On the USE of MAXIMUM LIKELIHOOD ESTIMATION for the ASSEMBLY of SPACE STATION FREEDOM

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ABSTRACT

The assembly of Space Station Freedom is performed by connecting a series of modules, carried aloft by the Space Shuttle, using a remote manipulation system (RMS). The numerous configurations involved necessitate efficient and accurate modeling of the structural and control dynamics. The inability to perform vibration tests of the assembled modules prior to launch dictate that parameter estimation be employed to refine the structural dynamics models using on-orbit data. Only then, can control system stability and performance be assured. Because of the reduced number of model parameters, distributed parameter modeling enable accurate estimation of model parameters. Distributed parameter models of the Solar Array Flight Experiment, the Mini-MAST truss, and Space Station Freedom assembly are discussed.

1. INTRODUCTION

Modeling of complex flexible spacecraft is an issue which has far reaching consequences in controller design and the subsequent spacecraft performance. Numerous difficulties in controlling flexible spacecraft have been attributed to inaccuracies in modeling [1]. With higher controller bandwidth, modeling issues assume a greater significance. Increased size and more demanding control specifications promise to make high performance control more difficult [2]. Current modeling schemes for the design and analysis of structural and control systems have several limitations [3]. The conventional approach of finite element modeling is to use thousands of simple spring elements which are void of dynamics in their interior. The computational cost and numerical inaccuracies involved in generating solutions to these equations impose a practical limit to the size (and consequently the accuracy) of these structural dynamics models. Although finite element models are adequate for problems with minimal control-structure interaction, high performance control systems will require increased accuracy in the models.

Distributed parameter modeling offers higher fidelity models of control-structural dynamics in a single set of equations, thereby eliminating the need and difficulties of reduced order approximations. The proposed approach represents flexible structural members by partial differential equations offering significant advantages in modeling, parameter estimation, and the integrated design of control/structural systems [4],[5], [6]. The present method differs from the finite element method in that a single super element can represent all the dynamics of a section of structure and produce the

forces and moments at its boundaries. These elements are then connected at their boundaries to form the model of the complete structure. Bishop [7] and Snowden [8] have studied applications in which a limited number of such elements have been connected to form three-element portal frames. The homogenization technique [9][10] for repetitive lattice trusses, is particularly useful. For spacecraft control applications, it is necessary to connect many distributed parameter elements to represent the structural dynamics of complex flexible spacecraft. Poelart's [11] DISTEL software enables several types of distributed parameter elements to be connected at their boundaries to form models of complex spacecraft. The PDEMODO software has similarly been successful in modeling the (1) Solar Array Flight Experiment (SAFE) [12], (2) Spacecraft Laboratory Control Experiment (SCOLE) [13], and (3) MiniMAST Experimental Facility [14][15][16][17].

This paper will discuss (1) the formulation of the structural dynamic models, (2) estimation of model parameters, (3) accuracy of results, and (4) discussion of modeling the assembly of Space Station Freedom.

2. NOMENCLATURE

A, B, C, D	- coefficients of shape functions
c	- model parameter vector
C	- damping coefficient
EA _z	- longitudinal stiffness
EI _x , EI _y	- bending stiffness
F	- force, force/moment input
G	- control system transfer function
GA _x , GA _y	- lateral shear
GI _ψ	- torsional stiffness
I	- moment of inertia, matrix
K	- dynamic stiffness matrix
L	- length of beam element
m ₁₀ , m ₁₈	- mass at bays 10 and 18, respectively
m'	- mass per unit length
M	- moment, system mass matrix
P _F , P _M	- force, moment matrix
Q _u , Q _s	- linear, angular deflection matrix
r, R	- attachment point, position vector
W	- weighting, error covariance matrix
T	- direction cosine matrix
u, u'	- linear and angular deformation
β	- eigenvalue coefficient
δ	- vector of differences
σ	- standard deviation
Θ	- state vector of coefficients
ω	- modal frequency
Ω	- angular velocity vector

3. FORMULATION OF STRUCTURAL DYNAMICS

The dynamics of structural elements are represented by their partial differential equations. These elements are then connected at their boundaries to form the structural model. The formulation uses a Newtonian or inertial frame of reference for the motion of the rigid bodies and the flexible beam super-elements.

Figure 1. Diagram of a Rigid Body Attached to its Reference Flexible Beam

Consider, as an example, a flexible beam attached to a rigid body as shown in Fig. 1. For the deflected beam

$$R_{attach}(t) = R_{attach}(0) + T_{beam}r + T_{beam}u(t) \quad (1)$$

where T_{beam} is a direction cosine matrix which performs the transformation between the rigid body coordinate system and the inertial frame. The acceleration of the body center-of-gravity is:

$$\ddot{R}_{c.g.}(t) = T_{beam} \ddot{u}(t) + R T_{beam} \ddot{u}'(t) \quad (2)$$

The forces and moments for each rigid body are related to the mode shapes of each flexible beam. In each case, one has to account for the different frames of reference and points of attachment. Equations of motion, in the s domain, for the linear and angular degrees of freedom for all of the bodies are comprised of the blocks:

$$m_j s^2 [T_j Q_{uj} + R_j T_j Q_{u'j}] - \sum_i [T_{beam-i} P_{Fi}] = 0 \quad (3)$$

$$I_j s^2 T_j Q_{u'j} - \sum_i [T_{beam-i} P_{Mi} + R_{beam-i} P_{Fi}] = 0 \quad (4)$$

For each case in which a rigid body has more than one beam attached, constraint equations are added to the system of equations. Assembly of the equations of motion and the constraint equations yields the system characteristic equation.

The partial differential equations for the transverse bending of a beam with Euler bending stiffness, Timoshenko shear, and axial force stiffness terms for lateral deflections in the x - z plane is given by:

$$m \ddot{u} + E I_y u'''' + G A \ddot{u}'' + F_0 u'' + C_1 \dot{u}'' + C_2 \dot{u}''' = F_x \quad (5)$$

where the dots and primes indicate differentiation with respect to time and distance, respectively. The

equation for bending in the y - z plane is similar.

Axial and torsion dynamics are represented by the wave equations:

$$m \ddot{u}_z - E A_z u_z'' + C_4 u_z' = F_z \quad (6)$$

$$I_\psi \ddot{\psi} - G I_\psi \psi'' + C_3 \dot{\psi} = M_z \quad (7)$$

The solutions of these partial differential equations for zero damping produce the sinusoidal and hyperbolic spatial equations which comprise the mode shape functions for bending in the x - z plane, bending in the y - z plane, axial elongation, and torsion:

$$u_x(z) = A_x \sin \beta_x z + B_x \cos \beta_x z + C_x \sinh \beta_x z + D_x \cosh \beta_x z \quad (8)$$

$$u_y(z) = A_y \sin \beta_y z + B_y \cos \beta_y z + C_y \sinh \beta_y z + D_y \cosh \beta_y z \quad (9)$$

$$u_z(z) = A_z \sin \beta_z z + B_z \cos \beta_z z \quad (10)$$

$$u_\psi(z) = A_\psi \sin \beta_\psi z + B_\psi \cos \beta_\psi z \quad (11)$$

These undamped mode shapes are expected to be good approximations to the exact solutions for low level of damping. The system or configuration mode shape consists of these functions, repeated for each beam element. Each element has two nodes and considering bending in two directions, torsion and elongation, 12 coefficients are needed for the solution. These coefficients of sinusoidal and hyperbolic functions serve as the state vector.

$$\Theta^T = [A_x \ B_x \ C_x \ D_x \ A_y \ B_y \ C_y \ D_y \ A_z \ B_z \ A_\psi \ B_\psi] \quad (12)$$

The linear and angular deflection, force and moment vectors can be expressed as:

$$u = [u_x \ u_y \ u_z]^T \approx Q_u(z) \Theta$$

$$u' = [-u_y' \ u_x' \ u_\psi']^T = Q_{u'}(z) \Theta \quad (13)$$

$$F_{attach} = [F_x \ F_y \ F_z]^T = P_F(z) \Theta$$

$$M_{attach} = [M_x \ M_y \ M_z]^T = P_M(z) \Theta \quad (14)$$

4. PARAMETER ESTIMATION

Next, maximum likelihood estimation is used to estimate the most probable parameter values given noisy experimental data. The difference between estimated and measured response is minimized iteratively. Convergence of the process yields the 'best' values of the unknown model parameters. The maximum likelihood estimator also provides a measure of uncertainty of each estimated model parameter. The Cramer-Rao bound [18][19] gives a minimum value of the standard deviation of the error in estimating the model parameter.

The maximum likelihood estimation technique is usually applied to transient response data as was done for the Mini-MAST truss in [16]. In this paper, however, it will be applied to static deflection data and frequency response measurements [17]. The reduced

number of unknown parameters give distributed parameter models a distinct advantage for parameter estimation using experimental data. The cost function:

$$J = \sum (\mathbf{y}_{\text{meas},n} - \mathbf{y}_{\text{model},n})^T \mathbf{W}^{-1} (\mathbf{y}_{\text{meas},n} - \mathbf{y}_{\text{model},n})$$

$$= \sum (\delta_n^T \mathbf{W}^{-1} \delta_n) \quad (15)$$

The modified Newton-Raphson technique [19] can be used to determine, iteratively, improved estimates of the model parameters, i.e.:

$$\mathbf{c}_{m+1} = \mathbf{c}_m + [\sum (\partial \delta_n / \partial \mathbf{c})^T \mathbf{W}^{-1} (\delta_n / \partial \mathbf{c})]^{-1} [\sum (\delta_n / \partial \mathbf{c})^T \mathbf{W}^{-1} \delta_n] \quad (16)$$

The sensitivity functions can be determined analytically or by using numerical difference approximations. The software PDEMODO can generate these functions for estimation or optimization purposes.

Normally, maximum likelihood estimation is applied to transient responses for ordinary differential equation and partial differential equation models [18][19]. Under the circumstances considered in this paper, however, it is necessary to determine the most likely model parameter values given two different types of measurements. For example, static deflection data can produce a measured beam stiffness, EI. Modal frequencies can be measured using frequency response data. A cost function which merges both types of data for bending in a single plane, can be written as:

$$J = \delta^T \mathbf{W}^{-1} \delta \quad (17)$$

$$J = \sum_k \sigma_k^{-2} (\omega_k, \text{meas.} - \omega_k, \text{model})^2 +$$

$$\sigma_{EI}^{-2} (EI, \text{meas.} - EI, \text{model})^2 +$$

$$\sigma_{m_{10}}^{-2} (m_{10}, \text{meas.} - m_{10}, \text{model})^2 +$$

$$\sigma_{m_{18}}^{-2} (m_{18}, \text{meas.} - m_{18}, \text{model})^2 +$$

$$\sigma_{m'}^{-2} (m', \text{meas.} - m', \text{model})^2 \quad (18)$$

The minimization of this cost function with the error covariance, W, yields maximum likelihood estimates of the model parameters.

5. PDEMODO SOFTWARE

In this way, information is used from static deflection tests and dynamic frequency response tests to determine the distributed parameters, EI, m', m₁₀, and m₁₈. The software PDEMODO will be used to relate the modal frequencies to the model parameter values in the estimation process. These parameters and several other parameters for the Mini-MAST Truss experiment were estimated in this way. The Mini-MAST truss was chosen because of its similarity to that proposed for Space Station Freedom. PDEMODO computer software enables modeling complex, flexible spacecraft consisting of three dimensional networks of flexible beams and rigid bodies. Each beam allows bending in two directions, torsion and elongation degrees of freedom. Further, rigid bodies can be

attached to beam ends at any angle or body location. PDEMODO is also capable of performing parameter estimation based on matching experimental modal frequencies and static deflection test data. The optimization features of PDEMODO is not available in DISTEL or BUNVIS [20].

- 3 Dimensional
- Flexible Beams (Bend, Twist, Elongate)
- Rigid Bodies (Full Inertia Matrices)

Figure 2. Generic Configuration

A typical configuration that can be modeled using PDEMODO is shown in figure 2. The power of this software lies in its distributed parameter modeling, parameter estimation, control system dynamics and parametric studies applications. The application of PDEMODO to the Mini-MAST modeling and parameter estimation will now be discussed.

6. RESULTS

The model of Mini-MAST [3] will consist of (1) body #1 to represent the cantilevered condition at its base, (2) beam #1 which runs from the base to bay 10, (3) body #2 located at bay 10, (4) beam #2 which runs from bay 10 to bay 18, and (5) body #3 at the tip, bay 18. The input file for PDEMODO [15] requires information concerning the topology of the configuration. The first column of matrix CONFIG identifies the beam. The second column identifies the inboard body, and the third column identifies the outboard body. A negative sign is used to designate the beam which a body uses as its reference axis, i.e.:

	Beam I.D.	Inboard Body	Outboard Body
CONFIG =	1	-1	-2
	2	2	-3

Body 1 uses the inboard end (z=0) of beam 1 as a reference axis. Bodies 2 and 3 use the outboard ends of beams 1 and 2, respectively, as shown in Fig. 3.

The physical characteristics of each body and each beam element are also required.

Figure 3. Schematic of the Mini-MAST Truss.

The parameter values for the two beams are given below, in English units.

$$\begin{array}{ll} L_1 = 66.24 \cdot 10/18 & L_2 = 66.24 \cdot 8/18 \\ EI_{X1} = 27565000. & EI_{X2} = 27565000. \\ EI_{Y1} = 27565000. & EI_{Y2} = 27565000. \\ EA_{Z1} = 10530000. & EA_{Z2} = 10530000. \\ EI_{Y1} = 2163465. & EI_{Y2} = 2163465 \\ m/L_1 = .1075 & m/L_2 = .1075 \\ I_Y/L_1 = .27 & I_Y/L_1 = .27 \end{array}$$

The mass characteristics of the first body represent the cantilevered condition at the base of the first beam. The mass characteristics of the second body located at bay 10 and the third body at bay 18, are:

$$\begin{array}{lll} m_1 = 10^9 & m_2 = 3.391 & m_3 = 10.36 \\ I_{xx1} = 10^{14} & I_{xx2} = m_2 \cdot 5^2 & I_{xx3} = m_3 \cdot 1.5^2 \\ I_{yy1} = 10^{14} & I_{yy2} = m_2 \cdot 5^2 & I_{yy3} = m_3 \cdot 1.5^2 \\ I_{zz1} = 10^{14} & I_{zz2} = I_{xx2} + I_{yy2} & I_{zz3} = I_{xx3} + I_{yy3} \end{array}$$

6.1 Modal Frequencies of the A Priori Model

PDEM0D was first used to generate the modal frequencies for the a priori model of the Mini-MAST truss.

Mode	Type	Frequency-Hz
1, 2	Bending #1	.7577
3	Torsion #1	3.941
4, 5	Bending #2	6.506
6	Axial #1	16.92
7	Torsion #2	20.41
8, 9	Bending #3	28.05
10	Torsion #3	42.61
11, 12	Bending #4	46.27
13	Axial #2	58.15
14	Torsion #4	60.30
15	Torsion #5	82.80
16, 17	Bending #5	85.65

Values of EI were derived from static deflection tests and were treated as measured parameters [15]. The modal frequencies derived from frequency response tests were also treated as measured values. Because of the involvement of model parameters in both types of data, it is necessary to determine the most likely parameter values. Figure 4. shows part of the frequency response results that was used to measure the modal frequencies [3].

Figure 4. Frequency Response Measurements

The resulting cost function for matching both static compliance data and the measured modal frequencies has the same form as equation [17] where:

$$\delta = \begin{bmatrix} \omega_{1meas} \\ \omega_{2meas} \\ \vdots \\ \omega_{12meas} \\ EI_{meas} \\ EA_{meas} \\ GI_{\psi meas} \\ m'_{meas} \\ m_{18meas} \\ m_{10meas} \\ I_{\psi/Lmeas} \\ r_{18meas} \\ r_{10meas} \end{bmatrix} - \begin{bmatrix} \omega_{1model} \\ \omega_{2model} \\ \vdots \\ \omega_{12model} \\ EI_{model} \\ EA_{model} \\ GI_{\psi model} \\ m'_{model} \\ m_{18,model} \\ m_{10,model} \\ I_{\psi/mmodel} \\ r_{18model} \\ r_{10model} \end{bmatrix} \quad (19)$$

The inverse error covariance, W^{-1} , is a diagonal matrix with elements equal to 1 divided by the square of the standard deviation. A standard deviation of 1 percent was used for the modal frequencies and mass characteristics, 2 percent for EI and GI_{ψ} , and 4 percent for r_{18} and r_{10} .

Next, PDEM0D was used to generate the sensitivity matrix, $\partial\delta/\partial c$, by calculating the changes in the modal frequencies for incremental changes in the model parameter. Then the "best" model parameter values were obtained from:

$$c_{best} = c_a \text{ protri} + [\partial\delta/\partial c]^T W^{-1} \partial\delta/\partial c \quad (19)$$

$$c_{best} = \begin{bmatrix} EI \\ EA \\ GI_{\psi} \\ m' \\ m_{18} \\ m_{10} \\ I_{\psi/L} \\ r_{18} \\ r_{10} \end{bmatrix} = \begin{bmatrix} 28620000. \\ 13120000. \\ 1988000. \\ .1077 \\ 9.915 \\ 3.397 \\ .235 \\ 1.512 \\ .4916 \end{bmatrix}$$

6.2 Modal Frequencies of the Estimated Model

The modal frequencies of the distributed parameter model using the estimated parameters are listed below:

Mode	Type	Measured Freq. Res	Adjusted Model
1, 2	Bending #1	.86	.79
3	Torsion #1	3.9	3.86
4, 5	Bending #2	6.2	6.64
6	Axial #1	-	19.2
7	Torsion #2	22.7	20.7
8, 9	Bending #3	30.9	28.6
10	Torsion #3	43.5	43.7
11, 12	Bending #4	40.	47.2
13	Torsion #4	67.2	61.5
14	Axial #2	-	65.1
15	Torsion #5	-	84.8
16, 17	Bending #5	69.5	87.6

There is fair agreement in the modal frequencies

except for the 1st, 4th and 5th bending modes. Inclusion of the effects of a support cable at the tip will raise the model values slightly. The use of a Timoshenko beam instead of the Bernoulli beam improved the match for the higher mode frequencies. In addition, the dynamics of the truss diagonal elements and their hinges are modeled by a coupled partial differential equation[17]. These changes greatly improve the accuracy of structural dynamic models.

6.3 Examples of Modeling Error

Not only simple structural configurations can be modeled using partial differential equations. The software PDEM0D makes it possible to model arbitrarily complex, flexible spacecraft. Figure 5. shows configurations modeled using PDEM0D.

Figure 5. Configurations Modeled Using PDEM0D.

There remains concern, however, as to the accuracy of partial differential equation models. The following tabulation gives some examples of modeling accuracy for a variety of structural configurations[7][12][17]. For the Solar Array Flight Experiment and Mini-MAST comparisons are made for the finite element and distributed parameter models.

Structural Configuration	Number of Modes	R.M.S. Error in ω	
		Finite El.	Dist. Param.
Symmetrical Frame	6	-	.5 %
Unsymmetrical Frame	6	-	1.4 %
Solar Array Flt. Exp	5	1.7 %	3.2 %
Mini-MAST Truss	5	6.4 %	4.0 %

It is clear that the partial differential equation models are competitive with the finite element models. In addition, modeling is underway for (1) the Evolutionary Model Experiment, (2) the LACE satellite, and (3) the Space Station. It is believed that the experience obtained in modeling these flexible structures will establish partial differential equation modeling as a practical and accurate approach.

7. SHUTTLE-RMS-SPACE STATION MODEL

The structural dynamics of the assembly configuration of Space Station Freedom involves modeling the Space Shuttle, its Remote Manipulation System (RMS), and the Space Station (Fig. 6.). The Space Shuttle and

Station cluster of modules can be considered rigid for frequencies less than 5 Hertz. The two booms of the RMS, Station truss, and solar arrays will be modeled as flexible beam elements. The software PDEM0D can be used to automatically assemble the dynamic equations of the Space Shuttle, RMS, and Space Station.

Because it is impossible to test the Space Station configurations prior to launch it is necessary to refine the models using on-orbit data. Bayes rule for conditional probability [19] provides the conditional likelihood function for additional data while considering the uncertainty of the a priori model. Repeated use of this approach will enable refining the model as additional data is available.

Figure 6. Space Station Freedom Assembly

It is also necessary to include in the model the dynamics of the control systems for the Shuttle, RMS, and Station. The insertion of the dynamics of these control systems involves adding terms to the system of equations previously developed for the structural dynamics. One advantage of this approach is that model order reduction is not required for closed system analysis. It is possible to synthesize control laws in this domain without resorting to the truncated modal model [21]. Starting with the open loop system of equations and substituting the feedback law, $F(s) = G(s)x$, the closed loop characteristic equation becomes:

$$Ms^2X(s) + C(s)sX(s) + K(s)X(s) - G(s)X(s) = 0 \quad (20)$$

The stability of the closed loop control-structure system is established by the eigenvalues of this equation. Note that $K(s)$ is a dynamic stiffness matrix which contains all of the dynamics of the flexible elements, albeit in irrational form.

8. CONCLUDING REMARKS

An approach of generating a family of dynamic models of the evolving Space Station Freedom has been described. The distributed parameter approach takes advantage of (1) the relatively small number of model parameters associated with partial differential equation models of structural dynamics, (2) maximum likelihood estimation using both prelaunch and on-orbit test data, (3) the inclusion of control system dynamics in the same equations, and (4) the incremental growth of the structural configurations.

Maximum likelihood parameter estimates for distributed parameter models were based on static compliance test results and frequency response measurements. Because the Space Station Freedom does not yet exist, the NASA Mini-MAST truss was used to test the procedure of modeling and parameter estimation. The resulting distributed parameter model of the Mini-MAST truss successfully demonstrated the approach taken. The same procedures can be used to generate and refine the family of configurations involved in the assembly of the Space Station Freedom.

Until recently the work needed to generate distributed parameter models of complex, flexible spacecraft was prohibitive. The computer program, PDEM0D, enables any configuration to be modeled which can be represented by a network of flexible beam elements and rigid bodies. The modeling process is well suited for the evolving Space Station Freedom, for the cases in which (1) the Space Station assembly is attached to the Shuttle, (2) the assembly is linked to the Shuttle through the RMS arm, and (3) the Space Station assembly is free of the Shuttle. The same software was used for parameter estimation for the Solar Array Flight Experiment and the NASA Mini-MAST truss using experimental data.

The resulting dynamic models of the evolving Space Station Freedom can be used to analyze the system stability and performance. This is done by imbedding the control system dynamics into the structural dynamic equations. The need to perform model order reduction is thereby eliminated. This control/structural system is particularly well suited for the optimization of both control and structural parameters.

Considerable work remains to be done in modeling, parameter estimation and control synthesis for distributed parameter models to realize all of these advantages for the assembly of Space Station Freedom.

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