A VSC Position Tracking System Involving a large Scale Pneumatic Muscle Actuator*

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Abstract

A variable structure controller (VSC) is developed to provide position tracking capability for a very large pneumatic muscle actuator, which has inherently nonlinear dynamics. After the controller design is completed, the internal dynamics of the closed-loop system are shown to be stable. Robustness and passivity issues are then discussed. The switching law is modified to account for the different dynamics through the use of a composite Lyapunov function which guarantees asymptotic tracking stability.

1 Introduction

A powerful means for augmenting strength or mobility assistance to humans is through the use of very large-scale pneumatic muscle (PM) actuators. Such systems (small in size) have been in existence for years [1-6] and found application in robotic devices when used in manipulation or precision tasks. When comparing hydraulic or electric motor actuation systems versus the PM device, its advantages include [1] having the highest power/weight ratio and power/volume ratio. For applications involving strength or mobility assistance to exoskeleton systems worn by humans completing tasks, the advantages of this technology include being sufficiently light-weight and Self-contained. The major disadvantage of the PM technology is its inherently nonlinear dynamics, which limits its ability to render precision tracking tasks.

This paper examines a very large-scale PM system for actuation purposes involved with a position tracking task. Modeling of the PM system is first conducted to understand the extent of its nonlinearities within the context of the use intended. For a position tracking application, a VSC system is then designed. To show asymptotic stability of the tracking error (with respect to a reference model trajectory), energy-related Lyapunov functions are utilized. Robustness issues are discussed to expand the operation of this device where uncertainties may exist in the parameters. Passivity is also examined as well as defining the class of possible reference inputs that can be tracked by this system.

2 Principle of Operation

Figure (1) illustrates the basic operation of the device under consideration. A thin-walled rubber balloon is inserted inside a very strong nylon or plastic braided material. This outside material has the ability to expand easily in the radial direction, but has large tensile strength and resistance to changes in the

longitudinal direction. For example, if a standard air source of 120 pounds per square inch (PSI) were used, the inside balloon would typically change its area approximately 4 square inches during inflation. The net force produced by the outside material on the environment in the longitudinal direction is given by:

$$\text{Net Force} = \text{Pressure} * \Delta \text{Surface Area of Balloon} \quad (1)$$

or

$$F = 120 \text{ PSI} * \Delta \text{Area} = 480 \text{ pounds} \quad (2)$$

Since the typical overall weight of an autonomous system of this type to produce this force is less than 1 pound, it would have a force/weight ratio of over $\frac{400}{1}$. Figure (2) illustrates such a system in operation at Wright-Patterson Air Force Base, Ohio, USA.

2.1 Method of Developing Position Control

The PM system (as a doubly antagonist muscle pair) could be used in either a linear translational or rotational sense to produce a force or moment (cf. Figures 3a-b). To activate the inflation, timed pressure pulses of one millisecond duration can be used which are electrically controlled by a solenoid. To deflate the PM, a bleed actuation can be accomplished via a solenoid action with dynamics that differ as compared to the inflation. Figure (4) shows a position tracking system description. The PM is the plant with a nonlinear characterization.

2.2 Modeling the PM System

Figure (5) illustrates a passive parallel element model used here. To determine $K(x)$, steady state measures are utilized; for $B(\dot{x})$, transient data are utilized.

2.3 Determination of $K(x)$ from Steady State Data

Figure (6) illustrates the response of the system under a positive pressure step input (22.1 PSI amplitude) for 6.25 seconds and then a negative step (deflation via bleed control) for an additional 6.25 seconds. The approximate inflation time constant is larger than its deflation counterpart. To determine $K(x)$, steady state data are used (0-70 PSI) with various loads varying from 0-300 pounds. The goal is to find $K(x)$ where:

$$F_{\text{STEADY STATE}} = K(x) x \quad (3)$$

Figures (7a-b) illustrates the regression assumed for specific polynomial fits which are discussed in the sequel.

2.4 Transient Measurements and Determining $B(\dot{x})$

Once the model (equation 3) is known, the transient force data are then used to characterize the viscous element $B(\dot{x})$. A three-step process is instituted:

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Step 1: Determine $K(x)$ from steady state data.

Step 2: Determine $F_{\text{TRANSIENT}}$ (correct for $K'(x)$) via:

$$F_{\text{TRANSIENT}} = F_{\text{TOTAL}} - F_{\text{STEADY STATE}} \quad (4)$$

Step 3: Regress $B(\dot{x})$ versus $\dot{x}$ using the relationship:

$$F_{\text{TRANSIENT}} = B(\dot{x}) \dot{x} \quad (5)$$

where the dot indicates time differentiation.

2.5 Experimental Paradigm to Obtain Model

Table I illustrates the experimental conditions to collect data to characterize the model. All combinations of loads with two types of pressure inputs were utilized.

Table I - Experimental Data Collected)

Loads	Pressures - Single Step	Step Up - Step Down
0 lbs.	0 PSI	0 PSI
25 lbs.	10 PSI	10 PSI
50 lbs.	20 PSI	20 PSI
100 lbs.	30 PSI	30 PSI
150 lbs.	40 PSI	40 PSI
200 lbs.	50 PSI	50 PSI
250 lbs.	60 PSI	60 PSI
300 lbs.	70 PSI	70 PSI

3 Identification Procedure Results

The data were analyzed in MS Excel 5.0 with successive polynomial fits made between the respective models assumed *a priori* and the model fit from the data. A summary of these models is now presented:

$$K_i(x) = 1.6x^2 - 10.9x + 27.1 \quad (6)$$

$$K_d(x) = 3.6x^2 - 20.7x + 47.23 \quad (7)$$

$$B_i(\dot{x}) = 0.04\dot{x}^2 - 1.3\dot{x} + 12.6 \quad (8)$$

$$B_d(\dot{x}) = 0.12\dot{x}^2 - 2.49\dot{x} + 14.48 \quad (9)$$

4 A Position Tracking VSC System

For brevity, the results are shown for only the inflation state of operation. The equations of motion of Figure (3a) using the model of Figure (5) are of the form:

$$M\ddot{x} + B(\dot{x})\dot{x} + K(x)x = \bar{u}\gamma \quad (10)$$

where the affine control $\bar{u}$ has units of pressure and the term γ is an effective area variable. For a unit mass, the equations of motion describing the position output of the PM system are:

$$\ddot{x} + B(\dot{x})\dot{x} + K(x)x = u \quad (11)$$

where $u = \bar{u}\gamma$. For simplicity of notation, this is written as:

$$\ddot{x} + f_1(\dot{x})\dot{x} + f_2(x)x = u \quad (12)$$

where, from equations (11,12) for inflation dynamics:

$$f_1(\dot{x}) = (0.04\dot{x}^2 - 1.3\dot{x} + 12.6) \quad (13)$$

$$f_2(x) = (1.6x^2 - 10.9x + 27.1) \quad (14)$$

We now comment on the reference model's bandwidth.

Remark:

The reference position trajectory $x_d(t)$ must have limited bandwidth bounded from above based from the approximate time

constants depicted in Figure (6). Starting with the larger time constant (inflation) and $\frac{1}{\tau_{\text{inflation}}} \approx 1$ Hz, the maximum bandwidth of the reference trajectory is to be selected to be no greater than $\frac{1}{10}$ Hz. One could characterize this via:

$$X_d [(p + \lambda_1)(p + \lambda_2)] = R(p) \quad (15)$$

where p is the Laplace variable, ($\lambda_1, \lambda_2 > 0$), with X_p and R (the input) the respectively transformed quantities.

4.1 The VSC Tracking Error System

From (12), the choice is made of the state variables via ($x_1 = x$ and $x_2 = \dot{x}$) resulting in:

$$\dot{x}_1 = x_2 \quad (16)$$

$$\dot{x}_2 = \ddot{x} = -f_1(x_2)\dot{x} - f_2(x_1)x + u \quad (17)$$

or

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -f_2(x_1) & -f_1(x_2) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \quad (18)$$

which is in canonical form with output measurement:

$$y(t) = x(t) = x_1(t) \quad (19)$$

4.1.1 Calculation of Relative Degree

To determine r = relative degree of the nonlinear equations (18, 19), $y(t)$ is differentiated once yielding:

$$\dot{y} = \dot{x} = \dot{x}_1 = x_2 \quad (20)$$

Differentiating again results in:

$$\ddot{y} = \ddot{x} = -f_1(x_2)\dot{x} - f_2(x_1)x + u \quad (21)$$

Since the term u appeared after two differentiations, the relative degree of the nonlinear system (18-19) is $r = 2$.

4.1.2 Construction of the Sliding Surface $S(t)$

Defining the position error vector:

$$\tilde{x}(t) = x(t) - x_d(t) \quad (22)$$

The time-varying surface $S(t) \in R^2$ is defined via:

$$s(x;t) := \left[\frac{d}{dt} + \lambda \right] \tilde{x}, \quad \text{with } \lambda > 0 \quad (23)$$

To determine the feedback linearization function, equation (23) is now differentiated [7].

4.1.3 Feedback Linearization - Determining $\hat{u}$

Differentiating $s(\cdot)$ from equation (23) yields the following:

$$\dot{s} = \ddot{\tilde{x}} + \lambda \dot{\tilde{x}} = \ddot{\tilde{x}} - \ddot{x}_d + \lambda \dot{\tilde{x}} \quad (24)$$

or

$$\dot{s} = -f_2(x_1)x_1 - f_1(x_2)x_2 + u - \ddot{x}_d + \lambda \dot{\tilde{x}} \quad (25)$$

The objective is to select $\hat{u}$ as the best approximation to achieve $\dot{s} = 0$, via:

$$\hat{u} = f_2(x_1)x_1 + f_1(x_2)x_2 + \ddot{x}_d - \lambda \dot{\tilde{x}} \quad (26)$$

Thus if the controller is on the sliding surface $S(t)$, then $\dot{s} = 0$ and the tracking error dynamics are a simple exponential. It is necessary to show that outside of the invariant set $S(t)$,

the control law drives the state space trajectories to the sliding surface.

4.1.4 Switching Conditions for the Controller

To develop a switching controller, equation (12) is rewritten as:

$$\ddot{x} = f_3(x, \dot{x}) + u \quad (27)$$

and the assumption is made that there exists a bound on the uncertainty in the estimate of f_3 denoted as $\hat{f}_3$ quantified as follows:

$$|\hat{f}_3 - f_3| \leq \Delta \quad (28)$$

Then the feedback linearization (affine) control u is specified via:

$$\hat{u} = -\hat{f}_3 + \ddot{x}_d - \lambda \dot{\tilde{x}} \quad (29)$$

($\lambda > 0$) where the monic polynomial ($\ddot{x} + \lambda \dot{\tilde{x}}$) is Hurwitz. The switching condition is selected as:

$$u = \hat{u} - K \operatorname{sgn}(s) \quad (30)$$

where K is a positive constant yet to be determined. It is noted that:

$$\operatorname{sgn}(s) = +1 \text{ if } s > 0 \quad (31)$$

and

$$\operatorname{sgn}(s) = -1 \text{ if } s < 0 \quad (32)$$

A Lyapunov function and the value of K that demonstrate the asymptotic stability of the tracking error controller can now be presented.

4.1.5 The First Lyapunov Function

In order to generate attractivity of the sliding surface (reachability, cf. Spong, Vidyasagar [8]), select:

$$V_1(s) = \frac{1}{2} s^2 \quad (33)$$

Then $V_1(s)$ satisfies the following conditions:

- (i) As $|s| \rightarrow \infty$, $\lim V_1(s) \rightarrow \infty$ (radially unbounded condition).
- (ii) $\dot{V}_1(s) > 0$ if $s \neq 0$ (positive definite property).
- (iii) $\dot{V}_1(s) < 0 \forall s \neq 0$ (negative definite property).

Proof

(i) and (ii) are obvious. For (iii), we calculate $\dot{V}_1$:

$$\dot{V}_1 = \frac{1}{2} \frac{d}{dt} s^2 = \dot{s} s = [f_3 - \hat{f}_3 - K \operatorname{sgn}(s)] s \quad (34)$$

or

$$\dot{V}_1 = (f_3 - \hat{f}_3) s - K |s| \quad (35)$$

The objective is to pick K sufficiently large to ensure that $\dot{V}_1 < 0$. From (28), K is selected to satisfy:

$$K \geq \Delta + \eta \quad (36)$$

where $\eta > 0$ and from (35, 36), this implies:

$$\dot{V}_1 = \frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s| \quad (37)$$

and $\dot{V}_1$ is negative definite if $|s| \neq 0$. $\square$

4.1.6 Stability of the Internal Dynamics (Inner Loop)

It is essential to examine if the control law specified via feedback linearization in Figure (8) maintains stable internal dynamics. For a nonlinear system with unstable internal dynamics, this is equivalent to having a nonminimum phase transmission zero and a perfect tracking controller is not appropriate (Isidori and Moog [9]). Theorem 1 demonstrates that both states are exponentially stable with equilibrium points at the

origin.

Theorem 1: $x_1(t)$ and $x_2(t)$ Are Exponentially Stable

Proof: If u is known precisely, then:

$$\dot{x}_1 = x_2 \quad (38)$$

$$\dot{x}_2 = -f_2(x_1) x_1 - f_1(x_2) x_2 + u \quad (39)$$

$$y = x_1 \quad (40)$$

But the feedback linearization law is:

$$u = f_2(x_1) x_1 + f_1(x_2) x_2 + \ddot{x}_d - \lambda \dot{\tilde{x}} \quad (41)$$

Then:

$$\dot{x}_1 = x_2 \quad (42)$$

$$\dot{x}_2 = \ddot{x}_d - \lambda [\dot{x} - \dot{x}_d] \quad (43)$$

is stable. This implies:

$$\dot{x}_2 + \lambda x_2 = \ddot{x}_d + \lambda \dot{x}_d \quad (44)$$

From the assumed form of $x_d(t)$ in equation (15), Lemma 1 establishes the fact that the derivatives of the reference trajectory $x_d(t)$ are of exponential order.

Lemma 1

From (15), the assumption is that $r(t)$ is a unit step function (and let $\lambda_1 \neq \lambda_2 > 0$), the following are true for $t > 0$:

$$x_d(t) = \frac{1}{\lambda_1 \lambda_2} + C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} \quad (45)$$

$$\dot{x}_d(t) = -\lambda_1 C_1 e^{-\lambda_1 t} - \lambda_2 C_2 e^{-\lambda_2 t} \quad (46)$$

$$\ddot{x}_d(t) = \lambda_1^2 C_1 e^{-\lambda_1 t} + \lambda_2^2 C_2 e^{-\lambda_2 t} \quad (47)$$

Proof

The result of (45) follows from equation (15) assuming zero initial conditions. For $t > 0$, successively differentiate equations (45) and (46) which results in (47). Now let:

$$\lambda_{\min} = \min [\lambda_1, \lambda_2] > 0 \quad (48)$$

Then using the triangle inequality, it follows that:

$$|\dot{x}_d(t)| \leq |\lambda_1 C_1| e^{-\lambda_{\min} t} + |\lambda_2 C_2| e^{-\lambda_{\min} t} \quad (49)$$

$$|\ddot{x}_d(t)| \leq |\lambda_1^2 C_1| e^{-\lambda_{\min} t} + |\lambda_2^2 C_2| e^{-\lambda_{\min} t} \quad (50)$$

Now let:

$$K_1 = |\lambda_1 C_1| + |\lambda_2 C_2| \quad (51)$$

$$K_2 = |\lambda_1^2 C_1| + |\lambda_2^2 C_2| \quad (52)$$

Then it follows that:

$$|\dot{x}_d(t)| \leq K_1 e^{-\lambda_{\min} t} \quad (53)$$

$$|\ddot{x}_d(t)| \leq K_2 e^{-\lambda_{\min} t} \quad (54)$$

Thus both $\dot{x}_d$ and $\ddot{x}_d$ are of exponential order. $\square$

Lemma 2 demonstrates that the state variable x_2 is also of exponential order.

Lemma 2 $x_2(t)$ satisfies: $|x_2(t)| < K_3 e^{-\lambda_{\min} t}$ where $0 < K_3 < \infty$.

Proof

Equation (44) is modified via:

$$\dot{x}_2 + \lambda x_2 \leq K_3 e^{-\lambda_{\min} t} \quad (55)$$

where:

$$K_3 = K_1 + K_2 \quad (56)$$

Without loss of generality, select $\lambda > \lambda_{\min}$. Multiplying the inequality (55) by $e^{\lambda t} > 0$ yields:

$$\dot{x}_2 e^{\lambda t} + \lambda x_2 e^{\lambda t} \leq K_3 e^{\lambda t} e^{-\lambda_{\min} t} \quad (57)$$

This can be written as a perfect differential:

$$\frac{d}{dt} [x_2 e^{\lambda t}] \leq K_3 e^{\lambda t} e^{-\lambda_{\min} t} \quad (58)$$

Integrating both sides of the inequality from 0 to t ($\lambda - \lambda_{\min} > 0$) yields:

$$x_2 e^{\lambda t} \leq \frac{K_3}{\lambda - \lambda_{\min}} e^{\lambda t} e^{-\lambda_{\min} t} + K_4 \quad (59)$$

Multiplying both sides of this inequality by $e^{-\lambda t} > 0$ yields the form:

$$x_2(t) \leq \frac{K_3}{\lambda - \lambda_{\min}} e^{-\lambda_{\min} t} + K_4 e^{-\lambda t} \quad (60)$$

Now let:

$$K_5 = K_4 + \frac{K_3}{\lambda - \lambda_{\min}} \quad (61)$$

Then, since $\lambda > \lambda_{\min}$, equations (60-61) clearly demonstrate that:

$$|x_2(t)| < K_5 e^{-\lambda_{\min} t} \quad (62)$$

□

The final lemma easily follows:

Lemma 3

The state $x_1(t)$ is also exponentially stable.

Proof

From the state equation (42), $x_1(t)$ is the first integral of the state $x_2(t)$ which is of exponential order. Integrating the inequality (62) yields:

$$|x_1(t)| \leq \left| \frac{K_5}{-\lambda_{\min}} \right| e^{-\lambda_{\min} t} \quad (63)$$

□

Thus, both states have equilibrium points at the origin and the internal dynamics are stable.

5 Passivity Issues

The objective is to demonstrate that the system and control law retain their passive characteristics. To show this, the equations of motion of the original PM system are rewritten as:

$$M \ddot{x} + f_1(\dot{x}) \dot{x} + f_2(x) x = u = F \quad (64)$$

where the input to the system is a force F and the output is $\dot{x}$, the PM velocity. It is desired to say something about this mapping in the sense of passivity. To accomplish this goal, consider the following energy-related Lyapunov function:

$$V_2 = \frac{1}{2} M \dot{x}^2 + \int_0^x f_2(\tau) \tau d\tau \quad (65)$$

This Lyapunov function represents the total kinetic and stored spring energy of the PM device. To calculate $\dot{V}_2$, the power in the system involving the mapping of F to $\dot{x}$ (using inflation dynamics for simplicity), we have:

$$\dot{V}_2 = M \dot{x} \ddot{x} + \frac{d}{dt} \left(\frac{1.6}{4} x^4 - \frac{10.9}{3} x^3 + \frac{27.1}{2} x^2 \right) \quad (66)$$

This can be written:

$$\dot{V}_2 = \dot{x} [-f_1(\dot{x}) \dot{x} - f_2(x) x + F] + 1.6x^3 \dot{x} - 10.9x^2 \dot{x} + 27.1x \dot{x} \quad (67)$$

or

$$\dot{V}_2 = \dot{x} [-f_1(\dot{x}) \dot{x} - f_2(x) x + F] + f_2(x) x [\dot{x}] \quad (68)$$

Finally, this can be put in the form:

$$\dot{V}_2 = F \dot{x} - [f_1(\dot{x}) \dot{x}^2] \quad (69)$$

where the right-hand side of (69) represents the input-output variables ($F, \dot{x}$) and the quantity in the square bracket is greater than zero (dissipative term) because $f_1(\dot{x}) > 0$. Thus a dissipative or passive mapping exists from the external input force F to the output velocity $\dot{x}$ since, in equation (69), $\dot{V}_2$ demonstrates a net power reduction due to this mapping. Also, the precise amount of power dissipated is given by the term $[f_1(\dot{x}) \dot{x}^2]$. □

Finally, it is noted that just because each control law (inflation and deflation) is stable by itself, the composite control law may not be stable and this issue needs to be considered. The switching methodology is now modified [10] to ensure that the composite controller provides asymptotic tracking stability.

6 A Modified Switching Law and a Composite Lyapunov Function

To develop a modified switching law, let the Lyapunov function for the inflation controller be described by:

$$V_i = \frac{1}{2} s^2 \quad (70)$$

where s is subjected to the inflation dynamics which are described via:

$$\ddot{x} + B_i(\dot{x}) \dot{x} + K_i(x) x = u_i \quad (71)$$

Also let:

$$V_d = \frac{1}{2} s^2 \quad (72)$$

be the Lyapunov function describing the deflation dynamics which satisfy:

$$\ddot{x} + B_d(\dot{x}) \dot{x} + K_d(x) x = u_d \quad (73)$$

To guarantee asymptotic tracking stability of the overall control scheme, a composite Lyapunov function is obtained independent of which control law is in use [10]. Theorem 2 demonstrates this result.

Theorem 2: Let V_c be a composite Lyapunov function (convex combination of the two V functions) specified via:

$$V_c = \delta V_i + (1 - \delta) V_d \quad (74)$$

where $0 < \delta < 1$. Then V_c is an appropriate composite Lyapunov function (with modified switching law) and satisfies.

(i) As $|s| \rightarrow \infty$, $\lim V_c(s) \rightarrow \infty$ (radially unbounded condition).

(ii) $\dot{V}_c(s) > 0$ if $s \neq 0$ (positive definite property).

(iii) $\dot{V}_c(s) < 0 \forall s \neq 0$ (negative definite property of $\dot{V}_c$).

Proof: (i) Since both $\delta > 0$ and $1 - \delta > 0$, and $\lim_{|s| \rightarrow \infty} V_i = \infty$ as well as $\lim_{|s| \rightarrow \infty} V_d = \infty$, it follows that $\lim_{|s| \rightarrow \infty} \delta V_i = \infty$ and $\lim_{|s| \rightarrow \infty} (1 - \delta) V_d = \infty$ and condition (i) is satisfied.

(ii) For $|s| \neq 0$, it has been shown that both $V_i > 0$ and $V_d > 0$. Again, since both $\delta > 0$ and $1 - \delta > 0$, it follows that $\delta V_i > 0$ and $(1 - \delta) V_d > 0$ and condition (ii) is satisfied.

(iii) To show $\dot{V}_c < 0$, a modification of the switching rule is required. Thus, as a result of the switching, the operating point moves down the respective Lyapunov functions (cf. Figure (9)). To show this result, with some small abuse of notation, equation (12) becomes:

$$\ddot{x} = f_{3j}(x, \dot{x}) + u_j \quad (75)$$

where ($j = i$ or d) for inflation and deflation dynamics, respectively. The uncertainties in the estimates of f_{3j} ($j = i$ or d) are represented via:

$$|\hat{f}_{3j} - f_{3j}| \leq \Delta_j \quad j = i \text{ or } d \quad (76)$$

Since $\dot{V}_c = \delta \dot{V}_i + (1 - \delta) \dot{V}_d$ with $\delta > 0$, $(1 - \delta) > 0$, the switching gain must guarantee both $\dot{V}_i < 0$ and $\dot{V}_d < 0$. By

Figure (4) - Position/Force Control Using A Pneumatic Muscle

Figure (7a) - Force Versus Delta L.
K Parameter Plot

Steady State Data Only

Figure (7b) - Calculated Viscous Force
Versus Velocity of Muscle Shortening
B Parameter Plot

Transient Data Only

Figure (5) - Stiffness-Visco Model to Characterize Pneumatic Muscle

Figure (6) - Pressure Step-Positive and then Negative
Maximum Pressure = 152 kPa

Figure (8) - Feedback Linearization for A Tracking Problem

Figure (9) - Switching Between the Two Lyapunov Function Surfaces

calculation, for ($j = i$ or d):

$$\dot{V}_j = \frac{1}{2} \frac{d}{dt} s^2 = \dot{s} s = [f_{3j} - \dot{f}_{3j} - K_j \operatorname{sgn}(s)] s \quad (77)$$

or

$$\dot{V}_j = [f_{3j} - \dot{f}_{3j}] s - K_j |s| \quad (78)$$

Thus the selection is made of:

$$K_j > \Delta_j + \eta, \quad (\eta > 0) \quad (79)$$

$\Rightarrow$

$$\dot{V}_j = \frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s| \quad (80)$$

Thus, to guarantee that $\dot{V}_j < 0$ for either $j = i$ or d , the modification of the switching law is to pick (at all times):

$$K_j = \max [K_i, K_d] \quad (81)$$

Hence the equilibrium at the origin is globally asymptotically stable. $\square$

7 Summary and Conclusions

A pneumatic muscle actuator was analyzed to provide position tracking even though its dynamics were inherently nonlinear and suffered from reduced bandwidth. In realizing a solution to this problem, the issues of robustness, passivity, and stability all had to be considered in the final design of the overall controller. The switching law required modification to account for the different dynamics that occur between the inflation and deflation modes of operation of the PM device.

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Figure (1) -
Pneumatic Muscle Concept

Figure (2) - The Physical System

Figure (3a) - Use of the PM System
In Linear Translation to Produce Force

Figure (3b) - Use of the PM System
In A Rotational Sense to Produce Torque