

A CONTROLLER DESIGN METHODOLOGY FOR SYSTEMS WITH PHYSICAL STRUCTURES: APPLICATION TO INDUCTION MOTORS

Romeo Ortega
Dept. of Electrical Engg.
McGill University
Montreal, P.Q.
CANADA

Gerardo Espinosa
DEPFI-UNAM
University of Mexico
P.O. Box 70-256, D.F.
MEXICO

Abstract. In the present paper we propose an extension of the controller design methodology used in robot motion control to solve an output tracking problem for a class of systems described by its Euler-Lagrange equations with less control actions than degrees of freedom. We show that the problem can be solved provided we can solve a set of algebro-differential equations that define the controller dynamics and the attainable trajectories. To illustrate the procedure we solve the induction motor torque regulation problem.

1. INTRODUCTION

On the development of nonlinear control theory the emphasis has been on general (state-space) systems neglecting the natural structures imposed by the physical character of the system. On the other hand, in modelling physical systems the underlying structure is exploited in a crucial way. Furthermore, some relevant applications, specifically the robotics problem, have shown that for control purposes it is advantageous to use the physical structure of the system under consideration in an explicit way. Already from (mathematical) dynamical systems theory it is well known that one cannot hope for one single theory covering all nonlinear systems; instead we have to concentrate on special subclasses. To make the theory relevant for engineering applications these subclasses must be restricted to "physically meaningful" systems, e.g. those which satisfy some energy balance principles. It seems therefore natural that an appropriate system theoretic framework for control of nonlinear systems should incorporate the systems physical structure on a fundamental level.

The present paper provides a modest contribution towards the development of a control theory for systems with physical structures as advocated in [14]. We propose an extension of the controller design methodology used for the robot motion control problem [17] to regulate the output for a class of systems described by its Euler-Lagrange equations with less control actions than degrees of freedom. To this end, we select first a closed loop target system, whose total energy function time derivative has the required sign. Then, we show that the target system can be attained provided we can solve a set of algebro-differential equations that define the controller dynamics and the attainable trajectories.

To illustrate the procedure we present an induction motor application. Induction motors have been extensively utilized in industrial controls. With increased use of digital techniques a trend towards wider drive ranges and higher reliability has recently been witnessed. This requires the development of new control techniques to insure a good dynamic behaviour and robustness of these highly nonlinear systems. This challenge, interesting from both practical and theoretical viewpoints, has prompted the recent publication of several applications of advanced control theory to electrical drives in the "control oriented" literature, e.g. [1-5], as well as in the "applications oriented" literature" [9,11,12]. In this paper we derive, using the methodology described above, a nonlinear state feedback control that insures global asymptotic regulation of the generated torque. Our solution is compatible with the "field orientation" ideas which have proven highly successful for AC drives control [1,4,9,11] and, in contrast with most existing solutions, is well defined in all operating points including motor start-up.

2. DESIGN METHODOLOGY

2.1 A class of nonlinear systems.
We consider dynamical systems described (in so-called symplectic form [7]) by

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + \phi(q, \dot{q}) = Mu, \quad y = h(q, \dot{q}) \quad (2.1a)$$

where $q \in \mathbb{R}^n$ is the generalized coordinates vector, $u \in \mathbb{R}^r$, $r \leq n$ is the vector of control actions, $M \in \mathbb{R}^{n \times r}$, $D = D^T > 0$, the vector ϕ , accounting for potential and dissipation energy effects, is of the form

$$\phi(q, \dot{q}) = B\dot{q} + Kq + g(q) \quad (2.1b)$$

with $B = B^T \geq 0$, $K = K^T \geq 0$, the matrices D and C are not independent and satisfy

A.1. $\dot{D} - 2C$ is skew-symmetric.

and $y \in \mathbb{R}^m$, $m \leq n$, is the output we want to regulate. That is, our control objective is to construct a control law that causes $y - y_d \rightarrow 0$ as $t \rightarrow \infty$ with $y_d \in \mathbb{R}^m$ a given bounded twice differentiable (\mathcal{C}^2) reference.

Remark 2.1. Systems described by (2.1) belong to the class studied in [14]. See this reference for an excellent tutorial account and [7] for physical motivation. The class contains most (electro) mechanical systems, including robots with flexible joints (section 5.3 in [15]) and induction motors.

Remark 2.2. Condition A.1 is not a technically motivated assumption, but simply a restatement of the fact that we are dealing with a physical system some of whose forces are workless. A key step in the modelling procedure is to select C using the Christoffel symbols so that A.1 holds. See section 2.2 in [15].

2.2. Target System.

The proposed design methodology follows closely the procedure used in the robot motion control problem [17] (see also [15]). Namely, from the systems total energy function we define first a target (not necessarily linear) system we want our closed loop dynamics to look like. Then, we establish the relationship between the controller dynamics and the desired references that must be satisfied to enforce this target dynamics.

At this point we find convenient to define the following signals

$$s := q - q_r; \quad q_r := q_d - \Lambda \int_0^t e(\tau) d\tau; \quad e := q - q_d \quad (2.2)$$

where $\Lambda = \Lambda^T > 0$ and q_d is an n -dimensional vector which, as will become clear below, is a state vector for the controller. In terms of these signals we define our target system as

$$D(q)\dot{s} + [C(q, \dot{q}) + B + K_V]s + K \int_0^t s(\tau) d\tau = 0 \quad (2.3)$$

where $K_V = K_V^T \geq 0$ is such that $B + K_V > 0$. Now, using (2.2) we can write (2.1) as the target system plus a perturbation as (the arguments of D , C and g will be omitted in the sequel)

$$D\dot{s} + [C + B + K_V]s + K \int_0^t s(\tau) d\tau = \psi \quad (2.4a)$$

$$\psi := Mu - [D\dot{q}_r + (C + B)\dot{q}_r + K\dot{q}_r + g] + K_V s \quad (2.4b)$$

2.3. Design Methodology.

Our motivation to define the target system is explained as follows. If we take the time derivative of the target systems "total energy"

$$H := \frac{1}{2} \left(\dot{s}^T D s + \left[\int_0^t s(\tau) d\tau \right]^T K \left[\int_0^t s(\tau) d\tau \right] \right) \quad (2.5)$$

along the trajectories of (2.4) we get

$$\dot{H} = -s^T (B + K_V) s + s^T \psi \leq 0 \quad (2.6)$$

Assume for the moment that $\psi = 0$. In this case, invoking standard arguments we can conclude that $e \rightarrow 0$ as $t \rightarrow \infty$ independently of the choice of q_d . In particular, we can choose q_d to solve $y_d = h(q_d, \dot{q}_d)$, thus we can insure $y \rightarrow y_d$ as $t \rightarrow \infty$. If in addition q_d is a bounded function then all signals are uniformly bounded.

The task of insuring $\psi = 0$ can be addressed as follows. First, notice that setting $\psi = 0$ in (2.4b) and replacing (2.2) we get an integro-differential equation for q_d as

$$(Dp^2 + \Lambda_1 p + \Lambda_2 + \Lambda_3 \int_0^t (\cdot) d\tau) q_d = Mu + v \quad (2.7)$$

where $p := d/dt$, v is a known function of q , \dot{q} and $\int q$ and the Λ_i are

$$\Lambda_1 := D\Lambda + C + B + K_V, \quad \Lambda_2 := (C + B)\Lambda + K + K_V\Lambda, \quad \Lambda_3 := K\Lambda$$

From invertibility of D it is clear that, for all u and v , (2.7) admits a solution in q_d . The key point is whether, for the given y_d , we can find solutions u and q_d of the integro-differential equation (2.7) which satisfy the algebraic equation $y_d = h(q_d, \dot{q}_d)$.

In a typical situation M will be a matrix with one 1 per column and 0 elsewhere. In this case it is natural to choose u that cancels the corresponding row and then solve the reduced set of integro-differential equations for q_d . These equations define the controller dynamics. This procedure is further detailed in the paragraph below.

2.4 An Output Regulation Problem.

At this point we find convenient to get a state space representation of (2.7) as

$$\dot{\lambda} = A_4 \lambda + B_1 (v + \bar{u}) \quad (2.8)$$

where we have chosen a state vector

$$\lambda := [\lambda_1^T, \lambda_2^T, \lambda_3^T]^T \in \mathbb{R}^{3n} \\ \lambda_1 := D \int_0^t q_d d\tau, \quad \lambda_2 := D q_d, \quad \lambda_3 := D \dot{q}_d \quad (2.9)$$

and the matrices A_4 and B_1 are given by

$$A_4 := \begin{bmatrix} \dot{D} D^{-1} & I_n & 0 \\ 0 & \dot{D} D^{-1} & I_n \\ -\Lambda_3 D^{-1} & -\Lambda_2 D^{-1} & \dot{D} D^{-1} - \Lambda_1 D^{-1} \end{bmatrix}, \quad B_1 := \begin{bmatrix} 0 \\ 0 \\ I_n \end{bmatrix}$$

Now, let us assume that $(Mu)^T = [u^T, 0]$. Also, we accordingly split λ_3 into $\lambda_{3u} = [\lambda_{3u}^T, \lambda_{3x}^T]^T$, where we use the subindices u and x to underscore the fact that these components will now be used as control and state signals respectively. To this end, notice that for any given λ and λ_{3u} we can select u so that the corresponding equations in (2.8) are satisfied. Therefore, with this choice of u , we can reduce our problem of solving the remaining equations in (2.8) to:

Output regulation problem. Consider the state space system

$$\dot{\lambda} = (A_4)_r \lambda + B_1 \lambda_{3u} + (B_1)_r v \quad (2.11)$$

where $(\cdot)_r$ denotes the operation of replacing the rows $2n+1$ up to $2n+r$ by zero entries, and B is a $(3n \times r)$ matrix with an r -dimensional identity matrix in the rows $2n+1$ up to $2n+r$. Find λ_{3u} such that the "output" $h(D^{-1} \lambda_2, D^{-1} \lambda_3)$, with h as in (2.1), tracks a desired reference y_d for all v .

■ ■ ■

From the preceding discussion the result below follows immediately.

Proposition 2.1. Consider the dynamic system (2.1) with A.1. Assume that for the given bounded output reference $y \in \mathbb{R}^m$, $m \leq n$, there exists a bounded function λ_{su} that solves the output regulation problem above. Under these conditions, the control law

$$u_1 = (Dp^2 + (C+B)p + K)q_r + q_i + (K_v)_i s \quad (2.12)$$

with q_r and s given by (2.2), (2.8)-(2.11) insures that the output tracking error $y - y_d \rightarrow 0$ as $t \rightarrow \infty$ with all internal signals uniformly bounded.

■ ■ ■

Remark 2.3 Finding conditions for solvability of the output regulation problem above is a far reaching question for which a solution, in the present level of generality, seems difficult to attain. Notice that A_4 and h are, in general, a function of q and \dot{q} . A significant contribution towards the understanding of this fundamental problem has been reported in [18]. Fortunately, in some specific applications (e.g., induction motors and robots with flexible joints) a solution can be obtained.

Remark 2.4. The integral term in (2.3) was proposed in [16] to address the problem of motion control of robots with flexible joints. In this case, the vector q is partitioned into q_1 -link angles and q_2 -motor shaft angles, and $y = q_1$. The problem is further simplified by the fact that the matrices D , C and B are block diagonal, thus an explicit solution for q_2 is available.

Remark 2.5. Global convergence of (2.4) can be still established relaxing the requirement $\psi \equiv 0$ to $\psi \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ or, as shown in Theorem 3 in [15], imposing a passivity condition to the map $s \mapsto \psi$. Current research is under way to exploit this additional flexibility.

3. APPLICATION TO INDUCTION MOTOR TORQUE CONTROL

3.1. Induction Motor Model.

In this section we apply our procedure to the following problem. Consider the nonlinear d-q model of a three-phase p pole pair induction motor [6]

$$V(u_3, \dot{q}_5) \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{bmatrix}$$

$$V := \begin{bmatrix} R_s + L_s p & -u_3 L_s & L_{sr} p & -u_3 L_{sr} \\ u_3 L_s & R_s + L_s p & u_3 L_{sr} & L_{sr} p \\ L_{sr} p & -(u_3 - p \dot{q}_5) L_{sr} & R_r + L_r p & -(u_3 - p \dot{q}_5) L_r \\ (u_3 - p \dot{q}_5) L_{sr} & L_{sr} p & (u_3 - p \dot{q}_5) L_r & R_r + L_r p \end{bmatrix}$$

$$j \dot{q}_5 = -b \dot{q}_5 + y; \quad y = h(q) := L_{sr} (\dot{q}_2 \dot{q}_3 - \dot{q}_1 \dot{q}_4) \quad (3.2)$$

where q_i , $i=1, \dots, 4$, are the d and q components of stator and rotor currents, u_1 and u_2 are the stator d and q-axis voltages, y the generated torque is the output we want to regulate, q_5 is the rotor angular speed and u_3 is the primary frequency. The parameters of the machine are R_s , R_r , L_s , L_r the rotor and stator resistances and self-inductances respectively, L_{sr} the mutual inductance, j the rotor inertia and b the damping coefficient. Notice that all parameters are positive.

Remark 3.1. As shown in any standard textbook, e.g. [6], this so-called d-q model can be obtained from the lumped element relationships between fluxes, currents and voltages of an idealized three phase machine, Kirchoff's laws and a nonlinear transformation (to a rotating frame). An important feature of this model is that the variables of interest appear as DC quantities. It can also be derived, as proposed in [13], see also [5], directly from the total energy function via Lagrange's equations. The latter approach is particularly useful since it provide us with the total energy function. From the control point of view the model (3.1), (3.2) is a fifth order nonlinear differential equation with states $q := [q_1, q_2, q_3, q_4, q_5]^T$, control inputs $u := [u_1, u_2, u_3]^T$ and output y .

Remark 3.2. Typically, the only variables available for measurement are the stator currents (q_1 and q_2) and the rotor speed (q_5). Furthermore, the rotor resistance (R_r) varies considerably with the motor temperature. Extensions of the basic design procedure to handle these cases are now available and will be reported elsewhere.

3.2. Symplectic Form.

A key quantity that determines the performance of the induction motor is the slip frequency defined as the difference between the synchronous speed of the stator field and the mechanical speed of rotation of the machine

$$\omega_s := u_3 - p \dot{q}_5 \quad (3.3)$$

A common procedure in practice is to fix the slip frequency at some desired value, say ω_{sd} , which is chosen so as to maximize the machines efficiency, see e.g. [4]. Here, we take the same approach and select

$$u_3 = \omega_{sd} + p \dot{q}_5 \quad (3.4)$$

We now show that with this choice of u_3 we can write the induction motor model in the form (2.1) and therefore apply the design procedure described above. To this end, we let $Mu := [u_1, u_2, 0, 0, 0]^T$ and set

$$D = \text{diag}\{\bar{D}, j\} \in \mathbb{R}^{5 \times 5}, \quad B = \text{diag}\{\bar{B}, b\} \in \mathbb{R}^{5 \times 5}$$

$$C = \begin{bmatrix} \bar{C} & -c \\ \bar{C}^T & 0 \end{bmatrix} \in \mathbb{R}^{5 \times 5}, \quad K = g = 0 \quad (3.5)$$

where we have defined

$$\bar{D} := \begin{bmatrix} L_{s2} & L_{sr2} \\ L_{sr2} & L_{r2} \end{bmatrix}; \quad \bar{C} := \begin{bmatrix} L_{s2} & L_{sr2} \\ L_{sr2} & L_{r2} \end{bmatrix} \omega_{sd};$$

$$\bar{B} := \begin{bmatrix} R_{s2} & 0 \\ 0 & R_{r2} \end{bmatrix}$$

$$c := [L_{s2}\dot{q}_2 + L_{sr}\dot{q}_4, -(L_{s1}\dot{q}_1 + L_{sr}\dot{q}_3), 0, 0]^T \quad (3.6)$$

with I_2 the 2x2 identity matrix and J_2 the 2x2 antisymmetric matrix

$$J_2 := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -J_2^T \quad (3.7)$$

It is easy to see that with this choice of D and C assumption A.1 holds and the induction motor model belongs to the class considered above.

3.3. Constant Reference Torque Case.

To apply the design procedure described above notice first, that our control objective is given in terms of q (currents) and not q , thus we can simplify the controller by setting $q_r = q_d$ ($\Lambda = 0$) in (2.2). With these choices equation (2.7) reduces to

$$D\ddot{q}_d + (C+B)\dot{q}_d = Mu \quad (3.8)$$

Remember that our problem is to find a solution of (3.8) such that $y_d = h(q_d)$ (3.1c) for the given y_d . One simple way to solve the problem, which is consistent with engineering practice, will be given first for the case of constant torque references. The general time-varying reference case is discussed in the next paragraph.

To this end, notice first that $h(\cdot)$ is independent of q_5 . Therefore, we can define q_{d5} from the fifth equation of (3.8)

$$J\ddot{q}_{d5} = -b\dot{q}_{d5} - (L_{s2}\dot{q}_2 + L_{sr}\dot{q}_4)\dot{q}_{d1} + (L_{s1}\dot{q}_1 + L_{sr}\dot{q}_3)\dot{q}_{d2} \quad (3.9)$$

Now, a design technique that has proven very successful in applications [1,9,11] is the field-oriented control that attempts to make the d-q coordinate frame coincide with the coordinate frame rotating synchronously with the rotor flux vector. This objective translates in our approach into selecting $q_{d5} = 0$. Furthermore, for constant output references we can restrict our attention to constant current references and the differential equations of (3.8) reduce to the simple algebraic equations

$$\dot{q}_{d2} = \frac{-L_r}{L_{sr}} \dot{q}_{d4}, \quad \dot{q}_{d1} = \frac{-R_r}{\omega_{sd} L_{sr}} \dot{q}_{d4}, \quad (3.10)$$

with the equation for the output reference

$$y_d = \frac{R_r}{\omega_{sd}} \dot{q}_{d4} \quad (3.11)$$

We are in position to present the main result of this section.

Proposition 3.1. Consider the induction motor model (3.1) and the given constant torque reference y_d . Define the desired constant currents q_{di} , $i=1, \dots, 4$ by (3.10), (3.11) and the nonlinear state feedback control law

$$\begin{aligned} u_1 &= R_{s1} \dot{q}_{d1} - \omega_{sd} (L_{s2} \dot{q}_2 + L_{sr} \dot{q}_4) - \\ &\quad - (L_{s2} \dot{q}_2 + L_{sr} \dot{q}_4) \dot{q}_{d5} - k_{v1} (\dot{q}_1 - \dot{q}_{d1}) \\ u_2 &= R_{s2} \dot{q}_{d2} + \omega_{sd} L_{s1} \dot{q}_{d1} + L_{sr} \dot{q}_1 \dot{q}_{d5} - k_{v2} (\dot{q}_2 - \dot{q}_{d2}) \\ u_3 &= \omega_{sd} + \dot{q}_{d5} \end{aligned} \quad (3.12)$$

where ω_{sd} is a (constant) desired slip frequency, $k_{vi} \geq 0$ and q_{d5} is the solution of (3.9). Under these conditions, the closed loop system insures global asymptotic output tracking with all signals bounded. Furthermore, the currents converge (exponentially) to their desired values.

Proof. From the developments above it is clear that q_d satisfies $y_d = h(q_d)$ and $\dot{y}_d \neq 0$. Thus, from Proposition 2.1 we have that $(q - q_d) \in \mathcal{L}_\infty$ and converges (exp.) to zero, consequently $y \rightarrow y_d$ as $t \rightarrow \infty$. Since $q_{di} \in \mathcal{L}_\infty$, for $i=1, \dots, 4$, we have that $q_i \in \mathcal{L}_\infty$ for $i=1, \dots, 4$. The proof that $q_5 \in \mathcal{L}_\infty$ follows noting from (3.9) that q_{d5} is the output of a stable first order filter with bounded input, thus $q_{d5} \in \mathcal{L}_\infty$. Exponential convergence follows from (2.5), (2.6), notice that $K=0$. Finally, the limit point of q_{d5} results from (3.9). ■

Remark 3.3. Two are the main drawbacks of the proposed design strategies. First, that rotor currents (or equivalently, fluxes) are not available for measurement. Second, that the rotor resistance varies considerably with the motor temperature. It is interesting to notice that this parameter is used in the control implementation only for the determination of the set point values. Therefore, imprecise knowledge of this parameter will only affect the steady state error and will not drive the system to instability.

Remark 3.4 The proposed control law (3.12) is very simple to implement and tune since it only contains two design parameters. The controller is always well defined, even in start-up.

Remark 3.5. It is clear from (2.6) that the convergence rate is determined by the natural damping of the motor. Notice that the smallest eigenvalue of $B + K_v$ is given by $\min(R_r, b)$. Therefore our design will be sensitive to these values.

3.4 Time-varying Reference Torque Case.

To analyse the general time-varying reference torque case we will follow also the "field-oriented" idea described above, that is we set $q_{d3}=0$. Further, we find convenient to choose $q_{d3}=\text{constant}$ to be defined below. With this choices the design equation (2.7) reduces to

$$L_{sr} \dot{q}_{d1} = L_{sr} \omega_{sd} \dot{q}_{d2} + L_{r} \omega_{sd} \dot{q}_{d4}$$

$$L_{sr} \dot{q}_{d2} = -L_{sr} \omega_{sd} \dot{q}_{d1} - R \dot{q}_{d4} \quad (3.13)$$

A simple calculation shows that the solutions of (3.13) satisfy

$$h(\dot{q}_d) = R \omega_{sd} (\dot{q}_{d4})^2 \cos \omega_{sd} t \quad (3.14)$$

with $h(\cdot)$ given by (3.2). It is clear then that with a suitable selection of ω_{sd} and q_{d4} it is possible to track sinusoidal torques of angular frequency ω_{sd} and arbitrary amplitude.

Unfortunately, we have not been able to find a satisfactory solution to the problem of reference signals with arbitrary internal model. One difficulty stems from the fact that in the output regulation procedure described above we are faced with critically stable zero dynamics and only "weak" relative degree zero, therefore the procedure of [18] is not directly applicable.

REFERENCES

- [1] W. Leonhard, "Microcomputer control of high-performance dynamic AC drives: A survey", *Automatica*, Vol. 22, No.1, pp. 1-19, 1986.
- [2] A. DeLuca, "Design of an exact nonlinear controller for induction motors", *IEEE Trans. Aut. Cont.*, Vol. 34, No. 12, pp. 1304-1307, 1989.
- [3] H. Kwatny and H. Kim, "Variable structure regulation of partially linearizable systems", *Syst. & Cont. Letters*, Vol. 15, No. 1, pp. 67-80, July 1990.
- [4] D. Kim, et al, "Control of induction motors via feedback linearization with input-output decoupling", *Int. J. Control*, Vol. 51, No. 4, pp. 863-883, 1990.
- [5] G. Espinosa and R. Ortega, "Nonlinear control of induction motors: An energy dissipation based approach", *Proc. IFAC Wkshp. Cont. Syst. Design*, Zurich, Switz., Sept. 1991.
- [6] S. Seely, "Electromechanical Energy Conversion", McGraw-Hill, 1962.
- [7] J. Skowronski, "Nonlinear Lyapunov Dynamics", World Sc. Press, 1990.
- [8] R. Ortega and G. Espinosa, "Passivity properties of induction motors", *Proc. IEEE-Ind. Appl. Soc. Conf.*, Mich. Sept. 1991.
- [9] Y. Hori and T. Umeno, "Robust flux observer based field orientation controller", 11th IFAC World Cong., Tallinn, USSR, Aug. 1990.
- [10] A. Isidori, "Nonlinear Control Systems: An Introduction", Springer-Verlag, 1989, Second Ed.
- [11] R. Lorenz and D. Lawson, "A simplified approach to continuous on-line tuning of field-oriented induction machine drives", *IEEE Trans. Ind. Appl.*, Vol. 26, No. 3, pp.420-424, 1990.
- [12] G. Verghese and S. Sanders, "Observers for flux estimation in induction machines", *IEEE Trans. Ind. Electr.*, Vol. 35, No. 1, pp. 85-94, February 1988.
- [13] J. Meisel, "Principles of electromechanical energy conversion", McGraw-Hill, 1966.
- [14] A. Van der Schaft, "Systems theory and mechanics", in *Three Decades of Mathematical Systems Theory*, Lect. Notes in Control & Inf. Sc., Vol. 135, Springer-Verlag, Berlin, pp. 426-452, 1989.
- [15] R. Ortega and M. Spong, "Adaptive motion control of rigid robots: a tutorial", *Automatica*, Vol. 25, No.6, pp. 877-888, 1989.
- [16] R. Lozano and B. Brogliato, "Adaptive control of robot manipulators with flexible joints", (Private Correspondence).
- [17] J. Slotine and W. Li, "On the adaptive control of robot manipulators", *Int. J. Robotics Res.*, 6(3), pp. 49-59, 1987.
- [18] A. Isidori and C. Byrnes, "Output regulation of nonlinear systems", *IEEE Trans. Aut. Control*, Vol. AC-35, No. 2, Feb. 1990, pp. 131-140.