thus  $\xi > 0$ .

#### IV. SIMULATION RESULTS

In this section, we carry out several sets of simulation to interpret our results. In the simulation, we distribute  $n$  CR nodes independently and randomly with a uniform distribution to approximate a Poisson point process. The transmission range of each node is independently generated from *Uniform*(60, 100). The *communicating* (active) and *sensing* (inactive) time of each CR node are independently generated from *Uniform*(0,  $\mathcal{T}_{on}$ ) and *Uniform*(0,  $\mathcal{T}_{off}$ ) respectively. And the probability of each channel *usable* for any node is set  $\mathbb{P}_u = 0.8$ .

Fig. 3(a) shows an example of the sufficient condition for percolation. Particularly, given  $\eta_{on} = 0.5$ , Eq. 1 in Theorem 1 shows that when  $\lambda > 1463$  (per  $km^2$ ),  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  percolates. Fig. 3(a) shows that the network percolate for  $\lambda = 700 < 1463$ , which proves that our theoretical result provides a condition for percolation of  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$ . From the existing work, we know that  $\lambda_{1c} \approx 144$  (per  $km^2$ ). Fig. 3(b) shows that  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  is not percolated for  $\lambda = 150$  (per  $km^2$ ), which validate our statement that when  $\lambda < \lambda_{1c}$ , the network is not percolated. Fig. 4 shows clearly how the messages disseminate in CR network when  $\lambda_{1c} < \lambda < \lambda_{2c}$ . Particularly, given  $\eta_{on} = 0.8$  and  $\lambda = 230$  (per  $km^2$ ), we find that  $G(\mathcal{H}_\lambda, 1)$  is percolated but  $G(\mathcal{H}_\lambda, f_r, \mathbb{P}_u, W(t))$  not by simulations. As expected, in this case, initially only a very small set of nodes will receive the message, as shown in Fig. 4(a). However, the message will be disseminated to more and more nodes gradually, as shown in Fig. 4(b) and Fig. 4(c). The average topology-incurred delay based on 100 independent simulations for different  $\eta_{on}$  is given in Fig. 3(c). In accordance with our theoretical analysis in Theorem 3, the topology-incurred delay increases almost linearly with the

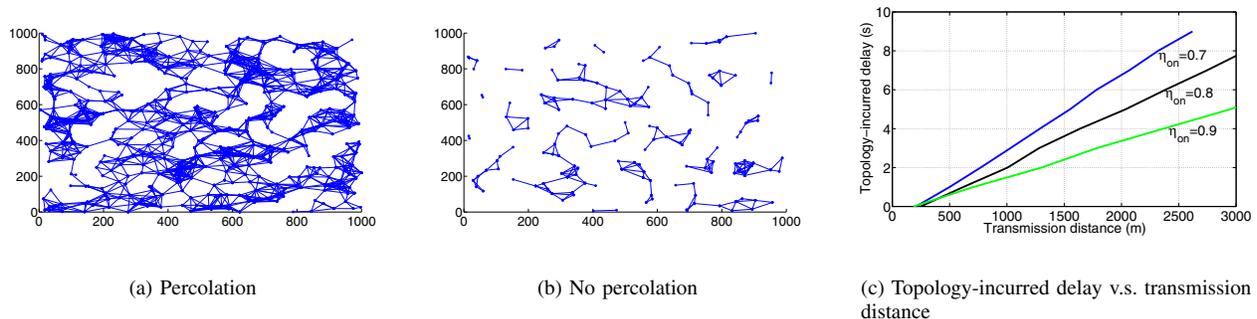


Fig. 3. Percolation conditions and topology-incurred delay.

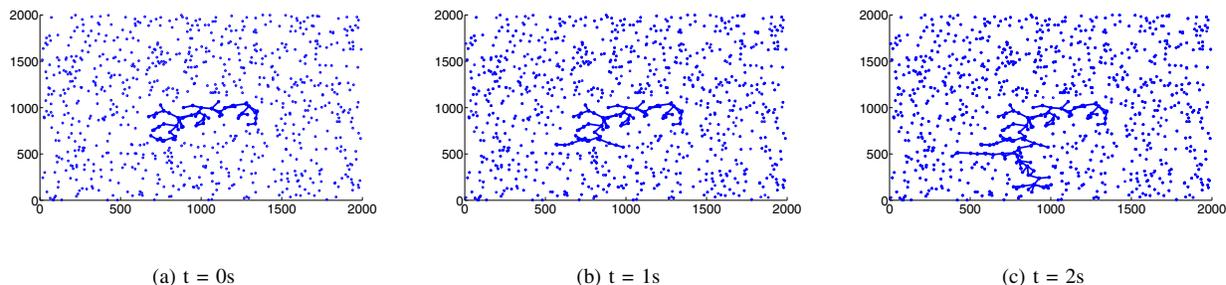


Fig. 4. Snapshots of packets disseminating gradually when  $\lambda_{1c} < \lambda < \lambda_{2c}$ .

transmission distance. And also, the delay will decrease as  $\eta_{on}$  increases. This is natural since larger  $\eta_{on}$  means more nodes are in *communicating* state and thus can be used to relay the information farther.

## V. CONCLUSIONS

In this paper, we studied how packets disseminate in large Cognitive Radio (CR) networks. Since *full connectivity* is hardly to achieve in CR networks considering the limited energy and opportunistically achievable channels, we first identified the sufficient condition for a percolated CR network, which ensures that the message can still be disseminated to the whole network. Then, we investigated the topology-incurred delay and found that for a percolated CR network, the topology-incurred delay scales linearly with the transmission distance.

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