INTERIM REPORT ON THE GEOGRAPHIC DEPENDENCY AND LATITUDE EFFECTS STUDY

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Abstract:

The Geographic Dependency and Latitude Effects Study is an effort underway at NRL to investigate the capability of the Global Positioning System (GPS) to disseminate Universal Coordinated Time as maintained by the U.S. Naval Observatory, UTC(USNO). It is assumed that the users will have access to the GPS Precise Positioning Service (PPS) and will use GPS in the passive mode. That is, they will not exchange data with another location to transfer time by the common view method, but rather will obtain UTC(USNO) via the GPS broadcast message. The study addresses the performance expected from this type of user on a global basis. USNO has demonstrated, through operation at their facility as a user, that the system elements can provide time to less than 25 ns. This seems to indicate that the final link with the user is the source of additional errors limiting performance. The other GPS system elements and the worldwide nature of operation for time dissemination has not been well studied and should not be overlooked. Since USNO, the GPS Master Control Station, and the primary GPS test range at Yuma are co-visible, the southern pacific area was chosen as the area of special interest to this study, being the more difficult for operational coverage.

In this study, the effects of the various GPS system error sources are being investigated for their influence on time dissemination. Data is being gathered from a variety of tracking and timing centers to attempt to identify and quantify the errors and their sources. The known influences of GPS satellite position error, propagation effects and the ability to maintain time at remote sites are being investigated. Trends in the error propagation of these different effects will be determined and limits imposed on capability and possible geographic dependency identified. Interim results and tentative conclusions will be presented on these efforts.

INTRODUCTION

This paper will discuss an investigation into

the errors associated with recovering Universal Coordinated Time as maintained by the Naval Observatory, UTC(USNO), by authorized users of the Global Positioning System (GPS). The eventual aim of the effort is for users who will have access to the GPS Precise Positioning Service (PPS) and will use GPS in the passive mode. That is, they will not exchange data with another location to transfer time by the common view method, but rather will obtain Universal Coordinated Time and maintained by the U.S. Naval Observatory (UTC(USNO)) via the GPS broadcast message. Consequently, the best accuracy from the system in this mode is desired. The identification of the error sources and their contribution to GPS time dissemination is the objective, with the aim of reducing or eliminating their effect.

GPS SYSTEM OVERVIEW

The GPS system is currently being deployed to provide a general purpose navigation, precise positioning and time dissemination service for military users. The system provides multi-satellite coverage over the entire globe and a combination of navigation signals. GPS is intended to be used as a real-time system for ground and air military platforms requiring accuracies to 10's of meters. The real-time requirement is met by using the coded signals to make near-instantaneous measurements to the GPS satellites. For fixed users, smoothing permits improved accuracies down to less than a meter. The Selective Availability/Anti-Spoofing (SA/AS) capability of the GPS system is

the basic difference between the Standard Positioning Service (SPS) and the Precision Positioning Service (PPS). The SPS is meant to be the standard service available to civilian and foreign users.¹ The PPS represents the full military capability of the system which must be accessed through secure cryptological means. These two schemes degrade or deny the system assets or accuracy for the Clear/Acquisition (C/A) only user. One aspect of SA is oscillator dither which dithers the basic clock frequency used to generate the C/A and Protected (P) code signals. The dither is an encrypted sequence which must be decoded by the user to obtain full accuracy and precision from the GPS signals. Anti-Spoofing (AS) provisions change the P code to another code known as the Y code.

GPS RECEIVERS

Receivers developed for use with GPS range from using only L_1 - C/A code only with one to five channels, a channel is designated for each GPS satellite being observed, to the full 5 channel military receiver. Sequential receivers have also been developed which sequence through a number of GPS satellites to provide the number observable for navigation. The most precise receivers developed for geodetic work operate predominately with the carrier frequencies. The coherent frequencies provided by the GPS satellites are used as a fine measure of the satellite range and range rate. This eliminates reliance on the less precise ranging information available from the signal code rates. For the timing user a single channel receiver is normally used since their are usually at a fixed, known location. If the position is not known accurately then GPS can be used to fix, or navigate, the receiver.

To solve for real-time position in the normal mode used with GPS, four satellite simultaneous code data is used. With a single channel receiver multiple measurements would be used to provide the solution. The code data measures the time delay a signal takes to propagate from the satellite to the user. The user can determine the range by,

$R = c(t_2 - t_1)$, where, R is the range, c is the speed of light, t_2 is the time of reception, and t_1 is the time of transmission. Ranging to three satellites simultaneously allows the determination of the user's three position coordinates. This process presumes that the satellites' and the user's receiver clock are synchronized, which is normally not the case. For this reason, the range determination is called pseudorange, which denotes unsynchronized clocks between the satellites (normally assumed to be synchronized) and user receiver (offset by some value b_u), and requires an additional satellite measurement to be able to solve for the synchronization error. The user offset is the measurement desired by the timing user. The pseudorange equation between the user, u , and multiple satellites (or measurements), i for $i = 1, 4$, is then,

$$(b_u + \Delta t_i)c = \rho_i + c(b_i + \Delta t_u)$$

where, $(b_u + \Delta t_i)c =$ pseudorange,

$\rho_i =$ true range to satellite i ,

$b_i =$ satellite clock offset, and

$\Delta t_u =$ propagation biases and noise.

Alternatively, this equation may be written as a function of the user's position, denoted as x_u, y_u, z_u , and the user clock offset, b_u .

$$(b_u + \Delta t_i)c = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} + c(b_i + \Delta t_u)$$

The solution of this equation is the navigation solution and the basis for the NAVSTAR system performance estimates. If the equation is rewritten in matrix form (cartesian coordinates assumed) as,

$$(R_u \cdot e_i - B_u) = (R_i \cdot e_i + B_i) - P_i + v_i$$

or,

$$\bar{G} \cdot \bar{x} = \bar{A} \cdot \bar{s} - P_i + v_i$$

where; $R_u =$ user's position vector,

$e_i =$ unit vector in direction of satellite i ,

$B_u = c b_u$, range equivalent user time bias,
 $P_i = c \Delta t_i$, measured pseudorange vector,
 $B_i = c b_i$, range equivalent satellite time bias,
 $v_i = c \Delta t_u$, range equivalent noise term.

and, the noise characteristics are of the form,

$$\text{Cov}(v_i) = E[v_i v_i^T] = \sigma^2 I$$

where, σ is the error in the individual ranging link. Then, the estimate of the parameter vector, x_i , is,

$$\hat{x}_i = [G_T \text{Cov}^{-1}(v_i) G]^T G^T [\text{Cov}^{-1}(v_i)] [\bar{A} \cdot \bar{s} - P_i]$$

and covariance of the estimated solution is,

$$\text{Cov}(\delta \bar{x}) = [G^T G]^{-1} G^T \text{Cov}(v_i) [(G^T G)^{-1} G^T]^T$$

and, from the noise characteristics above,

$$\text{Cov}(\delta \bar{x}) = [G^T G]^{-1} \sigma^2$$

The navigation accuracy estimate can be then described as the product of the basic ranging error to a satellite, σ , and a dimensionless constant, $[G^T G]^{-1}$, called GDOP for Geometric Dilution of Precision. GDOP is determined by the specific geometry of the GPS satellites to the user at the time of navigation solution. The basic ranging error, which has been called the User Range Error (URE) or User Equivalent Range Error (UERE), is the combination of the errors in the measurement of range between each satellite and the user receiver. URE is a combination of the actual range measurement errors due to instrumentation, residual ionospheric and tropospheric correction, orbital position and clock errors. This is a fundamental link in the time comparison between GPS and the time user. The URE is strongly influenced by both clock and orbit errors.

ERROR SOURCES

The URE and associated error contributions are illustrated in Table 1 (actual error contributions are highly dependent upon the equipment

used). The dominant errors appear to be atmospheric delays and multipath. However, the GPS satellite position and clock errors are functions of elapsed time since the last clock update or orbit prediction and represent the primary limiting factors on GPS system performance.

User error statistics are given and discussed in ICD-GPS-200² as a measure of system performance. In that document the user error is defined as URA (User Range Accuracy) and the difference between the Control Segment's prediction and the current Kalman estimate.

Table 2. GPS User Range Error	
Error Component	1 σ (m)
Satellite Orbit Error	1.5
Satellite Clock Error	1.5
Atmospheric Delays	2.4 - 5.2
Satellite Equipment Delays	1.0
Multipath	1.2 - 2.7
Receiver Noise and Resolution	1.5
TOTAL (RSS)	3.6 - 6.3

GPS Orbital Position Errors.

The GPS system relies on an accurate knowledge of the predicted satellite positions. Knowing the satellite position then permits accurate range determination to allow calculation of the user's position or passive time comparison. Since the position must be known in advance, the orbit predictability of the GPS satellites is the key factor in the operation of the system. A precision orbit determination process has been enacted by the Defence Mapping Agency (DMA) to provide the

most accurate satellite ephemerides for surveying and mapping purposes. That non-real-time process provides to most accurate positional information available for comparison and estimation of the predicted positional information broadcast to the users.

Satellite Clock Errors

The GPS satellite atomic clocks time offsets and rates are also predicted for real-time use. The coefficients broadcast in the navigation message are linear predictions of the particular clock's performance based on the observations from the GPS monitor stations and other data computed at the GPS Master Control Station. These calculations are performed from the passively observed pseudorange information collected at the GPS Monitor Stations. In this passive process, the ability to distinguish between a clock error and positional error cannot be done by measurement. Consequently, the computational models are used to separate and estimate the various errors for prediction. This clock/position ambiguity and the inability to measure clock performance independently leads to errors. In the error budgets and operation of the system clock and orbit predictability are roughly equal in effective magnitude.

Propagation Errors

A GPS signals passing through the earth's atmosphere encounter two distinct regions which alter the propagation time and direction, the ionosphere and troposphere. Propagation through the ionosphere experiences an effective change in the refractive index caused by the free electron plasma which makes up the ionosphere. The ray path followed by the radio signal is expressed as,

$$\text{Path Length} = \int_{\text{satellite}}^{\text{user}} n ds$$

where, n = refractive index, and ds is along the ray path.

For GPS frequencies the actual bending of the signal path is small and the primary delay is caused by phase or group delay of the signal. The deviation of the refractive index from unity is then $\Delta n = 1 - n$, which may be expressed as,

$$\Delta n = \frac{e^2}{8\pi^2\epsilon_0 m} \frac{N}{f^2} = \frac{40.3}{f^2} N$$

where, f = frequency (Hz), and N = electron content (electrons/m²).

The difference between the measured path length and the geometric path length is then,

$$\Delta P = \int \Delta n ds = -\frac{40.3}{f^2} \int N ds$$

The integral is taken along the signal path and represents the integrated electron density acting on the signal. By using two frequencies the integrated electron density function can be measured to first order. Higher order effects are not significant in general since the first order effects residuals are estimated to be on the order of 1-2 meters for high elevation angles.³ In general, the expression can be used to represent the signal path electron density anywhere in the world against a worldwide model of the ionosphere. This approach is used by the single frequency user of GPS against a standardized model whose coefficients are transmitted in the satellite data message.

Signal errors caused by propagation through the troposphere are independent of the signal frequency. The refractive index of the medium due to troposphere (n_T), is expressed as,

$$n_T = \frac{77.6}{T} (B + \frac{4810e}{T})$$

where, T = temperature (°K), B = total barometric pressure (millibars), and e = partial pressure of water vapor (millibars).

These two terms in this expression are the so-called "dry" (n_d) and "wet" (n_w) terms respective-

ly. The excess propagation path is given by,

$$P = 10^{-6} \int n_T ds$$

Since these terms are independent of frequency and dependent upon the metrological conditions along the signal path they must be modeled for the GPS user. Measurement of this effect is possible with water vapor radiometers which are large and expensive to be deployed for the modeling residual that results. A model would need to be applied to the GPS observations to account for these errors, which have been measured⁴ for elevation angles over 30° to be on the order of 20 cm. For lower angles the effect increases significantly due to the longer integrated path through the troposphere.

Instrumentation Errors

The receiving equipment and its installation effects the performance of navigation and time comparison users. The usually problems of receiving with the appropriate resolution are common. An additional factor affecting the time user is the calibration of the delays in the antenna lines and other distribution networks that may be employed at the receiving site. A delay in the antenna network would not cause a positioning error, for the multi-satellite case as long as the delay were common to all measurements, but could cause a timing error for the time comparison user. It is therefore important that the receiver and its supporting instrumentation be calibrated for delay and such variable timing problems. Comparisons in reception time made with timing pulses, such as a 1 pulse per second time mark, should ensure rise time and pulse integrity for proper resolution and comparison.

Multipath into the receiving antenna is a substantial problem, especially near large reflecting surfaces. These effects are common and can be large error contributors.

GPS TIME TRANSFER

The time comparison user has another complication in that the offset from GPS Time to UTC-(USNO) must likewise be known. The procedures for determining and providing the correction term to the GPS Control Segment for inclusion in the Navigation Broadcast message and earth

Figure 2 GPS SYSTEM TIME MAINTENANCE

orientation terms necessary for orbit determination are contained in ICD-GPS-202⁵.

There is an additional factor that limit UTC-(USNO) time recovery from GPS. This factor is operational philosophy, GPS has no requirement for time dissemination. To maintain time recovery better than the 100 nanoseconds level is not an overall system requirement. The maintenance of GPS Time relative to UTC(USNO) is to be kept to one micro-second and knowledge of the difference to 100 nanoseconds is therefore subject to operational expediency. Even though it has been demonstrated that the system is capable of maintaining knowledge of UTC(USNO) to less than 25 nanoseconds. Figure 1 shows the value of UTC broadcast by the GPS satellites compared to UTC(USNO).

Figure 2 shows the processes involved in maintaining and disseminating time within the GPS system. Time recovery as discussed here is Time Comparison (4) in figure 2. This is a result of the overall process, in which time must be maintained throughout. Each of the steps in the process are possible sources of systematic and noise-like errors.

The errors as discussed above in final link with the user is the source of additional errors limiting performance. There are two types of residual errors in the link to the users. The first, as seen in figure 3, is systematic errors observed in a pass of data. The points shown are the broadcast time, relative to a station clock, for one satellite as it passes from horizon to horizon. This systematic effect has been called the "bowing effect". The effect is the most extreme using a single frequency C/A receiver but is apparent in data from dual frequency receivers as well. By smoothing individual 13 minute ranging data points over a satellite pass and many passes, USNO has been able to recovery UTC(USNO) on a daily basis from the GPS message as shown in Figure 1 to less than 25ns for long periods. It is important to note that this level of time recovery is achieved by processing all data (<96 tracks per day), and that with limited data (<10 per day), the time recovery is likely to be closer to 100ns than 25ns. The bowing effect appears to be dealt with in the short term of a few days by the smoothing process, but the mechanism of the effect is not well understood. The principal cause for this effect has been attributed to propagation effects. However, some data, Figure 4, suggests a ramping effect not easily attributable to propagation. Possible long term effects have not been fully investigated.

To investigate a possible long term noise-like effect, an ensemble of clocks is needed since the behavior could be below the noise level of a moderately well behaved cesium. Figure 5 is data taken over the same timeframe from reliable clock ensembles in the western pacific. The magnitude of the effect varies between them, but

the frequency shifts appear to correlate. There are other sites in this region that show similar correlations. These data would normally be a measure of each site versus UTC(USNO) and the frequency performance would be considered to be due to individual sites. However, these sites are considered to be very stable. The correlation of the frequency shifts between the sites would generally tend to indicate that the movement was in either the GPS time base or UTC(USNO). USNO compared by common view to UTC(BIPM) shows no similarities with the western Pacific is shown during this period. The common view comparison should eliminate GPS time error contributions thereby tending to isolate the GPS time base as a possible source of the performance.

DATA COLLECTION

Data has been gathered for this investigation on actual performance for isolation of the error effects through the cooperation of USNO, the GPS Master Control Station, Defence Mapping Agency (DMA), Detachment C of the Naval Space Operations Center, the Australian Orroral Valley Observatory, and the Physics and Engineering Laboratory, Lower Hutt, New Zealand. These data included received GPS observations and data on the clocks used in data collection. Single frequency receivers were used in the overseas sites and a dual frequency at USNO. Precise ephemerides for the GPS satellites were provided by DMA and GPS broadcast data were obtained through the Cooperative International GPS Tracking Network computer center at the National Geodetic Survey.

Data collection from the various sources began 1 October 1990 with the intention of having coincident data from all sources. Prior to data collection site surveys were conducted at all sites to verify calibration and a clear understanding of all the parameters effecting the analysis of the data.

GPS TIME MEASUREMENT

The GPS time recovery by pseudoranges (PR)

time values at a stationary site can be expressed as,

$$t_{LC} - t_{GPS} = PR_M - PR_C$$

This produces the difference in clock values between the local clock (LC) used for the measured pseudorange (PR_M) and GPS time (t_{GPS}) as maintained by the satellite. The calculated pseudorange term (PR_C) is,

$$PR_C = c^{-1}P(x_G) - \Delta t_a + \Delta t_{sv},$$

$P(x_G)$ = Geometric range between SV position and receiver position, Δt_a = the ionospheric and tropospheric propagation correction, and $\Delta t_{sv} = t_{GPS} - t_{sv}$ (Actual time on the space vehicle (SV)) + Δt_{rel} (a correction for the relativistic time delay accounting for non-spherical nature of actual satellite orbit). The SV time is related directly to UTC-USNO), t_{UTC} by,

$$t_{UTC} - t_{GPS} = \Delta t_{GPS}$$

So that,

$$\Delta t_{sv} = t_{UTC} - \Delta t_{GPS} - t_{sv} + \Delta t_{rel}$$

The value of t_{LC} is the final desired value of the difference between a local clock and GPS time or UTC(USNO).

In normal system operation, the calculated term for the pseudorange is based on the data broadcast by the system. To examine the error components the DMA post processed data was used with the broadcast data. The post-processed derived pseudorange (PR_{DMA}) with the SV position information and the SV clock correction term is,

$$\begin{aligned} PR_{DMA} &= c^{-1}P(x_D) + \Delta t_D \\ &= c^{-1}P(x_D) + (t_{GPS} + t_{sv}) \end{aligned}$$

where, $P(x_D)$ = Position from post-processed tracking data, and Δt_D = The clock correction term derived from the post-processed estimate of clock offset.

The broadcast derived pseudorange (PR_B) provides predicted position information, predicted clock correction, and propagation corrections for ionospheric and tropospheric delays, so that,

$$PR_B = c^{-1}P(x_b) + \Delta t_b + \Delta t_a$$

where, $P(x_b)$ = SV broadcast position, Δt_b = the broadcast SV clock correction term, which is the predicted time error (t_e) between SV clock and t_{GPS} , and Δt_a = the propagation correction term.

The measured pseudorange (PR_M) contains the composite terms,

$$PR_M = c^{-1}P(x_B) + \Delta t_b + \Delta t_{LC} + \Delta t_{a_a} + E$$

where, Δt_{LC} = local clock offset, Δt_{a_a} = actual propagation delay, and E = other residual error including instrumentation effects.

DATA COMPARISON

These measurement data sets may be combined for comparison and examination of the components. The pseudoranges from the broadcast and the post-processed should, in theory, be equivalent, as,

$$PR_B - PR_D = \Delta P(x_B - x_D) + c(\Delta t_b - \Delta t_D).$$

These data being a function of the position vectors and clock offset of the SV clock from GPS Time. Comparing the radial position component of these two data sets, which should be the most sensitive component, for GPS SV3 and SV16 are shown in figures 6 and 7. The residual error in these figures shows a significant error in the broadcast positions. These errors appear for all SV's examined, and longer data sets are to be investigated.

The SV clock comparison term above for SV 11, 16, 20 and 21 are shown in figures 8 to 11, respectively. These data are residuals to a linear fit to the post-processed data, and a linear term was used because the broadcast clock term are likewise linear. Higher order models will be used

in further comparisons. The spread in these data suggests some difficulty in estimating the clock term. The magnitude of the errors vary between the SVs, but the minimums appear to be on the order of 10-15 nanoseconds over the prediction interval. These data further suggest that the post-processed solution is quite noisy and not indicative of clock-like behavior. This behavior being indicated data from the USNO receiver was used for comparison and verification of the post-processed solution. One comparison is shown in figure 12. In these data the USNO appears to have a different daily rate, but overall, following the general trend of the clock. Further investigation showed that if the relativistic correction (for orbit eccentricity) were removed from the USNO observations the data shows a better correlation as shown in figure 13. The residuals of the data from figure 13 are shown in figure 14.

The orbital and clock comparisons shown in figure 15 demonstrate the correlation between the orbital position and clock error ambiguity present in this passive system. These data also show the overlap period used in the post-processing to prevent discontinuities between the orbit fit periods. In this overlap period, the clock residual differences at the same observation time are the result of the orbit fits made in the two successive weeks using largely different data. The variation between the two fits could be a significant error contributor coupled with other sources.

These comparisons suggest the presence of structured signals in the error components for orbit and clock offset, exclusive of propagation effects. These error signals that would be unseen by the timing user, directly effect the short term performance and possible introduce a long term feature.

Data taken by participating sites to contribute to component error resolution compared to UTC as broadcast by GPS are shown in figures 16 through 19. The long term structure in these data do not appear to correlate with GPS Time or UTC(USNO) variations, although the analysis is not complete. These sites are in general using an

ensemble of clocks and the data is to be examined. Independent calibrations or comparisons are being developed for comparative analysis.

PRELIMINARY CONCLUSIONS/RESULTS

Data collection and analysis for this effort is continuing. Considerable analysis and data correlation is yet to be performed. However, the preliminary conclusions made are that the Bowing effect may have major contribution from a number of different contributors. Orbit and clock residual error signatures imposed on the pseudorange residuals appear to be the large contributors. Analysis of the propagation factors which would classically be expected to produce a bowing type effect has yet to be fully investigated.

It does seem reasonable to conclude that the spread of data and error growth in orbit, clock and possibly propagation factors is larger as observed in the southern pacific area. Data spread and noise from the data there appears to bear this out. A spectrum analysis of the error structure is to be examined in looking for a long term error structure.

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Figure 1, Universal Coordinated Time as Disseminated by GPS vs UTC(USNO)

Figure 3, Single Frequency Receiver at GPS Master Control Station Observation of SV21

Figure 4, Single Frequency Receiver at GPS Master Control Station
Observation of SV21

Figure 5, Observations of SV09 at Two Sites in the Southern Pacific

Figure 6, SV03 Radial Position Difference (Broadcast vs DMA)

Figure 7, SV16 Radial Position Difference (Broadcast vs DMA)

Figure 8, SV11 Clock Residuals (Broadcast vs DMA)

Figure 9, SV16 Clock Residuals (Broadcast vs DMA)

Figure 10, SV20 Clock Residuals (Broadcast vs DMA)

Figure 11, SV21 Clock Residuals (Broadcast vs DMA)

Figure 12, SV11 Clock Offset to GPS Time, DMA and USNO
Observations corrected for Relativity

Figure 13, SV21 Clock Offset to GPS Time, DMA and USNO
Observations Uncorrected for Relativity

Figure 14, SV21 Clock Offset Residuals to GPS Time, DMA and USNO Observations Uncorrected for Relativity

Figure 15, SV11 Radial Position Difference and Clock Offset to GPS Time Residuals in Nanoseconds

Figure 16, New Zealand Site vs UTC as Disseminated by GPS

Figure 17, Orroral Valley vs UTC as Disseminated by GPS

Figure 18, GPS Master Control Station Receiver vs UTC as Disseminated by GPS

Figure 19, Hawaii Site vs UTC as Disseminated by GPS