

Trajectory Generation for Aircraft Subject to Dynamic Weather Uncertainty

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Abstract—Determining safe aircraft trajectories that avoid hazardous weather regions and other aircraft while efficiently using the available airspace is an important problem. Although tactical weather forecast maps have been available, their use in automated aircraft trajectory generation has not been fully explored or implemented. We consider aircraft trajectory generation using forecast data available from the Corridor Integrated Weather System product. The forecasts, updated at regular intervals, are used to design no-fly regions. We propose a receding horizon trajectory generation based on the dynamic nature of the forecast. Our method uses current environment information and incrementally updated forecast to update and optimize the trajectories. Simulations are shown based on forecast weather and indicate the advantage of taking into account time-varying forecasts in the planning horizon.

I. INTRODUCTION

The use of weather forecasts in trajectory planning and control of aircraft has clear importance and applicability. Aside from causing delays and inefficiencies costing the United States economy billions of dollars [19], weather is a contributing factor to thirty percent of all aviation accidents [1]. In recent years weather forecasts with increasing accuracy and resolution for the national airspace have become available, and their utilization in air traffic control has also begun recently. Currently, in a typical commercial flight, aircraft routes are planned in advance, using fixed predefined waypoints. During the flight, the weather map along the route is evaluated. In some cases, pilots reroute their flights to avoid storms, while in other cases, the air traffic controllers, who have access to detailed weather forecasts, may issue warnings or commands to the pilot to avoid flying through dangerous storms. In either case, the available forecast information is not yet used in an automated way to plan the aircraft trajectories in their en-route phase.

Flying through well-defined routes connecting fixed waypoints simplifies the problem of conflict resolution and air traffic management; however, it also does not efficiently use the airspace. The Next Generation (NextGen) air traffic project envisions accommodating an increased amount of air traffic safely and efficiently [5]. One limiting factor in increasing the capacity of the airspace is the controller's workload. Hence, to achieve the NextGen objective, we propose to automate some of the air traffic control tasks such as separation between aircraft and between aircraft and hazardous weather. In order to implement this concept, an

important problem that needs to be addressed, in addition to equipping aircraft with appropriate sensing and communication technologies, is decentralized conflict resolution of aircraft in the presence of storms and hazardous weather. This work is an initial step in automatic use of forecast data to plan paths that avoid hazardous weather and conflict with other aircraft.

A. Previous Research

If one thinks of regions with storms or hazardous weather as obstacles in the airspace, then the problem of control of multiple aircraft through hazardous weather is similar to the problem of control of multiple agents through obstacles. There is extensive literature on this topic and numerous algorithms have been developed to address safe path planning and control of autonomous agents. The authors in [10] consider collision avoidance control of multiple aircraft and develop an algorithm for addressing general constrained nonlinear multi-objective optimization problems in a decentralized way. The authors in [14] develop decentralized collision-free feedback control laws using potential function methods to steer agents to their destinations while avoiding constraints. This formulation assumes a stationary known environment and no constraints on the input. The authors in [25] combine the above method with a model predictive control to implement a collision-free controller for multiple aircraft. In works such as [8], [20], [2], [27], the trajectory planning and control are addressed using a receding horizon framework, which can accommodate a changing environment. Given the amount of research on trajectory optimization and control, discussing all of these algorithms is not the scope of this work. However, it becomes clear from the previous work that one of the main remaining challenges in air traffic control is appropriately utilizing the weather information in the existing trajectory generation methods.

Given the availability of forecast data with increasing resolution, there has been growing interest in using this available data for air traffic management. The authors in [9], [15] determine stochastic motion models for wind based on forecast and aircraft measurements and utilize this information in estimation and prediction of aircraft state. The authors in [13] assume static weather regions, and design en-route routes which avoid hazardous weather. The extension to dynamic weather constraints is considered in [23]. The authors in [18] utilize weather forecast in the terminal space. They develop machine learning algorithms to find stochastic weather maps from the deterministic forecasts and use this information to determine aircraft routes that are robustly safe to fly through. Additionally, the Route Availability

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Planning Tool (RAPT) has been developed to help air traffic controllers assess the availability of departure routes prior to aircraft take-off to better utilize the airspace [24].

The above works on integrating forecast into air traffic management either focus on terminal airspace or consider the availability of fixed routes or regions. Motivated by the concept of each aircraft optimizing its own flight profile, in our formulation, we do not consider fixed routes but rather allow each aircraft to optimize its trajectory while avoiding dynamic impassible weather regions and other aircraft. We consider en-route flight areas which have a much more significant impact on overall system delays than terminal flight [4], [7]. The work of [7] also found that storms cause more significant delay when they pass over areas away from airports, near the borders of en-route and terminal airspaces. While previous work has considered aircraft separation, for example [28], [29], [16], [25], we extend these works by integrating weather information. One of the initial works on use of weather information in en-route aircraft planning is in [21]. However, this formulation assumes probabilistic storm appearances and these probabilities are not based on real forecast data. The work of [22] presents recent research that considers integrating the weather forecast with aircraft control. In this work, only static weather is considered and a cost for a model predictive control law based on a Hamilton-Jacobi-Bellman approximation of the minimum-time to reach the destination point is presented. Inspired by this work, we consider the additional complexity of a dynamic weather forecast in our formulation.

Our contributions are using a forecast product to account for the dynamic forecasts in a systematic way and to estimate dynamic of the forecast during the time intervals between the forecast updates. Based on the dynamic forecast, we define a receding horizon trajectory planner for aircraft with a cost function that is computationally simpler than that of [22]. We also compare our simulation results with actual weather data and outline several promising directions for future research. This report is organized as follows: in Section II we describe the problem under consideration and set the mathematical framework of the model. In Section III we describe our approach in use of the forecast data in a receding horizon trajectory generation framework. In Section IV we show the results of simulations for a section of airspace using the forecast and actual weather data. Finally, we conclude the report in Section V and discuss directions for our future work.

II. PROBLEM STATEMENT

The guidance and navigation of an aircraft is a hierarchical procedure in which a high level trajectory planner designs a feasible trajectory for the aircraft based on a given reference path or initial condition and final destination of the aircraft. Due to computational limitations, this high level planner typically assumes a low fidelity model of the aircraft. The low-level controller or autopilot then considers a high fidelity model and provides the control commands, such as thrust and bank angle, to ensure trajectory tracking [8]. An architecture

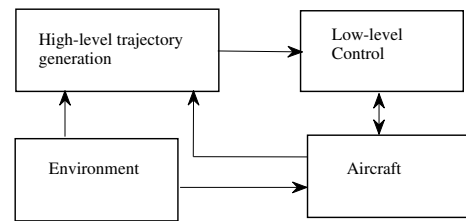


Fig. 1. Hierarchical structure of aircraft navigation and control.

for aircraft navigation and control is shown in Figure 1. In the figure shown, the environment represents the weather and wind conditions, and other nearby aircraft. Currently, the weather information available from forecasts is not taken into account in a systematic way in the trajectory generation block. Our focus is on improving this link.

Most current approaches in trajectory generation either do not use weather forecast data, or only consider the forecast available at the time of trajectory generation. Due to the movement of the storms and hazardous weather or changes in wind profiles, the resulting trajectories are often either suboptimal or unsafe. Additionally, planning may be overly conservative and lead to unnecessary ground holding if a good forecast is not used.

The ability of receding horizon control framework in taking into account changes in the environment and plant makes it very suitable for the problem at hand. Receding horizon control has been applied both for high level trajectory generation and low level vehicle control [11], [8], [20]. In [30] receding horizon trajectory generation based on idea of differential flatness is considered for complex nonlinear systems. The authors in [26] utilize a receding horizon control for trajectory generation and control of multiple UAVs based on vision information. The authors in [12] develop a decentralized receding horizon control for stabilizing multi-agent systems with decoupled dynamics, constraints, and cost functions. The authors in [2] apply receding horizon control and graph representations of obstacles in the environment to address the trajectory optimization of aerial vehicles.

We use the receding horizon framework for trajectory generation. We focus on trajectories in two dimensions and as such, we do not consider resolving conflicts between aircraft or hazardous weather through changing altitude. There are few reasons for this approach. First, if there are bad weather regions such as storms, depending on the storm height, a change in altitude will not necessarily resolve the conflict. Second, in busy sections of airspace, aircraft descend and ascend at various altitudes; a change in the assigned altitude of aircraft may cause potential conflicts. Additionally, a change in altitude may be accompanied by passenger discomfort, and thus altitude changes will typically be slower maneuvers.

A. Problem Model

Let x_1 and x_2 denote the position of the aircraft in the horizontal plane, ψ denote the orientation of the aircraft with respect to the x_1 axis, v denote the speed of the aircraft, and

ω denote the rate of change ψ . We use the unicycle model to describe the equation of motion of each aircraft:

$$\begin{aligned} \dot{x}_1^i(t) &= v^i \cos \psi^i(t) \\ \dot{x}_2^i(t) &= v^i \sin \psi^i(t) \\ \dot{\psi}^i(t) &= \omega^i(t) \end{aligned} \quad (1)$$

Here, we use superscript i to denote the parameters of aircraft i , for $i = 1, \dots, M$. We use the above model instead of a more realistic high dimensional model of aircraft dynamics in order to simplify computation of trajectories. In the typical hierarchical framework of Fig. 1, the output of the trajectory generator is the desired path of the aircraft parameterized by x_1 and x_2 , as well as the desired input ω and/or v which achieve this path. These parameters are then passed to the low-level controller which uses the high fidelity aircraft dynamic model in order to stabilize the aircraft around the desired trajectory.

Given a reference trajectory which is planned in advance considering the origin and destination of aircraft as well as the aircraft parameters, a cost is formulated such that deviations from the reference trajectory are penalized. Let $x^i = [x_1^i, x_2^i, \psi^i]^T$ denote the aircraft state, and $u^i = u^i([0, T_f])$ denote the input to aircraft i over the time horizon of interest $[0, T_f]$. Here, we assume $u^i = \omega^i$. We define the cost to penalize the L_2 norm of the tracking error of the reference path:

$$J(u^1, \dots, u^M) = \sum_{i=1}^M \int_0^{T_f} \|x^i(t) - x_d^i(t)\|^2 dt \quad (2)$$

In a decentralized formulation, each aircraft would minimize its own cost rather than minimizing the sum of all aircraft costs. Although we do not address this formulation in the current paper, we note that a multi-objective game theoretic framework can be introduced to address the decentralized case [10]. We formulate a constrained optimization problem to minimize the above cost, while ensuring avoidance of other aircraft and hazardous weather:

$$\begin{aligned} \min_{u^1, \dots, u^M} \quad & J(u^1, \dots, u^M) \\ \text{s.t.} \quad & \dot{x}^i(t) = f^i(x^i(t), u^i(t)) \quad \forall i \\ & \underline{u} \leq u^i(t) \leq \bar{u} \quad \forall i \\ & g_c(x^i(t), x^j(t)) \leq 0 \quad \forall i, j \mid i \neq j \\ & g_e(x^i(t), t) \leq 0 \quad \forall i, \forall e = 1, \dots, E_t \end{aligned} \quad (3)$$

In the above, f^i represents aircraft dynamics defined in (1), \underline{u} and \bar{u} are the constraints on input magnitude, g_c denotes the collision avoidance constraint and g_e denotes the constraint on avoiding time-varying hazardous weather. Each of these constraints is enforced throughout the time $[0, T_f]$ and these terms will be described in more detail below.

The protected zone of each aircraft is defined as a cylinder, centered at the aircraft with radius of $R = 5$ nautical miles and height of 2000 ft. If an aircraft enters the protected zone of another aircraft, a conflict referred to as loss of separation occurs. In the two dimensional framework, we impose the

loss of separation constraint between each pair of aircraft at every time through the constraint:

$$g_c(x^i, x^j) = R^2 - ((x_1^i - x_1^j)^2 + (x_2^i - x_2^j)^2)$$

We will assume that the hazardous weather at each time is represented by a number of ellipses labeled by $e = 1, \dots, E_t$, in the 2-dimensional plane. Each ellipse is parameterized by its center $[x_1^e, x_2^e]^T \in \mathbb{R}^2$ and the matrix $A^e \in \mathbb{R}^{2 \times 2}$. Hence, the constraint for aircraft i to avoid this hazardous weather region takes the form

$$g_e(x^i, t) = 1 - [x_1^i - x_1^e, x_2^i - x_2^e] A^e [x_1^i - x_1^e, x_2^i - x_2^e]^T$$

The dependence on time in the above is due to change of weather over time. In the next section, we describe what defines a hazardous weather and how the forecast information is used to define the constraint function g_e .

III. APPROACH

A. Utilizing Dynamic Weather Forecast

One factor used to determine whether or not a region of airspace is safe to fly through is the Vertically Integrated Liquid (VIL) measurements. These measurements represent the amount of precipitation in a column of air, measured by NextGen radars. The Corridor Integrated Weather System (CIWS) product [6] provides VIL numbers in a 1 km by 1 km gridded form for the United States. These VIL measurements can be quantized into 6 levels, with levels 3 and higher indicating a recommended no-fly zone for aircraft [19].

To create weather maps for routing, a geographic region of interest, time, and date are selected and the current and forecast VIL quantities are uploaded as pixel images. Then, the VIL data is converted into binary fly and no-fly zones based on the VIL quantization levels. The binary zones are segmented into disjoint storms, with each storm containing no pixels adjacent to any other storms. These storms are then enclosed by minimum volume bounding ellipses. This ensures that all the storms' pixels are accounted for while creating a conservatively larger storm in the more algorithmically friendly form of an ellipse. In our algorithm, storms are these elliptical over-approximations of impassible weather. Other approximations such as bounding polygons could be used at the expense of computational speed. This CIWS product, which comes with additional weather data, is currently used by air traffic controllers for tactical routing [31]. By using a weather product that is already available for air traffic control, our algorithms are viable for testing and comparison with current methods. Figure 2 shows the impassible regions segmented into individual storms, their polygonal convex hulls, and their elliptical over approximations created from the raw VIL data.

Every 5 minutes, the CIWS product provides 2 hours of 5 minute forecasts (24 forecasts) for the entire United States airspace. In addition, these forecasts are updated at 5-minute intervals. Figure 3 shows the time variation of the forecasted storms. Here, we tracked storms in the window

Binary Segmented Weather with Convex Hulls and Bounding Ellipses

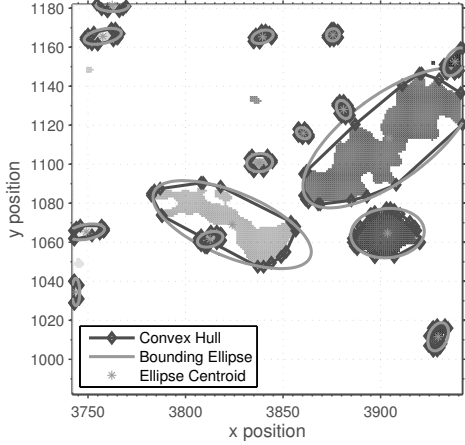


Fig. 2. Impassible regions from VIL data segmented into storms, each a different shade of gray. Convex hulls of storms were used to create the minimum volume bounding ellipses.

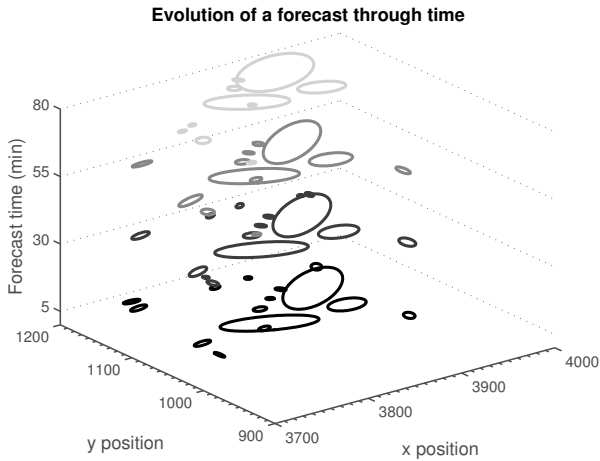


Fig. 3. The minimum volume bounding ellipses are tracked over time based on the dynamic forecasts.

of Figure 2 over 80 minutes of time. Although the forecast remains constant during the 5 minute update intervals, in reality the storms and hazardous weather regions move due to wind. There is no general closed form motion or uncertainty model for these forecasts that one could take into account while planning trajectory between the forecast updates. In order to account for the movement of storms we identify ellipses corresponding to storms in subsequent forecasts. From this, we determine the rate of movement and growth of the ellipses. This rate is then used to interpolate the location of the storms during intervals of time for which there is no forecast available. We automatically have identified and tracked storms that are persistent in the window of interest.

B. Receding Horizon Trajectory Generation

In order to implement a receding horizon trajectory generation algorithm, we discretized the dynamics and cost function using Euler forward integration. In the receding horizon framework, a planning horizon n_p is chosen as the

number of discrete steps for which the trajectory planning is performed. Let the sequence of inputs to aircraft i for a planning horizon starting at t be denoted by $u^i(t:t+n_p) = [u^i(t), u^i(t+1), \dots, u^i(t+n_p)]$. Starting at an initial time index 1 and with the initial state $x^i(1) = x_0^i$, the optimization problem is solved over the planning horizon n_p to provide inputs $u_o^i(t:t+n_p)$, $i = 1, \dots, M$. Then, for an execution horizon of n_e , the first n_e inputs $u_o^i(t:t+n_e)$ are applied for each aircraft. The horizon is shifted by n_e time units and the optimization step is repeated, this time, with initial condition set to $x^i(t+n_e)$. By repeating the optimization, one is able to consider variations in the environment or the aircraft motion. For the cases in which a receding horizon optimization is implemented to approximate infinite horizon optimal control problems with constant state constraints, it decreases the computational burden while ensuring stability and feasibility under appropriate formulation of a terminal cost and a terminal constraint [17].

In our current formulation, we choose the execution horizon as the interval for which the current weather forecast update is available. Hence, the execution horizon is set to 5 minutes. Given the desired destination point for an aircraft $[x_{1d}^i(T_f), x_{2d}^i(T_f)]^T$, at every receding horizon iteration at time t with initial condition on dynamics given by $x^i(t)$, we compute the shortest distance to reach the desired point, as the line segment joining $[x_1^i(t), x_2^i(t)]^T$ with $[x_{1d}^i(T_f), x_{2d}^i(T_f)]^T$. We let $[x_{1d}^i, x_{2d}^i]^T$ be the point along this line segment that is reached in the planning horizon of n_p steps. Let $u^i = u^i(t:t+n_p)$, the receding horizon cost at time t based on the tracking error cost in (2) is then defined as:

$$J_t(u^1, \dots, u^M) = \sum_{i=1}^M (x_1^i(t+n_p) - x_{1d}^i)^2 + (x_2^i(t+n_p) - x_{2d}^i)^2 \quad (4)$$

Let K denote the number of execution steps in a planning horizon of n_p , i.e. $K = \frac{n_p}{n_e}$. The receding horizon optimization algorithm is described below:

Algorithm 1 Receding Horizon Trajectory Generation using Dynamic Forecast

- Given. $t = 1$, and initial condition x_0^i , for $i = 1, \dots, M$.
- Step 1. Set up the optimization problem
 - a. initialize $x^i(t) = x_0^i$.
 - b. get weather constraint ellipses $e = 1, \dots, E_l$ from the forecasts at times $l = t, t+n_e, \dots, t+Kn_e$.
 - Step 2. Find u_o^i , for $i = 1, \dots, M$ as the minimizer of the cost in (4).
 - Step 3. Apply $u_o^i(t:t+n_e)$ to aircraft i , for $i = 1, \dots, M$. Evaluate $x^i(t+n_e)$.
 - Step 4. Set $x_0^i = x^i(t+n_e)$, $t = t+n_e$, and go to Step 1.
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If aircraft i reaches a pre-defined neighborhood of its destination, then it is removed from the optimization in the subsequent iterations. The algorithm terminates when all aircraft reach their destinations.

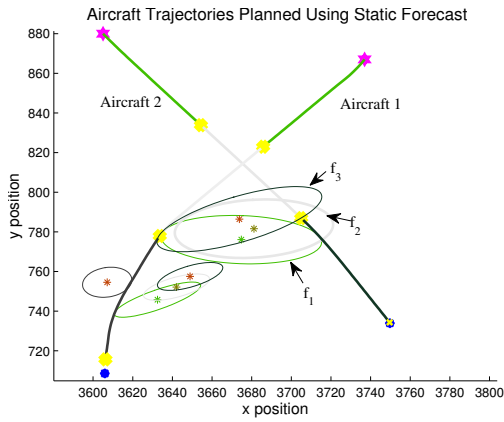


Fig. 4. Planned trajectories, starting at the top points and moving towards the target points at the bottom, using static forecast data. The labels f_1 , f_2 , f_3 show the minimum bounding ellipses for a predicted storm at times 1, 6, and 11 minutes. Similarly, the time variation of the forecast for the other storms is shown through using different line formats. The circles indicate aircraft states at the end of each iteration of the receding horizon optimization. The trajectories generated using static forecast use the predictions at time 1. Consequently, aircraft 2 has a potentially unsafe trajectory.

IV. RESULTS

We considered a section of airspace centered at latitude 30° and longitude 86.5° coordinates, near the gulf coast of Florida. We used the weather forecast data for 01/07/2009, a day in which there were storms observed in the region under consideration. The aircraft speed was set to 230 nautical miles per hour, which is a typical airspeed for the en-route portion of the flight. We used a TOMLAB nonlinear solver for implementing the iterations of Algorithm 1.

The simulations for a two aircraft scenario using static and dynamic forecasts are shown in Figures 4 and 5 respectively. Here, the planning horizon was set to 15 minutes. Each 5 minute portion of the trajectory as well as the forecast is shown with a different line format and labeled in order to highlight the dynamic nature of the forecast and the trajectories generated. From these figures, one can see the movement of the no-fly zones which represent the storms. Observe from Fig. 4 that if the static weather forecast, labeled by f_1 for the largest storm in the window, is used the path of aircraft 2 will intersect the storm forecasted at the second time interval, labeled by f_2 in this figure, while the trajectories planned using the dynamic forecast will remain safe as shown in Fig. 5.

Figure 6 shows the same trajectories obtained from the dynamic forecast plotted over the weather ellipses obtained from the actual weather recorded for the day under consideration. The comparison of the weather ellipses indicates that there is relatively significant error between the forecast and actual data for the largest storm in the second time horizon. While aircraft 2 has a safe trajectory despite this error, aircraft 1's trajectory enters the no-fly zone for a short period of time. However, the advantage of using dynamic forecast is still clear because re-routing the aircraft to avoid

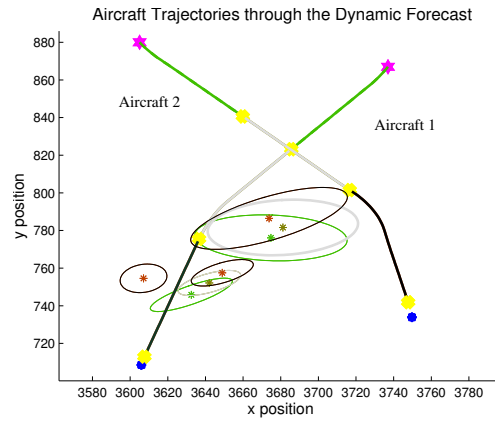


Fig. 5. Planned trajectories using dynamic forecast. In this case, the trajectories remain safe with respect to the updated forecasts.

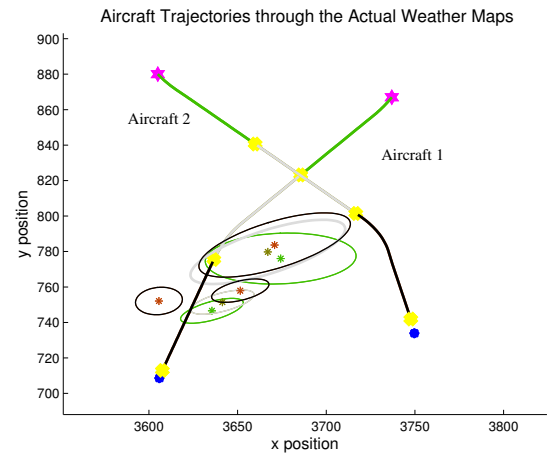


Fig. 6. Trajectories through the actual weather storms. Here, aircraft 1's trajectory enters the no-fly region due to discrepancy between forecasted and actual weather.

the hazardous weather could be achieved through deviating the aircraft from its planned path by a small amount.

Based on these simulations, we are motivated to quantify the differences between the forecast and the actual weather data and utilize this information to design robust trajectories. Several additional future research directions are detailed in the next section.

V. CONCLUSIONS AND FUTURE WORK

We utilized a dynamic weather forecast product to formulate a receding horizon framework for designing safe aircraft trajectories in the en-route portion of flight. We showed our methods with simulations based on weather forecast data and actual weather maps. Our simulations confirm the advantage of taking into account dynamic versus static forecasts in a systematic way in trajectory generation.

In the future, we plan to deal with the complexities of storm evolution including merging, splitting, dissipating, and growing behavior. In addition to the VIL precipitation levels used in this study to characterize the no-fly zones, we plan to utilize other weather factors that affect the pilot's decision in flying through a region of airspace, such as types and heights

of storms [3]. We plan to take into account the uncertainty in the forecast through a stochastic or robust trajectory generation method. Finally, in order to envision the optimized aircraft profile concept in the near future, a provably safe decentralized version of the trajectory generation should be implemented.

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