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MEASUREMENT OF VOLTAGE NOISE IN CHEMICAL BATTERIES

Chadwick K. Boggs, Alan D. Doak, F. L. Walls
National Institute of Standards and Technology
325 Broadway Boulder, CO 80303

ABSTRACT

Ultra low noise voltage sources are often required in measurement systems and other applications. Common voltage regulators have performed inadequately in some applications. As an alternative, battery cells have been used. Of the various types, Hg cells have been credited with the best performance. However, actual values for the voltage noise in batteries have not, to our knowledge, been reported. In this paper a measurement system capable of measuring voltage noise below -200 dBV/Hz is discussed and its ability to characterize experimentally high performance voltage references is explored. The results of such measurements on common batteries are presented, and potential applications are considered.

I. Introduction

Chemical batteries have often been used when power supply noise is a significant problem. It is widely known that chemical batteries provide noise performance that is superior to common regulators and regulated power supplies. It is not, however, widely known exactly what performance can be realized with batteries, and what performance differences exist between battery types. Such knowledge is significant in some applications and yields insight into the fundamental noise processes present in batteries. In this paper, chemical battery noise performance is explored and measurements are presented. The voltage noise and current noise measurement techniques developed in these experiments have very high resolution and may be applied in other similar measurement systems.

II. Fundamental Noise

Johnson Noise

The fundamental source of voltage noise across any resistor is Johnson noise, given by [1,2]*

$$V_{noise} = \sqrt{4hfR\Delta\nu/[\exp(hf/kT)-1]}, \quad (1)$$

where k = Boltzmann's constant, T = temperature in kelvin, R = resistance in ohms, h is Planck's constant, f is frequency in Hertz, and $\Delta\nu$ is the measurement bandwidth.

At low frequencies Johnson noise can be approximated by

$$V_{noise} \cong \sqrt{4kTR\Delta\nu} \quad (2)$$

This fundamental noise process is dependent only upon temperature and resistance.

Shot Noise

Broadband current noise is found when two conditions are satisfied: (1) the charge carriers are quantized; and (2) the arrival times are random. These conditions cause current fluctuations of magnitude

$$i_{shot} = \sqrt{2qI\Delta\nu}, \quad (3)$$

where q is the charge quantization (in this case the charge of an electron) in coulombs, I is the bias current in amperes, and $\Delta\nu$ is the measurement bandwidth in Hertz. Current noise of this type is commonly called shot noise.

III. Voltage Noise

Voltage Noise Measurement System

To see voltage fluctuations in batteries, a measurement system with a resolution of approximately -200 dBV/Hz is required. This was accomplished by the use of two low noise amplifiers and a two channel cross-correlation technique[3], shown in Fig. 1. The two independent and similar amplifiers provided ~50 dB of gain with a noise floor of -180 dBV/Hz. The amplifiers boosted the noise from the batteries to approximately -150 dBV/Hz, which was just above the noise floor of the Fast Fourier Transform (FFT) spectrum analyzer used. The output noise from the amplifier was, however, -130 dBV/Hz which obscured the noise signal from the batteries.

To recover the battery signal, cross-correlation was performed between the two channels. The cross-correlation operation caused the noise from the independent amplifiers to average to zero as $1/\sqrt{n}$ (n = number of averages) revealing the underlying correlated voltage noise from the batteries. Two batteries were used to double the battery noise power and permit the use of high gain DC coupled amplifiers. The limiting factor in this technique was the time taken to average the data. A practical resolution of -206 dBV/Hz at 1 kHz was reached after 100,000 averages. This noise floor was sufficient to measure the voltage noise of batteries. The effect of cross-correlation in this system is described by

$$\overline{(V_1)_N \cdot (V_2)_N} = (V_{B1})_N \cdot (V_{B2})_N + \frac{Sys1_N \cdot Sys2_N}{\sqrt{N}} \quad (4)$$

The terms are as labeled in Fig. 1.

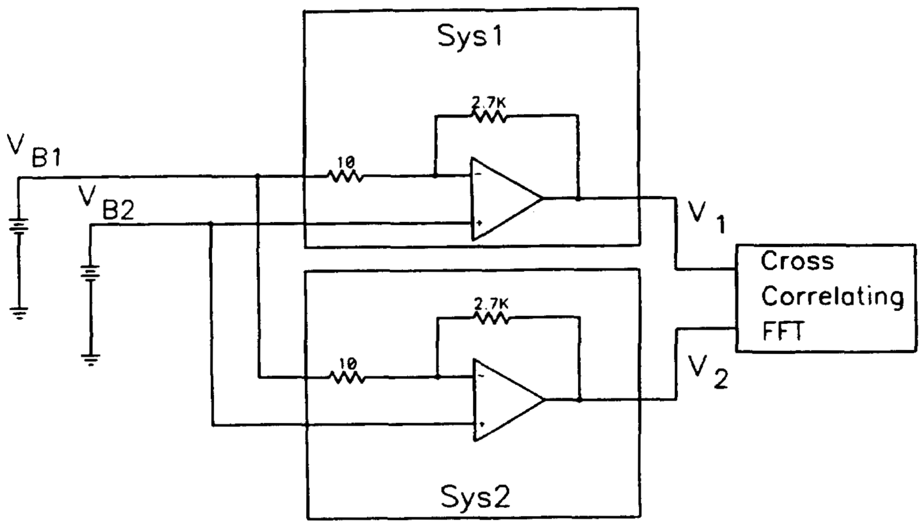

Figure 1: Voltage Noise Measurement System

Measurement of Chemical Battery Noise Voltage

Figs. 2 and 3 depict the results of measurement on various types of batteries. The top line in Fig. 3 shows the noise floor of the measurement system without cross-correlation, and the bottom line shows the ultimate noise floor of the system obtained through cross correlation.

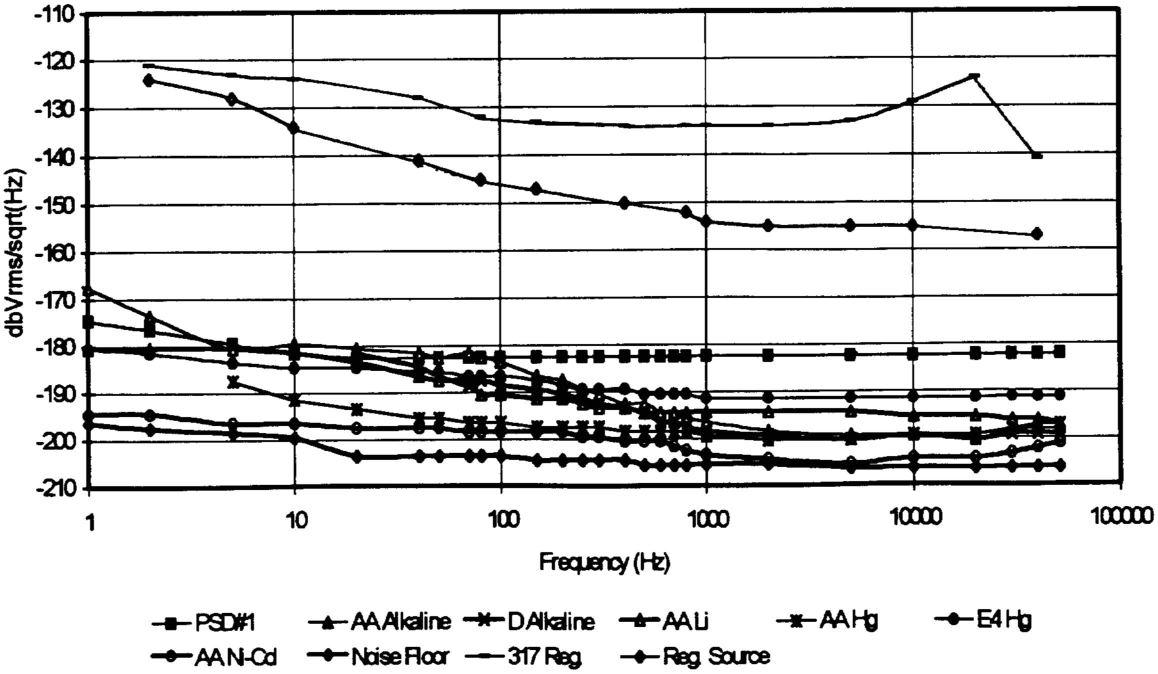

Figure 2: Voltage Noise Measurements

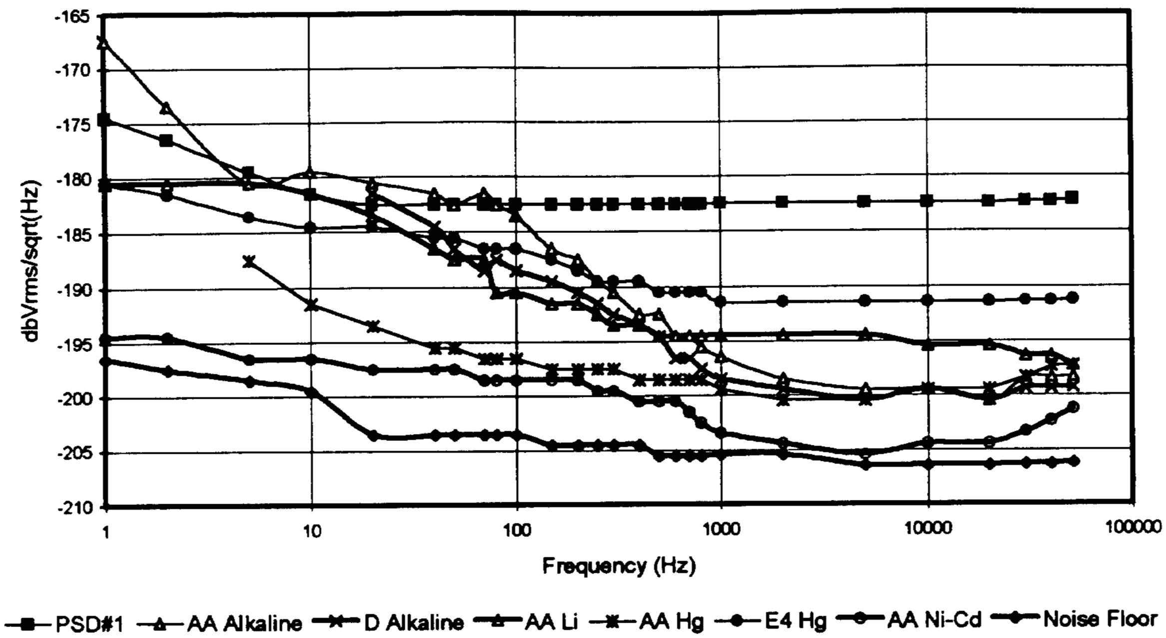

Figure 3: Battery Voltage Noise Measurements

Our results show that Ni-Cd batteries have the best noise performance of any of the chemical batteries tested. A value of -205 dBV/Hz at 10 kHz was recorded. In fact, measurements this close to the system noise floor were possibly inflated due to the addition of noise from the measurement system.

Different types and sizes of batteries produced different voltage noise. The battery's capacity influenced voltage noise. As can be seen from the graph, there is a slight difference between the noise in the AA and D size batteries. It was also found that the voltage noise of batteries was constant over the range of current from trickle charge to ~1mA discharge.

Noise Processes In Chemical Batteries

Batteries have very low internal resistance, generally less than 1 Ω. A value of 0.20 Ω was measured for Ni-Cd batteries, corresponding to a Johnson noise of -205 dBV/Hz at room temperature. As can be seen from Figs. 2 and 3, this agrees closely with the measured broad band voltage noise of the Ni-Cd batteries. This agreement suggests that the dominant broadband noise process in batteries is the Johnson noise from the internal resistance of the batteries.

This conclusion suggests that larger capacity batteries, which generally have smaller internal resistances, would produce a lower voltage noise. Voltage noise was found to be independent of bias current.

IV. Current Noise

The current noise of a battery is directly related to its voltage noise by Ohm's Law (see Fig. 4). In our measurements, battery voltage noise was found to be independent of bias current for typical current levels. Thus, the noise in a bias current from a battery is inversely proportional to the load impedance across the battery, $i = \frac{v}{Z}$. The current noise, then, should be known once the voltage noise is determined.

A battery can be modeled by the circuit in Fig. 4. The battery is represented by a dc voltage, V, an ac noise voltage, v, and an internal resistance, r. In normal applications, the battery's load, R, is much greater than the battery's internal resistance. The current noise in this model is $i = \frac{v}{r+R} \approx \frac{v}{R}$, R >> r. As discussed previously, it was found that v was approximately described by Eq. 2. This produces a current noise, i, that is small compared to shot noise.

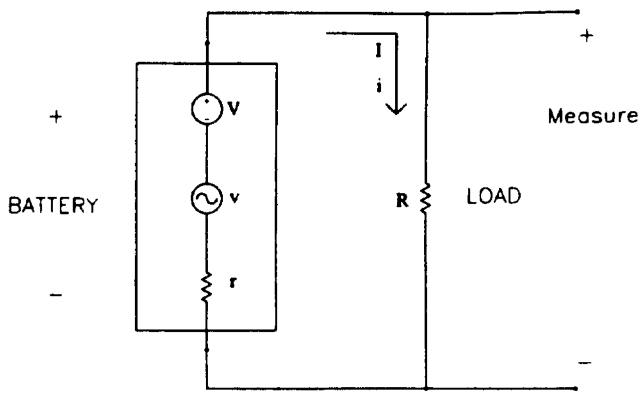

Figure 4: Low Frequency Chemical Batter Circuit Model

Batteries suppress shot noise by correlating the arrival times of charge carriers at the battery terminals. This can be shown by looking at an example of typical values in the model in Fig.4: V = 1.2 V, r = 0.2 Ω, R = 1000 Ω. If shot noise, i_{shot}, was dominant in the current noise, the result would be

$$v = i_{shot}R = R\sqrt{2q\frac{V}{R}\Delta v} = \sqrt{2qVR\Delta v} \quad (5)$$

For the example values, v = -154 dBV/Hz assuming shot noise. However, we measured $v \cong \sqrt{4kTr\Delta v} = -204\,\text{dBV/Hz}$ which is nearly 50 dB below the shot noise in 1mA.

It can be shown that for any power supply consistent with the model in Fig. 4, and R >> r, the shot noise will induce a voltage noise greater than the Johnson noise of r. Assuming shot noise is present, v is given by Eq. 5. If the voltage noise is dominated by the Johnson noise in r (note that the noise of r||R is roughly that of r), v is given by Eq. 2 as $v \cong \sqrt{4kTr\Delta v}$. Shot noise dominates whenever Eq. 5 is greater than Eq. 2, or

$$\sqrt{2qVR\Delta v} > \sqrt{4kTr\Delta v},$$
$$VR > \frac{2kTr}{q},$$
$$V > \frac{2kT}{q}\frac{r}{R},$$
$$V > 0,\ R >> r. \qquad (6)$$

In the above model shot noise would be expected to dominate over Johnson noise for source voltages greater than 52 mV at T = 300 K and r=R.

Current Noise Measurement System

To verify the model upon which the previous analyses were made, direct measurements of the current noise in batteries were made with the measurement system shown in Fig. 5. The transconductance of this circuit is

$$T = \frac{V_{out}}{i} = Z(K), \qquad (7)$$

where Z is the impedance of the circuit inductor, and K is the gain of the non-inverting amplifier.

An inductor was used to obtain high impedance with low Johnson noise. At high frequencies the input noise of this system was dominated by the Johnson noise of the 825 Ω resistor, yet the load impedance was on the order of 1 MΩ. A load impedance on the order of 1 MΩ produced a transconductance of 140 dBΩ. This means that the input current fluctuations were amplified by 140 dB and converted to voltage for measurement.

Input Noise of Measurement System

Measurements confirmed that the broadband input voltage noise of the system in Fig. 5 was white from 20 Hz to at least 100 kHz at -136 dBV/Hz. The broadband input current noise was

$$i = \frac{V_{out}}{T} = (-136 - 140)\ \text{dBA/Hz} \qquad (8)$$
$$= -276\ \text{dBA/Hz}$$
$$= 16\ \text{fA/Hz}$$

This agrees with the calculated Johnson noise from the 825 Ω resistor. At low frequencies, the input Johnson noise drops to that of the 203 Ω resistance in the inductor. However, at frequencies below 20 Hz the input noise becomes dominated by power law processes that raise it above that due to the Johnson noise of the input resistors.

To predict accurately the performance of the physical circuit we need to consider the inductive impedance more carefully. A real inductor of 1 H has large parasitic capacitance shown by the transfer function in Fig. 6. The impedance of our inductor was ideal to approximately 5 kHz. The parallel parasitic capacitance resonated at 13 kHz, and beyond 20 kHz the impedance dropped significantly from ideal.

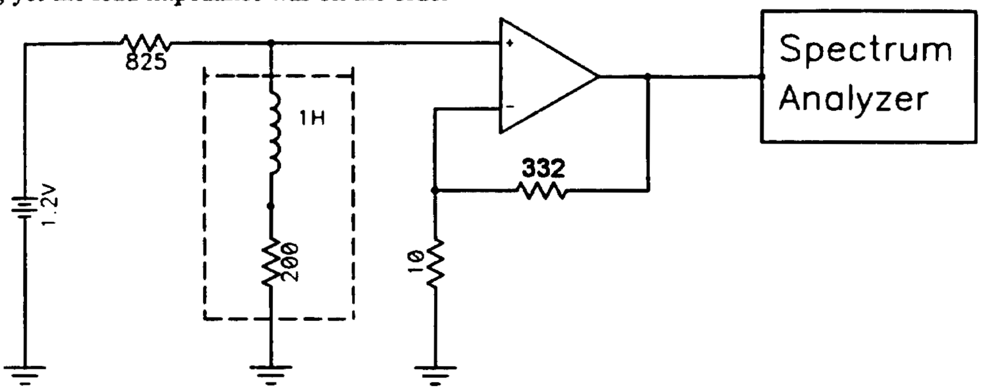

Figure 5: Current Noise Measurement System

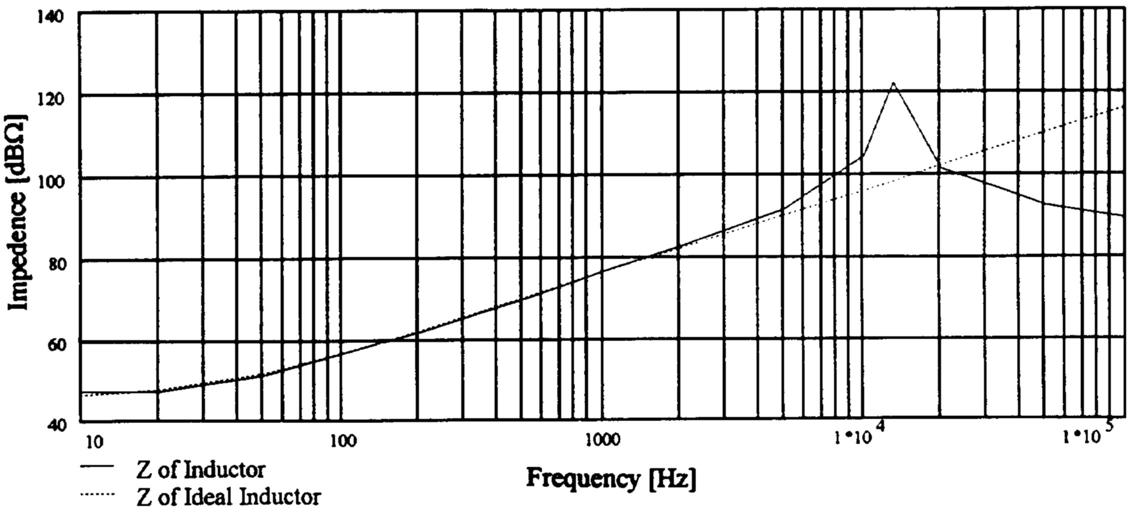

Figure 6: Effect of Parasitic Capacitance

Replacement of the battery in Fig. 5 with a high impedance current source made possible direct measurement of the system's broad band transconductance, from which the actual inductor impedance was calculated. The input current noise floor of the system is shown in Fig. 7. As a reference, shot noise for a 1 mA bias current is also plotted.

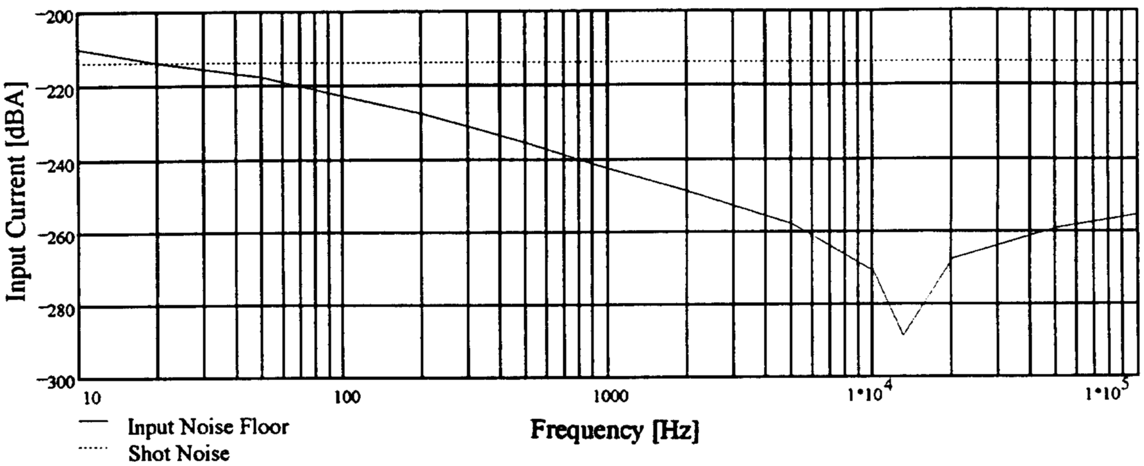

Figure 7: Measurement System Input Noise Floor

<u>Measurement of Current Noise in Battery Bias Current</u>

Measurement of chemical batteries in the system in Fig. 5 confirmed shot noise suppression. Data from typical battery measurements were identical to data from measurements of the system's input noise floor within the accuracy of the measurements. That is, a 1.2 V battery provided 1 mA of bias current with no noise to the resolution of our measurement system, which was over 40 dB below shot noise at resonance. The calculation of current noise from measured voltage noise suggested that the actual current noise was up to 95 dB below shot noise.

V. Conclusion

Very high resolution voltage measurement systems were constructed using two similar amplifiers and a cross-correlation method. This operation is built into many modern high end digital spectrum analyzers, and can resolve up to 25 dB. This approach, however, requires approximately 10^5averages.

Using this low noise measurement system we have characterized the voltage noise of 5 different battery types and compared them to a popular voltage regulator and a high performance power supply. We found that the voltage noise of the chemical batteries measured was many decades smaller than that of traditional power supplies. The lowest noise battery tested was a AA Ni-Cd with Vn = -205 dBV/Hz at 1 kHz. Different battery types exhibit vastly different noise voltage. In the batteries measured the broad band noise voltage appears to be approximately equal to the Johnson noise of their internal resistance.

We have made measurements of the current noise in battery driven bias currents. We show that chemical batteries correlate the arrival times of charge carriers at their terminals, thus suppressing current noise to well below shot noise. For the AA Ni-Cd batteries tested, the current noise in a 1 mA bias current was more than 50 dB below shot noise. The fundamental relationships between voltage and current noise were explored, and a simple model for circuit elements with noise was demonstrated.

The unique characteristics and superior noise performance of chemical batteries suggest interesting applications. A steady state arrangement may be possible in which the battery is not drained. The long term stability of other components might, through the charging circuitry, be able to enforce long term stability upon batteries, thus providing a low noise voltage reference that will last for years. In some applications such as biasing networks it may be possible to replace noisy capacitor/resistor chains with Ni-Cd batteries. The use of chemical batteries is already widespread in low noise applications and more precise knowledge of their performance will aid design and analysis.

Acknowledgments

We thank Hugh Robinson for the original suggestion to investigate the absence of shot noise in bias currents from chemical batteries. We also thank Franklin Ascarrunz for suggestions on low noise measurement circuits.

References

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