

SPACESTRUCTURE CONTROL USING  
MOVING BANK MULTIPLE MODEL ADAPTIVE ESTIMATIONRobert W. Lashlee, Jr.\* and Peter S. Maybeck  
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Colorado Springs, CO 80840**ABSTRACT**

This research investigates the feasibility of moving-bank multiple model adaptive estimation algorithms as applied to flexible spacestructure control. Moving-bank multiple model adaptive estimation/control is an attempt to reduce the computational loading associated with the implementation of a fullscale multiple model adaptive estimator/controller. This research shows that the moving-bank controller performs nearly identically to a benchmark controller and substantially better than a fixed-bank controller with a coarse discretization level that covers the entire range of parameter variation.

**I. INTRODUCTION**

A problem exists in some applications of estimation and control which is caused by the uncertainty of parameters in mathematical models. These uncertain parameters reduce the accuracy which can be expected from the system model and algorithms based on that model. Their values can be constant yet unknown, slowly varying, or changing abruptly. These changes in parameters often necessitate the identification of parameters within the mathematical model during a real-time control problem. In a system adequately represented by a linear stochastic state model, a bank consisting of a Kalman filter for each possible parameter value (full-bank MMAE algorithm) can be used to alleviate the problem of uncertain parameters. It is assumed that the uncertain parameters can take on only discrete values; either this is reasonable physically or discrete values are chosen from the continuous parameter variation range. For each Kalman filter, a conditional probability that its parameter is "correct", given the measurement history, is computed. These conditional probabilities are based on the characteristics of the residuals of each Kalman filter and are used as a weighting factor for the state estimate produced by each Kalman filter [14:365-369].

However, one basic problem of the full bank MMAE algorithm is the number of Kalman filters which must be computed simultaneously. For example, if a system had two uncertain parameters and if each of these parameters can take on ten different values, then 100 Kalman filters need to be implemented. As can well be imagined, this is an extremely high computational burden [12:1876]. A method proposed by Maybeck and Hentz is to implement a small number of estimators in a "moving bank" [3;12]. For instance, one might take the current best estimate of the uncertain parameters, and implement only those estimators (and controllers) that most "closely" surround the estimated value in parameter space. For the case of two uncertain

parameters requiring 100 separate filters, the three discrete values of each parameter that most closely surround the estimated value can be selected, requiring only 9 separate filters instead of 100; see Fig. 1. As the parameter estimate changes, the choice of filters could change, resulting in a "move" of the bank of 9 filters. Hentz [12] applied the moving-bank MMAE to a simple but physically motivated two-state system model and was able to demonstrate performance equivalent to the full-bank MMAE algorithm (and also equivalent to a benchmark of an estimator or controller artificially given knowledge of the true parameters), with an order of magnitude less computational loading. Karnick [4;5] showed that, although the use of a moving bank may provide enhanced state estimation performance, similar performance could have been obtained from a fixed bank estimator with a coarse space

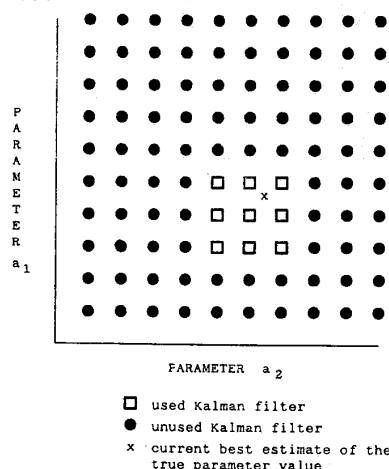


Fig. 1. Moving-Bank Multiple Model Adaptive Estimator

discretization that covered the range of parameter variation. This research is directed towards determining the cause of the problem Karnick experienced with the moving-bank algorithm and fine tuning the moving-bank algorithm.

**II. MULTIPLE MODEL ADAPTIVE ESTIMATION**

Let  $a$  denote the vector of uncertain parameter in a given linear stochastic state model for a dynamic system. These parameters can affect the matrices defining the structure of the model or depicting the statistics of the noises entering it. In order to make simultaneous estimation of states and parameters tractable, the continuous range of  $a$  values is discretized into  $K$  representative values. If we define the hypothesis conditional probability  $p_k(t_i)$  as the probability

that  $a_k$  assumes the value  $a_k$  (for  $k=1, 2, \dots, K$ ) conditioned on the observed measurement history to time  $t_i$ :

$$p_k(t_i) = \text{Prob}(a=a_k | Z(t_i) = Z_i) \quad (1)$$

then it can be shown [2, 9, 11] that  $p_k(t_i)$  can be evaluated recursively for all  $k$  via the iteration:

$$p_k(t_i) = \frac{f_{z(t_i)|a, z(t_{i-1})}(z_i | a_k, z_{i-1}) p_k(t_{i-1})}{\sum_{j=1}^K f_{z(t_i)|a, z(t_{i-1})}(z_i | a_j, z_{i-1}) p_j(t_{i-1})} \quad (2)$$

Notationally, the measurement history random vector  $Z(t_i)$  is made up of partitions  $z(t_1), \dots, z(t_i)$  that are the measurement vectors available at the sample times  $t_1, \dots, t_i$ ; similarly, the realization  $Z_i$  of the measurement history vector has partitions  $z_1, \dots, z_i$ . Furthermore, the minimum mean square error estimate of the state is the probabilistically weighted average:

$$\begin{aligned} \hat{x}(t_i^+) &= E(x(t_i) | Z(t_i) = Z_i) \\ &= \sum_{k=1}^K \hat{x}_k(t_i^+) p_k(t_i) \end{aligned} \quad (3)$$

where  $\hat{x}_k(t_i^+)$  is the state estimate generated by a Kalman filter based on the assumption that the parameter vector equals  $a_k$ .

Thus, the multiple model adaptive filtering algorithm is composed of a bank of  $K$  separate Kalman filters, each based on a particular value  $a_1, \dots, a_K$  of the parameter vector, as depicted in Fig. 2. When the measurement  $z_i$  becomes available at  $t_i$ , the residuals  $r_1(t_i), \dots, r_K(t_i)$  are generated in the  $K$  filters and used to compute  $p_1(t_i), \dots, p_K(t_i)$  via Eq. (2). Each numerator density function in Eq. (2) is given by:

$$\begin{aligned} & \left( \frac{1}{(2\pi)^{m/2} |A_k|} \right) \exp \left[ -\frac{1}{2} r_k^T(t_i) A_k^{-1}(t_i) r_k(t_i) \right] \\ & \text{where } m \text{ is the measurement dimension and} \end{aligned} \quad (4)$$

where  $m$  is the measurement dimension and

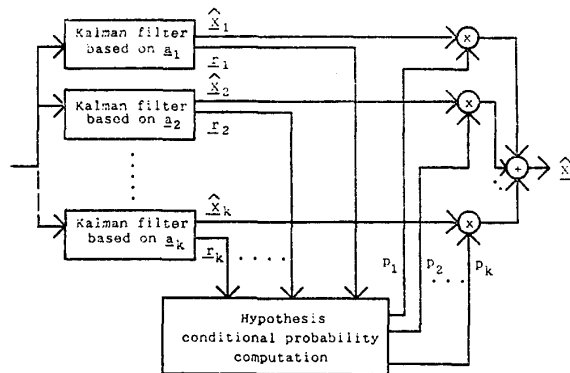


Fig. 2. Multiple Model Adaptive Estimator

$A_k(t_i)$  is the residual covariance calculated in the  $k$ -th Kalman filter within its gain computation. The denominator in Eq. (2) is simply the sum of all the computer numerator terms and thus is the scale factor required to ensure that the  $p_k(t_i)$ 's sum to one.

One expects the residuals of the Kalman filter based upon the "best" model to have mean squared value most in consonance with its own computed  $A_k(t_i)$ , while "mismatched" filters will have larger residuals than anticipated through  $A_k(t_i)$ . Therefore, Eqs. (2), (3), and (4) will most heavily weight the filter based upon the most correct assumed parameter value. However, the performance of the algorithm depends on there being significant differences in the characteristics of residuals in "correct" vs. "mismatched" filters. Each filter should be tuned for the best performance when the "true" values of the uncertain parameters are identical to its assumed value for these parameters. One should specifically avoid the "conservative" philosophy of adding considerable dynamics pseudonoise, often used to open the bandwidth of a single Kalman filter to guard against divergence, since this tends to mask the differences between "correct" and "mismatched" models.

The moving-bank multiple model adaptive estimator is identical to the full-bank estimator just described except that the  $K$  does not correspond to the total number of possible parameter vector values. Instead, it is the smaller number of elemental Kalman filters maintained with the bank. Which particular  $K$  filters are in the bank at a given time is determined by a decision mechanism. The decision mechanism used in this research is probability monitoring [1, 3, 4, 12]. In this mechanism, the conditional hypothesis probabilities, generated by Eq. (2), are monitored. If the conditional hypothesis probability associated with an elemental filter is larger than a previously determined threshold, the bank is centered on that filter. Maybeck and Hentz [3, 12] found this decision logic to provide the best performance of the decision logics they investigated.

### III. TWO BAY TRUSS [6]

The rotating two bay truss (see Fig. 3) approximates a space structure that has a hub with an appendage extending from the structure. The mass of the hub is large relative to the mass of the appendage, and the hub can be rotated to point the appendage in a commanded direction. The truss was originally developed to study the effects of structural optimization on optimal control design [17]. Also, a similar model was used to research active control laws for vibration damping [10].

The structure consists of 13 rods which are assumed to be constructed of aluminum, having a modulus of elasticity of  $10^7$  psi and weight density of  $.1 \text{ lb/in}^3$  [17]. Nonstructural masses with a mass of  $1.294 \text{ lb-sec}^2/\text{in}$  are located at positions 1, 2, 3, and 4 as shown in Fig. 3. The nonstructural mass is very large compared to the structural mass so as to achieve the low frequencies associated with large space structures [10].

The general second-order differential equations which describe the forced vibration of a large space structure with active controls and  $n$  frequency modes can be written in physical coordinates as [10, 17].

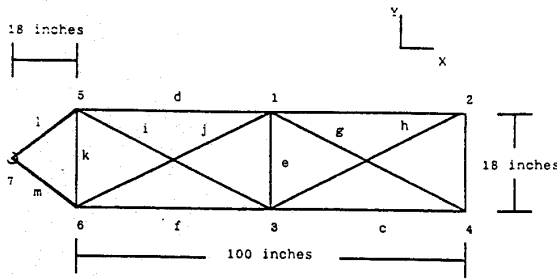


Fig. 3. Rotating Two-bay Truss Model

$$\ddot{M}r(t) + \dot{C}r(t) + Kr(t) = -bu(t) - gw(t) \quad (5)$$

where:

- M - constant nxn mass matrix,
- C - constant nxn damping matrix,
- K - constant nxn stiffness matrix,
- u(t) - vector of length m representing actuator input,
- b - nxm matrix identifying position and relationship between actuators and controlled variables [9],
- w - vector of length r representing dynamics driving noise, where r is the number of noise inputs,
- g - nxr matrix identifying position and relationships between dynamics driving noise and controlled variables.

The mass and stiffness matrices are obtained from finite element analysis [18]. The control system is assumed to consist of a set of discrete actuators, and the external disturbances and unmodeled control inputs are represented by white noise. The state representation of Eq. [5] can be written as:

$$\dot{x} = Ax + Bu + Gw \quad (6)$$

$$x = [r^T \quad \dot{r}^T]^T \quad (7)$$

and the open-loop plant matrix A, the control matrix B, and the noise matrix G are given by:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}_{2n \times 2n} \quad B = \begin{bmatrix} 0 \\ -M^{-1}b \end{bmatrix}_{2n \times m} \quad (8)$$

It is assumed that the noises enter the system at the same location as the actuators (B matrix = G matrix). Measurements are assumed available from accelerometers (located at nodes 1 and 2) and gyros (located at the hub). The output signals of the accelerometers are integrated once to provide velocity measurements and once again to provide position measurements. Therefore, the position and velocity measurements are assumed co-located. It is further assumed that the measurements are noise corrupted due either to deficiencies in the model of the sensor or some actual external measurement noise. The measurements are modeled as:

$$z = \begin{bmatrix} H & 0 \\ 0 & H' \end{bmatrix} x + v \quad (9)$$

px2n

where p is the number of measurements, v is an uncertain measurement disturbance of dimension p and modeled as a white noise [13:114], H is the position measurement matrix, and H' is the velocity measurement matrix. The velocity and position measurement matrices are identical for this application because of co-location of the velocity and position sensors.

Modal analysis transforms the system into a set of decoupled modal equations [10, 16, 17, 18]. In order to achieve decoupling, the damping matrix is assumed to be a linear combination of the mass and stiffness matrices [8]. It is assumed that uniform damping exists throughout the structure. The damping coefficient of  $\xi = 0.005$  is chosen for implementation because it is characteristic of damping associated with large space structures [10].

#### Parameter Variation

A 10 by 10 point discretization in parameter space is created by considering two physically motivated parameter variations. Initially it is assumed that the four non-structural masses vary -50% to +40% from the nominal value in discrete steps of 10%, and the entire stiffness matrix is varied -20% to +16% from the nominal value in discrete steps of 4%. The mass variation can be physically related to fuel being depleted or shifted to a different section of the space structure. The change in the stiffness matrix can be associated with structural fatigue in the rods or a failure of a member within the structure itself. The initial discretization proved to yield unsatisfactory parameter estimation performance [9]. Therefore, a space discretization study was performed to determine an optimal discretization level. In the final discretization level, the non-structural masses vary -50% to +50% in a nonlinear fashion, while the entire stiffness matrix varies -50% to +40% in a nonlinear fashion [9] see Fig. 8. This new space discretization level aids in parameter identification.

#### Order Reduction

The method of order reduction based on singular perturbations [6,7,10,15:219] is used to reduce the system model from 24 to 6 states. The method of singular perturbations assumes that faster modes reach steady state essentially instantaneously. The deterministic system is reformulated as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad (10)$$

$$z = [H_1 \quad H_2]x \quad (11)$$

The  $x_1$  states are to be retained and  $A_{11}$  and  $A_{22}$  are square matrices. The high frequency modes are eliminated by assuming

steady state is reached instantaneously in these modes ( $x_2 = 0$ ). It can be shown that, for the state equations developed previously (Eqs. (6 - 9)), the reduced order system model is:

$$\dot{x}_1 = A_r x_1 + B_r u \quad z = H_r x_1 + D_r u \quad (12)$$

where

$$A_r = A_{11} \quad (13)$$

$$B_r = B_1 \quad (14)$$

$$H_r = H_1 \quad (15)$$

$$D_r = (-H_2 A_{22}^{-1} B_2) \quad (16)$$

$D_r$  is the only term in Eq. (16) that is dependent upon terms associated with the modes assumed to reach steady state instantaneously. The other reduced-order matrices are calculated simply by truncating those states associated with  $x_2$ . Numerical problems in the calculation of  $D_r$  can be avoided by reforming Eq (16). It can be shown that [4:55-56]:

$$D_r = \begin{bmatrix} H_2 [-w_2^2]^{-1} b' \\ 0 \end{bmatrix}_{p \times 1} \quad (17)$$

where  $p$  is the number of measurements and  $b'$  is the non-zero portion of the  $B_2$  matrix which has the same form as the  $B$  matrix in Eq. (8). The inverse of  $w_2^2$  is easily calculated since the matrix is diagonal.

#### IV. PERFORMANCE ANALYSIS

A ten run Monte Carlo analysis is used to evaluate the performance of a moving-bank multiple model adaptive estimator/controller. The rotating two bay truss discussed previously is used, with the uncertain parameters being the non-structural mass and the stiffness matrices. The parameters were discretized into a 10 x 10 point parameter space yielding a 100-filter full-bank estimator. By comparison, the moving-bank estimator will implement a subset of 9 filters, corresponding to a 3 x 3 array of points within the full grid.

For all algorithms, only steady state constant-gain elemental filter (and controller) gains are considered. The purpose of this investigation is to assess the effectiveness of a moving-bank estimator or controller, using probability monitoring decision logic discussed previously, under realistic conditions on the "true" parameter. For the purposes of this investigation, the "true" parameter is considered constant and equal to one of the discretized values; it may lie outside the initial location of the 3 by 3 moving bank. The ability of the moving-bank estimator to provide adequate state estimation accuracy is the primary criterion of performance; the main objective is to design a good adaptive state estimator, not a parameter identifier.

This research concentrates on two areas. The first is to determine the cause of the moving-bank MMAE performance degradation with system model complexity seen in past research efforts, and the second is to fine tune the moving-bank MMAE and MMAC. The first step in

analyzing the problems experienced in past research efforts with the moving-bank MMAE is to determine appropriate values for the noise covariance ( $R$ ) and dynamics noise strength ( $Q$ ). The next step is to reduce the system model to two states for both the filter and truth models in order to determine if Hentz's [3;12] successes with a two state model could be reproduced for the truss model. Following this study, the system model is increased to six states. Then the performance of the 6-state MMAE is determined to find out if system complexity seriously affects the moving-bank MMAE algorithm.

The fine tuning of the MMAE consists of determining an appropriate space discretization level so that the individual filters have distinguishable performance characteristics. Finally, a controller study is conducted to determine the performance of the MMAC.

#### V. ANALYSIS RESULTS

##### Two-State Model

The goal of the two-state model investigation was to determine if Hentz's work [3] could be duplicated for the large space structure model. Hentz showed that, for a two-state filter model matched against a two-state truth model for simulation of the "real world", the center of the moving-bank would move in the direction of the true parameter. However, it is important to note that optimal performance of the moving bank was not a goal of this study. In the Monte Carlo simulations, a dither signal (square wave) with a magnitude of 300 and frequency of 30 rad/sec is used to excite the system. In order to determine the trends in the movement of the bank, a record is needed of the estimated true parameter [6]. This record is acquired by initializing a 10 x 10 matrix of elements to zero and then adding one at each sample time to the matrix element corresponding to the estimate of the true parameter. Entries are recorded from  $t = 1.0$  to the end of the run. Recording is not started at  $t = 0.0$  in an attempt to avoid transient effects. In addition, the moving-bank algorithm uses the smallest discretization level and is initially centered at the (5,5) parameter point (i.e., in the middle of the parameter space). In Fig. 4, the mass parameter is pulled in the correct direction but the stiffness parameter is not pulled in any direction. This and the state estimation performance are directly comparable to Hentz's results [3,12].

##### Six-State Filter

Since Hentz's work is duplicated for the one-bending mode model, it is desired to determine if the performance of the moving-bank algorithm degrades for a more complex system. Therefore, the state dimension of both the filter model and the "truth" model are increased to six. The data shown in Fig. 5 is acquired using the same technique as in Fig. 4. Fig. 5 shows a definite pulling of the center of the moving-bank toward the true parameter point. In analyzing Fig. 5 as compared to Fig. 4, it appears that the six-state system performed better than the two-state system, which would indicate that the moving-bank algorithm performed better as the system complexity increases. However, the  $Q$  and  $R$  values are chosen specifically for tun-

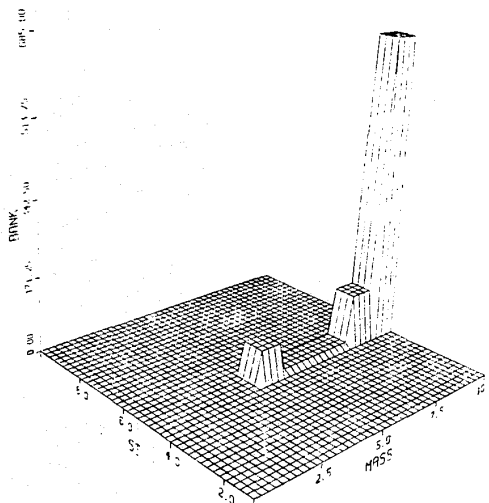


Fig. 4. Two-State Model Bank Location Time History; True Parameter at Mass = 9, Stiffness = 2

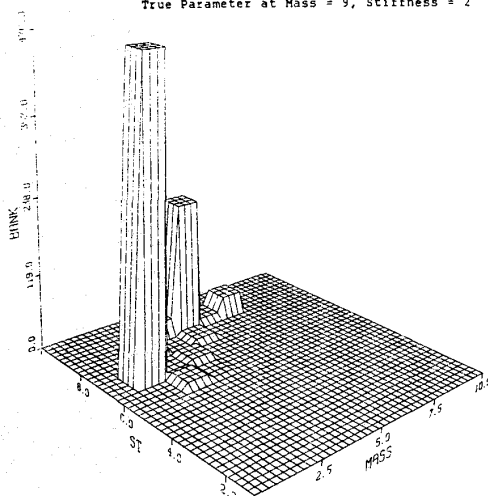


Fig. 5. Six-State Model Bank Location Time History; True Parameter at Mass = 2, Stiffness = 7 Old Discretization Level

ing the six-state model. These same  $Q$  and  $R$  values are used in the two-state model study, therefore the  $Q$  and  $R$  values are not tuned for the two-state model study. However, the goal of the two-state model study is to show the trend of the center of the bank, not to obtain optimal performance, and so no lengthy retuning process is accomplished.

The performance of the six-state filter indicates that system complexity does not appreciably affect the performance of the moving-bank parameter estimation. This is true as long as the measurements are precise enough; if  $R$  matrix entries are too large, the ability to move the bank effectively through parameter space is lost, and estimation performance degrades because the excessive measurement noise masks the effects of different assumed parameter values in the elemental filters. Sensitivity studies indicated much less pronounced impact from  $Q$  variations, even allowing for order-of-magnitude changes on matrix elements.

However, the six-state model still has a performance problem. Recall in Fig. 5, the

moving bank's estimate of the true parameter did not lock onto the actual true parameter value. The corresponding state estimation errors for the (2,7) parameter point are unstable (error standard deviations grow to  $10^{24}$  by the end of the 10 sec simulation). However, the moving-bank algorithm's estimate of the true parameter did lock onto the actual true parameter value. The corresponding state estimate error plots for the (9,9) parameter point are stable. This indicates that there is a strong correlation between the ability of the moving-bank algorithm to estimate the true parameter and its ability to provide precise state estimates. Couple this correlation with the fact that the old space discretization level was determined by intuition, and this indicates that a space discretization level study must be performed to determine an appropriate space discretization level.

#### Space Discretization Study

The space discretization study was accomplished by monitoring the rms error of the state variable estimates from a nonadaptive filter, as the true parameter and the filter parameter are moved apart. A new discretization level is determined, and the benefits of the new space discretization level can be seen in Fig. 6 (compare to Fig. 5). The moving bank's parameter estimate is now locking onto the actual true parameter. In addition, the state estimation error plot produced for the (2,7) parameter point is now stable.

#### Controller Study

A controller study is performed to compare the performance of the moving-bank multiple model controller with a fixed-bank controller and a benchmark controller. The fixed-bank controller uses the largest possible discretization level in parameter space and the bank is not allowed to move. The benchmark controller uses one filter given artificial knowledge of the true parameter value. A dither signal is used to excite the system for 0.5 sec, during which time an open-loop adaptive estimator operates, which is then followed by multiple model adaptive control (see Fig. 7; controller gains in this figure were evaluated by linear-quadratic regulator synthesis).

The transients are short lived and stable, therefore in the remainder of this paper the post-transient performance is given in the figures. Plots are generated for the benchmark controller, the moving-bank controller, and the fixed-bank controller, for true parameter value at the point (7,6). These plots show that the moving-bank and the benchmark controllers perform nearly identically. This similarity is expected because the moving-bank algorithm locks onto the actual true parameter. Therefore, nearly all the probability weighting is being applied to the filter that corresponds to the true parameter value (i.e. essentially the same as the benchmark controller). These figures also show that the fixed-bank controller performs much worse than the moving-bank controller. The reason for this poor performance is twofold. First, it was initially assumed that the fixed-bank controller would put most of its probability weight on the four filters that surround the true parameter point. Instead, it put nearly all of its

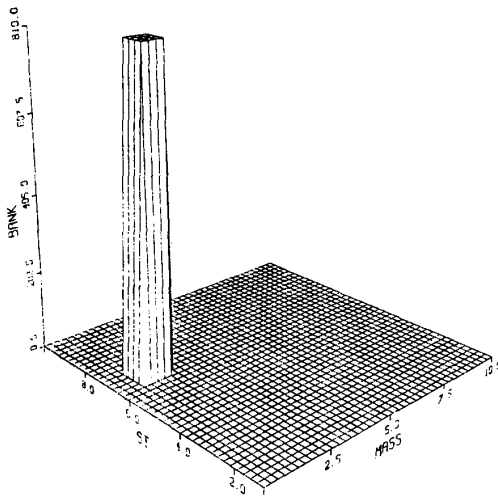


Fig. 6. Six-State Model Bank Location Time History; True Parameter at Mass = 2, Stiffness = 7 New Discretization Level

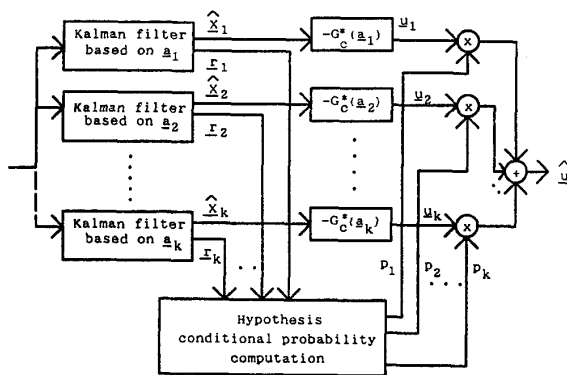


Fig. 7. Multiple Model Adaptive Controller

probability weight (i.e., 1) on one filter and bounced this heavy weight factor nonsymmetrically throughout the nine filters in the fixed bank configuration. In addition, the state and control quadratic cost weighting matrices (used for controller gain synthesis) were all initially tuned for the (7,6) parameter point. Therefore, these  $X$  and  $U$  matrices are not tuned for the locations in parameter space that correspond to the filter in the fixed-bank controller.

To show the need for tuning the  $X$  and  $U$  matrices for each parameter point, the true parameter point is set to (5,5) and the  $X$  and  $U$  matrices for the (7,6) parameter point are used. The performance of the moving-bank controller is unstable. However, when the  $X$  and  $U$  matrices are tuned for the (5,5) parameter point, the moving-bank controller is stable.

#### Conclusions and Recommendations

The values of the measurement noise covariance ( $R$ ) play an extremely important role in the performance of the moving-bank algorithm. A range of admissible measurement precisions exists, beyond which the effective movement of the bank in parameter space is seriously impaired. In fact, this was the predominant problem with the moving-bank algorithm performance in past research efforts.

Once the moving bank is being pulled in the right direction in parameter space through proper choice of accurate sensors, additional performance can be achieved by determining the optimal space discretization level. The correct discretization level can transform a poorly operating moving bank into an effectively operating one.

After the best moving-bank MMAE has been designed, the next step is to enhance the controller performance of the MMAC. For the large spacestructure model, the moving-bank MMAC needs to have the state weighting matrix and control weighting matrix of the linear quadratic regulator synthesis tuned for each parameter point. The fixed-bank MMAC performs as well as the moving-bank MMAC if the true parameter corresponds to one of the nine filters contained in the fixed-bank MMAC, but, the fixed-bank MMAC performance degrades substantially if the true parameter does not correspond to one of those filters. Thus, the moving-bank algorithm inherently has greater performance potential, since it does not require such a coarse discretization of parameter space in order to yield a practically implementable controller.

The most important recommendation by far is that the robustness to unmodeled effects be investigated. This would be accomplished by keeping the filter dimensioned at six, but increasing the "truth" dimension to 16 states or more. It is also recommended that the state weighting matrix and control weighting matrix be tuned for each parameter point.

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