

Robust Stability and Performance of Adaptive Jitter Suppression in Laser Beam Pointing

Saeid Jafari, Petros Ioannou, Ben Fitzpatrick, and Yun Wang

Abstract—This paper studies the robustness and performance of an adaptive scheme for suppression of laser beam jitter in the presence of unmodeled dynamics in a continuous-time formulation. To demonstrate the level of performance that can be achieved in the case of known disturbances, at first we consider the ideal case (non-adaptive) when a complete information about characteristics of the noise-corrupted periodic disturbance is available. Then the adaptive case when the disturbance is unknown with time-varying characteristics is investigated. The results show under what conditions jitter suppression can be achieved and what parameters contribute to the performance of the adaptive control scheme. Numerical simulations are given to demonstrate the efficacy of the robust adaptive control law.

I. INTRODUCTION

The applications of laser systems have grown significantly in recent decades, including areas such as communications [1], [2], detection and ranging [3], medicine [4] and defense [2], [5], [6]. Of particular interest in many laser applications are problems of beam control, ranging from pointing and tracking to wavefront control. Performance of beam control systems is often adversely affected by difficult-to-characterize disturbances that arise from the medium of propagation, structural vibrations in the platform, or other external factors [7]. In particular, much of the jitter in laser beam control is due to periodic disturbances whose frequencies and amplitudes are unknown and could vary with time. If the frequencies of these disturbances were fixed, well-known internal model control techniques could be used to attenuate their effect. In the case of unknown and time changing frequencies the corresponding internal model of the disturbances involves unknown parameters. In such case, adaptive control techniques could be used to estimate on line the parameters of the internal model and use them in the feedback loop for disturbance rejection.

The problem of adaptive rejection of unknown sinusoidal disturbances has been already investigated in many applications such as acoustic noise cancelation, mechanical vibration control, wavefront control in adaptive optics, and active tracking in laser systems [8]–[14]. In [15], adaptive rejection of band-limited disturbances with unknown frequencies is studied through an internal model principle. Recently, in [16]–[18], adaptive suppression of unknown and time-varying sinusoidal disturbances is studied. In the

area of laser point and tracking, a variable-order recursive least-squares lattice filters is used in [19] for suppression of laser beam jitter and it is shown that by using a high-order adaptive controller the output-error variance in the presence of complex jitter having multiple physical sources and multiple bandwidths can be significantly reduced. A frequency-weighting method is introduced in [20] that improves the performance of the approach of [19]. The efficacy of the proposed scheme and performance improvement in jitter reduction are demonstrated by experiment. The problem of unknown time-varying periodic disturbance rejection is investigated in some other references such as [21], [22]; however, no robustness analysis is provided in the presence of unmodeled dynamics. It is known that adaptive techniques such as these may fail in some circumstances [23]. Applying adaptive control in the demanding applications of beam control requires theoretical justification and guaranteed performance for integration into advanced laser platforms.

In this paper, it is aimed to propose a robust adaptive control scheme for disturbance rejection in laser beam jitter control and show analytically its stability and performance in the presence of unmodeled dynamics. Despite the numerous application of similar adaptive control schemes there is analysis that shows that such schemes are robust in the presence of modeling errors. We begin with the ideal non-adaptive case in which we have complete information about the characteristics of the disturbance. We therefore first examine an optimum design that completely rejects disturbances in an ideal environment where full information is available. Such a design provides the form of the controller based on which we design the adaptive control scheme by using the certainty equivalence principle.

This paper is organized as follows. In Section II, the problem is defined and formulated. Section III presents the main results of the paper and simulation results are given in Section IV. The paper is finally concluded in Section V.

II. A BEAM CONTROL SYSTEM MODEL – A CONTINUOUS-TIME FORMULATION

Consider stable plants of the form

$$\begin{aligned} y(t) &= G(s)[u(t)] + d(t) \\ &= G_0(s)(1 + \Delta_m(s))[u(t)] + d(t), \end{aligned} \quad (1)$$

in which u is the control input, d is an unknown and unmeasurable bounded disturbance, y is the measured output. The plant $G(s)$ is typically not perfectly known. We model $G_0(s)$ as a known estimate of $G(s)$ (possibly non-minimum phase), with $\Delta_m(s)$ representing an unknown multiplicative

S. Jafari and P. Ioannou are with the Department of Electrical Engineering, University of Southern California, {sjafari, ioannou}@usc.edu.

B. Fitzpatrick and Y. Wang are with Tempest Technologies Inc., {fitzpatrick, yun.wang}@tempest-tech.com.

uncertainty such that $G_0(s)\Delta_m(s)$ is stable. In many, if not most, beam control applications, the system has already been stabilized with fixed-gain controller (often a simple integrator), and the control system (1) is an augmentation of the baseline original control design [13], [19], [20].

We assume that the disturbance can be written as

$$d(t) = d_s(t) + v(t), \quad (2)$$

where $d_s(t) = \sum_i a_i \sin(\omega_i t + \varphi_i)$, the parameters a_i , ω_i , and φ_i are all unknown and possibly time varying, and $v(t)$ is a bounded noise disturbance.

It is aimed to design a filter $K(s)$ to minimize the norm of output signal y . Figure 1 illustrates the closed-loop system in which the disturbance rejection is based on the model plant $G_0(s)$.

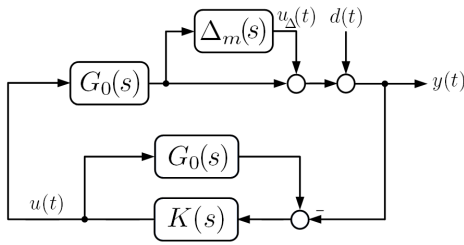


Fig. 1. Structure of the closed-loop system

We assume $\Delta_m(s)$ is such that $G_0(s)K(s)\Delta_m(s)$ is strictly proper with stable poles. Therefore, a sufficient condition for stability of the closed-loop system is that

$$\|G_0(s)K(s)\Delta_m(s)\|_\infty < 1, \quad \forall s, \quad (3)$$

that is if the unmodeled dynamics satisfies a norm bound condition, the BIBO stability of the closed-loop system is ensured. Since

$$y(t) = (1 - G_0(s)K(s)) [d(t) + u_\Delta(t)],$$

it is clear that with $K(s) = G_0^{-1}(s)$ the output signal $y(t)$ will be zero for any $d(t)$ and for all $\Delta_m(s)$'s which satisfies (3); however, such a filter is not realizable. In the next section, we undertake the design and analysis of a simple filter structure for disturbance suppression.

III. CONTROLLER DESIGN FOR DISTURBANCE REJECTION

The experimentally demonstrated effectiveness of least-squares based algorithms [13], [19], [20] for the beam control problem of Section II suggests that performance bounds and stability theory should be attainable. In this section, we provide such conditions along with the requisite analysis. We begin with an examination of a fixed-gain controller in the presence of known disturbance characteristics, whose results we then use to establish stability for the adaptive control problem.

A. Non-adaptive Control Design: Known Disturbance Case

This subsection considers the ideal case when a complete knowledge about the disturbance is available to investigate the level of performance can be achieved (which is the best one). Then, we consider a same architecture to study unknown disturbance case.

Assume $d_s(t)$ in (2) is known with the generating polynomial $D_s(s)$ whose roots are all on the imaginary axis. It is well known that if the zeros of the sensitivity transfer function $S(s)$ given by

$$\begin{aligned} y(t) &= S(s)[d(t)] \\ &= \frac{1 - G_0(s)K(s)}{1 + G_0(s)K(s)\Delta_m(s)} [d_s(t) + v(t)] \end{aligned} \quad (4)$$

contains the roots of $D_s(s)$, then the effect of $d_s(t)$ on $y(t)$ will be eliminated.

Let $K(s)$ be of the form

$$K(s) = \frac{\Theta(s)}{\Lambda(s)} \triangleq \frac{\theta_N s^N + \dots + \theta_1 s + \theta_0}{\Lambda(s)} = \theta^T \alpha(s), \quad (5)$$

where $\Lambda(s) = (s + \lambda)^{N+1}$, $\lambda > 0$ is a design constant, $\theta = [\theta_0, \theta_1, \dots, \theta_N]^T \in \mathbb{R}^{N+1}$ is the constant coefficient vector of the filter and $\alpha(s) \triangleq \frac{1}{\Lambda(s)} [1, s, \dots, s^N]^T$. Let $G_0(s) = \frac{B(s)}{A(s)}$, where $A(s)$ is a monic Hurwitz polynomial of degree n and $B(s)$ is a polynomial of degree at most n . Then, the sensitivity transfer function (4) can be written as

$$S(s) = \frac{A(s)\Lambda(s) - B(s)\Theta(s)}{A(s)\Lambda(s) + B(s)\Theta(s)\Delta_m(s)}, \quad (6)$$

where $\Theta(s) = \theta_N s^N + \dots + \theta_1 s + \theta_0$. Now, the objective is to find parameter vector θ so that the norm of $y(t)$ is minimized.

Lemma 1: There exist polynomials $\Theta(s)$ and $L(s)$ so that the zeros of the sensitivity transfer function (6) contain the roots of polynomial $D_s(s)$, i.e., $A(s)\Lambda(s) - B(s)\Theta(s) = D_s(s)L(s)$, if and only if $B(s)$ and $D_s(s)$ are coprime and $N + 1 \geq n_d$, where $n_d = \deg(D_s(s))$.

Proof: The proof is straightforward and is omitted due to the space limitation.

If $N + 1 = n_d$, there exists a unique θ satisfying the conditions of Lemma 1; however, the norm of the sensitivity transfer function $S(s)$ may be much larger than one, especially when $B(s)$ has roots close to those of $D_s(s)$. This situation is not desirable, because it leads to noise amplification. Moreover, the stability condition (3) may not be satisfied even with a relatively small size unmodeled dynamics.

By choosing a higher order filter, $N + 1 > n_d$, we have infinite number of θ vectors for which we can completely reject the sinusoidal part of the disturbance. At this point, we may look for vectors θ from this set that minimize the

objective function

$$J(\theta) = \left\| \frac{B(s)\Theta(s)}{A(s)\Lambda(s)} \right\|_{\infty}, \quad (7)$$

$$\text{s.t. } A(s)\Lambda(s) - B(s)\Theta(s) = D_s(s)L(s).$$

The existence of a minimizer is the subject of the following lemma.

Lemma 2: Consider the closed-loop system shown in Figure 1. Assume the filter is of the form (5) and the output disturbance can be represented as in (2). If $B(s)$ and $D_s(s)$ have no common factors and $N + 1 > n_d$, then there exists a $\theta = \theta^*$ with $|\theta| \leq r$, for some $r > 0$, for which the effect of $d_s(t)$ on $y(t)$ is completely rejected and the cost function (7) is minimized.

Proof: Due to the space limitation, the proof is omitted.

Lemma 2 shows the existence of constant parameters for complete disturbance rejection of the class defined by (2). The values of θ^* can be simply found by solving an LMI optimization problem. Although for a given plant model under the aforementioned conditions with $N = n_d - 1$ we can completely reject the sinusoidal terms, the norm of $S(s)$ and $J(\theta^*)$ may be relatively large which leads to noise amplification and stability margin reduction. Therefore, we may need to choose a larger N to make the norm of $S(s)$ and $J(\theta^*)$ and hence $y(t)$ as small as possible.

Theorem 1: Consider the closed-loop system shown in Figure 1 with the filter (5). Assume the frequency components of $d_s(t)$ are known (i.e., internal model of $d_s(t)$, $D_s(s)$, is known), $N + 1 \geq n_d$, where $n_d = \deg(D_s(s))$, and θ^* is a solution of the optimization problem (7). If the inequality

$$\|G_0(s)K(s, \theta^*)\Delta_m(s)\|_{\infty} < 1 \quad (8)$$

holds for all s , then

$$\limsup_{t \rightarrow \infty} \sup_{\tau \geq t} |y(\tau)| \leq \left\| \frac{1 - G_0(s)K(s, \theta^*)}{1 + G_0(s)K(s, \theta^*)\Delta_m(s)} \right\|_1 v_0, \quad (9)$$

where $v_0 = \sup_t |v(t)|$. In addition, in the absence of the noise (i.e., when $v_0 = 0$), the plant output $y(t)$ goes to zero exponentially fast with time.

Proof: The proof is omitted due to lack of space.

It is clear from Theorem 1 that when the frequencies of the sinusoidal part of the disturbance are known, the effect of $d_s(t)$ on $y(t)$ can be completely rejected.

With these results in hand, we turn now to the problem of primary interest: rejection of periodic components with unknown frequencies, amplitudes, and phases.

B. Adaptive Control Design: Unknown Disturbance Case

In practice, the statistics of the disturbance signal are unknown and time-varying, so an adaptive control scheme is often required to provide high performance suppression of the effects of disturbance on the plant output. In this

subsection, our aim is to design and analyze an adaptive filter of the form

$$K(s, \hat{\theta}) = \hat{\theta}^T(t)\alpha(s)$$

in the feedback loop. Clearly, if the characteristics of the disturbance are known, we can compute a minimizer θ^* to minimize the output norm by rejecting the sinusoidal components of the disturbance; however, since $d(t)$ is unknown, an adaptively tuned filter $K(s, \hat{\theta})$ is designed to minimize the output variance, canceling to the extent possible the disturbance. In this case, we have the following relationships.

$$\begin{aligned} y(t) &= G(s)[u(t)] + d(t), \\ z(t) &= y(t) - G_0(s)[u(t)] \\ &= d(t) + u_{\Delta}(t), \\ u(t) &= -\tau_0 K(s, \hat{\theta})[z(t)] \\ &= -\tau_0 \hat{\theta}^T(t)\alpha(s)[z(t)] \\ &= -\hat{\theta}^T(t)w(t), \end{aligned} \quad (10)$$

where $w(t) = \tau_0 \alpha(s)[z(t)]$ and τ_0 is the reciprocal of the high-frequency gain of the nominal plant $G_0(s)$ which is used for normalization. Then the plant output can be written as

$$\begin{aligned} y(t) &= G(s)[u(t)] + d(t) \\ &= G_0(s)[u(t)] + z(t) \\ &= -G_0(s)[\hat{\theta}^T(t)w(t)] + z(t). \end{aligned} \quad (11)$$

Applying the Swapping Lemma [24], we have

$$\begin{aligned} G_0(s)[\hat{\theta}^T w] &= \hat{\theta}^T G_0(s)[w] \\ &\quad + G_{0c}(s)[(G_{0b}(s)[w^T])\dot{\hat{\theta}}], \end{aligned} \quad (12)$$

where $G_{0c}(s) = -\mathcal{C}_0^T(sI - \mathcal{A}_0)^{-1}$, $G_{0b}(s) = (sI - \mathcal{A}_0)^{-1}\mathcal{B}_0$, and the quadruple $(\mathcal{A}_0, \mathcal{B}_0, \mathcal{C}_0, \mathcal{D}_0)$ represents a minimal realization of $G_0(s)$. Thus, from (11), (12) we obtain

$$\begin{aligned} y(t) &= -\hat{\theta}^T(t)G_0(s)[w(t)] - G_{0c}(s)[(G_{0b}(s)[w^T(t)])\dot{\hat{\theta}}(t)] \\ &\quad + z(t) \\ &= -\hat{\theta}^T(t)G_0(s)[w(t)] + \theta^{*T}G_0(s)[w(t)] \\ &\quad - \theta^{*T}G_0(s)[w(t)] - G_{0c}(s)[(G_{0b}(s)[w^T(t)])\dot{\hat{\theta}}(t)] \\ &\quad + z(t) \\ &= -\tilde{\theta}^T(t)G_0(s)[w(t)] \\ &\quad + (1 - \tau_0 G_0(s)K(s, \theta^*)) [z(t)] \\ &\quad - G_{0c}(s)[(G_{0b}(s)[w^T(t)])\dot{\hat{\theta}}(t)], \end{aligned}$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta^*$; therefore, we can write

$$y(t) + \eta_0(t) = -\tilde{\theta}^T(t)\phi(t) + \eta(t), \quad (13)$$

where

$$\begin{aligned} \phi(t) &= G_0(s)[w(t)] \\ \eta_0(t) &= G_{0c}(s)[(G_{0b}(s)[w^T(t)])\dot{\hat{\theta}}(t)] \\ \eta(t) &= (1 - \tau_0 G_0(s)K(s, \theta^*)) [z(t)] \\ &= (1 - \tau_0 G_0(s)K(s, \theta^*)) [d_s(t) + v(t) + u_{\Delta}(t)]. \end{aligned} \quad (14)$$

From (13), (12), and (11) we have

$$\begin{aligned}\theta^{*T}\phi(t) + \eta(t) &= y(t) + \hat{\theta}^T(t)\phi(t) + \eta_0(t) \\ &= y(t) + G_0(s)[\hat{\theta}^T(t)w(t)] \\ &= z(t).\end{aligned}$$

Thus, we have the following parametric model where the unknown parameter θ^* appears linearly with known signal vector $\phi(t)$.

$$\zeta(t) \triangleq z(t) = \theta^{*T}\phi(t) + \eta(t).$$

Since $\eta(t) = (1 - \tau_0 G_0(s)K(s, \theta^*)) [d_s(t) + v(t) + u_\Delta(t)]$ and that the zeros of $1 - \tau_0 G_0(s)K(s, \theta^*)$ contains the disturbance modes, then the modeling error term $\eta(t)$ is due noise and unmodeled dynamics.

This parametric model can be used to develop a wide class of robust parameter estimators using the approach in [24], [25] as follows: If $\hat{\theta}(t)$ is the estimate of θ^* at time t , the predicted value of the signal $\zeta(t)$ can be generated as

$$\hat{\zeta}(t) \triangleq \hat{\theta}^T(t)\phi(t).$$

We define the normalized estimation error by

$$\begin{aligned}\varepsilon(t) &= \frac{\zeta(t) - \hat{\zeta}(t)}{m^2(t)} = \frac{-\hat{\theta}^T(t)\phi(t) + \eta(t)}{m^2(t)} \\ &= \frac{y(t) + \eta_0(t)}{m^2(t)},\end{aligned}\quad (15)$$

where $m(t)$ is a normalizing signal designed to guarantee that $|\phi(t)|/m(t)$ and $|\eta(t)|/m(t)$ are bounded.

To develop an adaptation scheme for θ , we consider the robust pure least-squares algorithm [24], [25]

$$\dot{P}(t) = -P(t) \frac{\phi(t)\phi^T(t)}{m^2(t)} P(t) \quad (16a)$$

$$\varepsilon(t) = \frac{\zeta(t) - \hat{\theta}^T(t)\phi(t)}{m^2(t)} \quad (16b)$$

$$\dot{\hat{\theta}}(t) = P(t)\varepsilon(t)\phi(t), \quad (16c)$$

where $P(0) = P_0 = P_0^T > 0$ and $m(t) > 0$ is the normalizing signal given by

$$\begin{aligned}m^2(t) &= 1 + n_d(t) \\ \dot{n}_d(t) &= -\delta_0 n_d(t) + |z(t)|^2,\end{aligned}\quad (17)$$

where $n_d(0) = 0$ and $\delta_0 > 0$ so that $G_0(s)$ is analytic in $Re[s] \geq -\frac{\delta_0}{2}$.

Projection can be used in (16c) to guarantee that $\hat{\theta}(t) \in \mathbb{S}$, $\forall t$, where \mathbb{S} is a compact set defined as

$$\mathbb{S} = \{\hat{\theta} \in \mathbb{R}^{N+1} \mid g(\hat{\theta}) \triangleq \hat{\theta}^T \hat{\theta} - \theta_{\max}^2 \leq 0\},$$

where $\theta_{\max} > 0$ is chosen so that the coefficient vector of the optimum filter, θ^* , belongs to \mathbb{S} . Since we do not have a priori knowledge on the norm of θ^* , θ_{\max} must be chosen sufficiently large. An alternative to projection is a fixed σ -modification where no bounds for $|\theta^*|$ are required [24]. An additional constraint that is often used in practice to ensure

that the control signal u is within certain limits $\pm u_{\max}$ is a saturation block inserted right after $K(s, \hat{\theta})$.

In view of the above considerations, we write the adaptive closed-loop system as

$$\begin{aligned}y(t) &= G(s)[u(t)] + d(t) \\ z(t) &= y(t) - G_0(s)[u(t)] \\ \phi(t) &= \tau_0 G_0(s)\alpha(s)[z(t)] \\ \dot{P}(t) &= -P(t) \frac{\phi(t)\phi^T(t)}{m^2(t)} P(t) \\ \varepsilon(t) &= (z(t) - \hat{\theta}^T(t)\phi(t))/m^2(t) \\ \dot{\hat{\theta}}(t) &= \begin{cases} P\varepsilon\phi & \text{if } \hat{\theta} \in \mathbb{S}_0 \\ & \text{or if } \hat{\theta} \in \delta\mathbb{S} \text{ and} \\ & (P\varepsilon\phi)^T \nabla g \leq 0, \\ P\varepsilon\phi - P \frac{\nabla g \nabla g^T}{\nabla g^T P \nabla g} P\varepsilon\phi & \text{otherwise,} \end{cases} \\ u(k) &= -\tau_0 \hat{\theta}^T(t)\alpha(s)[z(t)]\end{aligned}\quad (18)$$

where

$$\begin{aligned}\mathbb{S}_0 &= \{\hat{\theta} \in \mathbb{R}^{N+1} \mid g(\hat{\theta}) = \hat{\theta}^T \hat{\theta} - \theta_{\max}^2 < 0\}, \\ \delta\mathbb{S} &= \{\hat{\theta} \in \mathbb{R}^{N+1} \mid g(\hat{\theta}) = \hat{\theta}^T \hat{\theta} - \theta_{\max}^2 = 0\},\end{aligned}$$

and the normalizing signal $m(k)$ is given by (17).

In implementing the above scheme we also need to monitor the covariance matrix $P(t)$ to make sure it does not become small in some directions i.e. its minimum eigenvalue is always greater than a small positive constant. Such modifications are presented and analyzed in [24] and other references on adaptive control. The performance of this adaptive system can be guaranteed as is detailed in the following theorem in which it is assumed that the characteristics of the disturbance is unknown, but there exists a θ^* (unknown to designer) to achieve complete sinusoidal disturbance rejection, i.e., the conditions given in Lemma 2 are satisfied.

Theorem 2: Consider the closed-loop system (18) and assume $G_0(s)\Delta_m(s)$ is analytic in $Re[s] \geq -\frac{\delta_0}{2}$, for some $\delta_0 > 0$. If

$$\bar{\theta}_{\max} \|\tau_0 \alpha(s)\|_1 \|G_0(s)\Delta_m(s)\|_1 < 1, \quad \forall s \quad (19)$$

where $\bar{\theta}_{\max}$ is an upper bound for $\|\hat{\theta}(t)\|_1$, then all signals in the closed-loop system are uniformly bounded and the plant output satisfies

$$\frac{1}{T} \int_t^{t+T} |y(\tau)|^2 d\tau \leq c \|Q(s)G_0(s)\Delta_m(s)\|_{2\delta_0}^2 + cv_0^2 + \frac{c}{T}$$

for any T and t and some finite constant c independent of T, t , where $Q(s)$ is a transfer function analytic in $Re[s] \geq -\frac{\delta_0}{2}$. In addition, in the absence of modeling error and noise (i.e., when $\Delta_m(s) = 0$ and $v_0 = 0$), the adaptive law guarantees the convergence of $y(t)$ to zero.

Proof: The proof is omitted due to lack of space.

Theorem 2 states that the average energy of the plant output y is of the order of modeling error and the amplitude of the noise.

In the next section we present some simulation results to illustrate the performance of the adaptive control scheme.

IV. SIMULATION RESULTS

Consider the following model for the nominal plant

$$G_0(s) = \frac{0.0018(s - 0.1442)(s + 710.2)(s - 960)}{(s + 162.4)(s^2 + 1781s + 8.357 \times 10^5)} \quad (20)$$

which is a reduced-order model for a 19th-order transfer function $G(s)$ obtained by applying an identification algorithm to input-output data from the full-order plant. Figure 2 shows the bode plots of the nominal model $G_0(s)$ and actual full-order model $G(s)$.

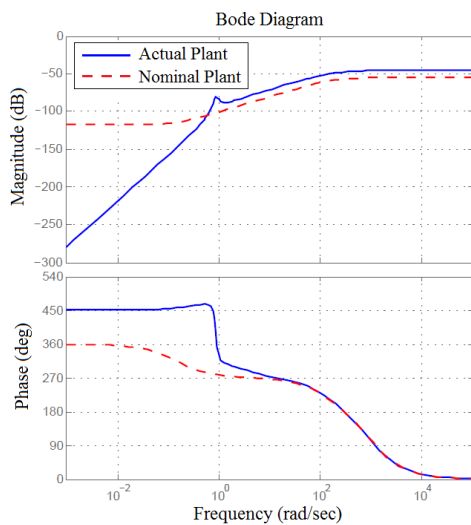


Fig. 2. Bode plot of the actual full-order model $G(s)$ and that the nominal model $G_0(s)$.

It is assumed that the disturbance $d_s(t)$ has four sinusoidal terms with frequencies $\omega_1 = 37.3$ rad/sec, $\omega_2 = 75.6$ rad/sec, $\omega_3 = 111.9$ rad/sec, and $\omega_4 = 303.5$ rad/sec and $|v(t)| \leq 0.05, \forall t$. Then, $D_s(s) = (s^2 + \omega_1^2)(s^2 + \omega_2^2)(s^2 + \omega_3^2)(s^2 + \omega_4^2)$. Since $D_s(s)$ and the numerator of $G_0(s)$ have no common factor, complete sinusoidal disturbance rejection is possible with $N \geq n_d - 1 = 7$.

Consider the plant (20) and assume $\delta_0 = 0.01$, $P(0) = 100I$, $N = 50$ and assume that the characteristics of the disturbance are not known to the controller. Figure 3 shows the output of the closed-loop system (18). Clearly, if the frequencies of the disturbance change the LTI controller will not work anymore and the closed-loop performance will be destroyed. However, in the adaptive scheme no information on the disturbance is required and in the face of any changes in the statistics of disturbance the controller updates its parameters and makes the output norm as small as possible.

Now, assume at $t = 20$ sec, three new sinusoidal terms with frequencies $\omega_5 = 53$ rad/sec, $\omega_6 = 203$ rad/sec, and $\omega_7 = 507$ rad/sec are added to the disturbance. Then for $t > 20$ sec, the disturbance $d(t)$ has seven unknown frequencies.

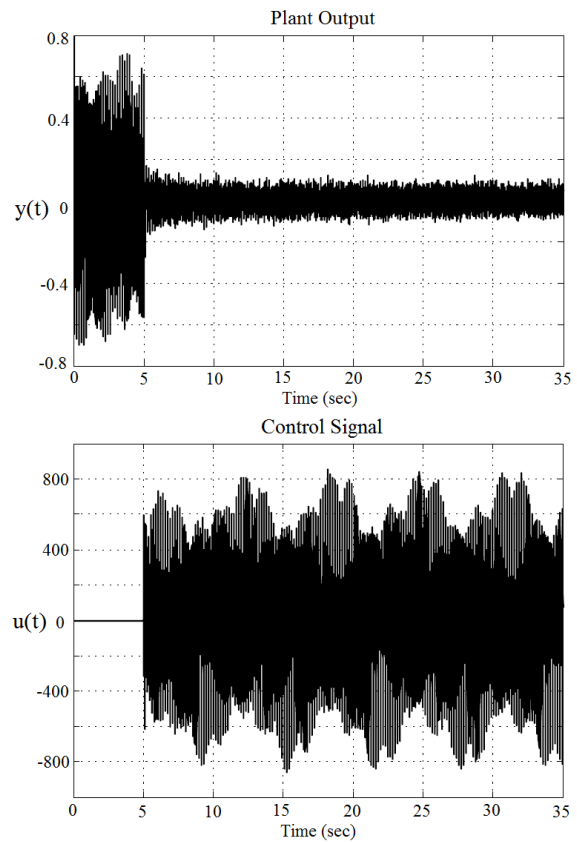


Fig. 3. The output of the full-order plant $G(s)$; the control is applied at $t = 5$ sec. Since $P(0)$ is large enough, the adaptive control law quickly identifies and rejects the sinusoidal disturbances.

Figure 4 and Figure 5 show simulation results. In Figure 4, the controller parameters are frozen at $t = 20$ sec, so for $t > 20$ sec we have an LTI closed-loop system; thus adding the three new sinusoidal terms to the disturbance at $t = 20$ sec destroys the performance. In Figure 4, the controller remains adaptive and after a short transient period minimizes the output norm.

V. CONCLUSIONS

This paper presents stability and performance analysis of an adaptive control scheme for jitter suppression in laser beam pointing in the presence of unmodeled dynamics. Algorithms such as the one analyzed in this paper have been in use in a number of experimental and prototype systems [13], [19], [20], but theoretical results establishing guaranteed performance bounds and stability criteria have been lacking. The results presented herein provide theoretical justification of algorithms that are coming into use in advanced laser beam control systems and also offer insight into reasons for failures in such systems. In future efforts we plan to generalize the analysis to the MIMO case and to study other types of disturbances of interest in beam control applications and examine experimental results in light of these results.

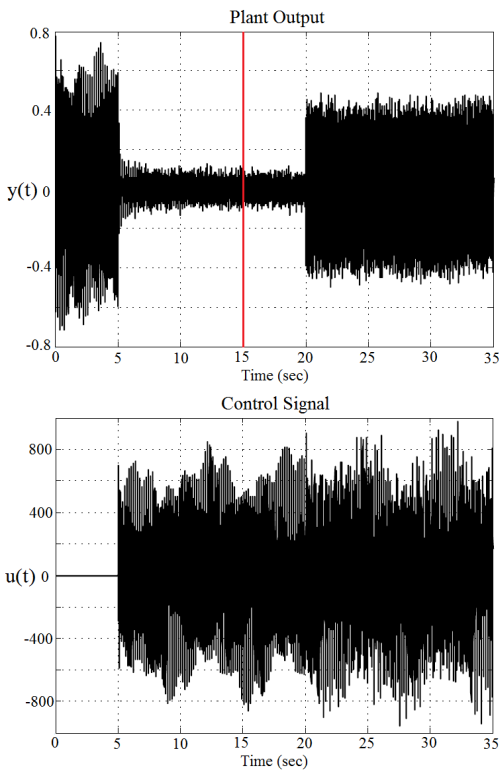


Fig. 4. The output of the full-order plant $G(s)$; the control is applied at $t = 5$ sec. The parameters of the adaptive controller are frozen at $t = 15$ sec, so for $t > 15$ sec, the closed-loop system is LTI. At $t = 20$ sec, three new sinusoidal terms are added to the disturbance. This destroys the closed-loop performance.

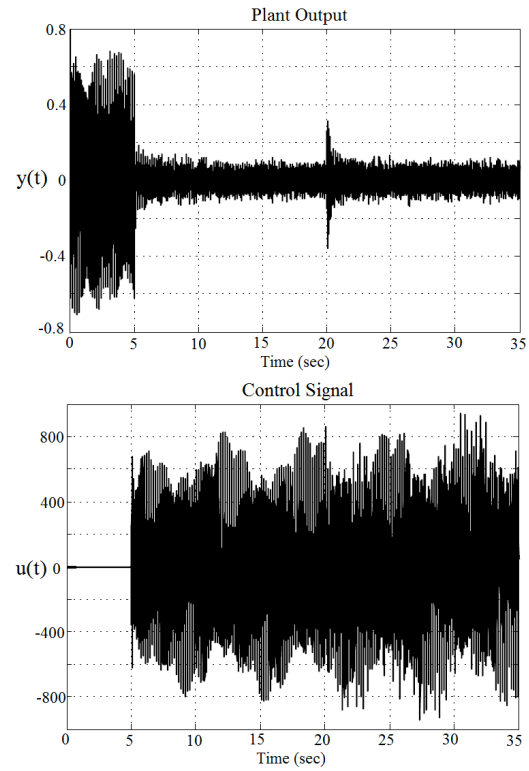


Fig. 5. The output of the full-order plant $G(s)$; the control is applied at $t = 5$ sec. The controller is adaptive for all t . At $t = 20$ sec, three new sinusoidal terms are added to the disturbance which are identified and rejected quickly by the adaptive control law.

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