High Performance Phase Rotated FD-MC-CDMA To Exploit Full Diversity

Zhiping Zhang¹, Lin Zhang², Yang Qu¹, John Ellinger³ and Zhiqiang Wu²

¹Department of Electrical Engineering, Wright State University

²School of Information Science and Technology, Sun Yat-sen University

Sensors Directorate, Air Force Research Laboratory³

Email: Zhang.176@wright.edu

Abstract—FD-MC-CDMA is an attractive frequency domain CDMA system that provides high performance in multi-path fading channels by exploiting both diversity gain and multi-user detection (MUD) gain. In FD-MC-CDMA, by decomposing the entire subcarrier set into multiple non-contiguous subcarrier sets, the number of interfering users within each subcarrier set is significantly reduced. As a direct result, an optimal maximum likelihood MUD receiver can be implemented at low complexity. In this paper, we first revisit previously developed phase rotation spreading code design to bring signal space diversity to conventional MC-CDMA systems. Similarly, due to the binary nature of the spreading codes, full diversity is not always exploited in FD-MC-CDMA system. Then we combine the phase rotated spreading code design scheme with FD-MC-CDMA system to exploit full diversity. With the phase rotated spreading codes and the exploitation of full diversity, the new FD-MC-CDMA system offers significant performance enhancement compared with the original FD-MC-CDMA system along with conventional MC-CDMA system. Simulation results over multi-path fading channels confirm the performance gain of the proposed scheme.

I. INTRODUCTION

FD-MC-CDMA [1]- [3] is a frequency based multiple access architecture that combines the best parts of *frequency division multiple access* (FDMA) and multi-carrier code division multiple access (MC-CDMA) [4]- [7] to simultaneously exploit frequency diversity and minimize *multiple access interference* (MAI). It is well-known that MC-CDMA can achieve high diversity gain in frequency selective fading channels by simultaneously transmitting over N multiple subcarriers, and separating the signal into subcarriers at the receiver providing for excellent BER performance. The achieved performance gain degrades in the presence of MAI. On the other hand, FDMA can completely avoid MAI by assigning each user a unique subcarrier with the trade-off of reduced performance in fading channels due to no frequency diversity gains at the receiver.

Similar to MC-CDMA, FD-MC-CDMA system has N subcarriers. However, instead of using all N subcarriers, they are divided into groups. Typically, fading channels in spread spectrum wireless systems (such as MC-CDMA) demonstrate an M -fold frequency diversity over the transmit bandwidth, where M is, for example, 2, 3 or 4. Mathematically, this

Fig. 1. FD-MC-CDMA: $N = 32$ carriers, $M = 4$ fold diversity

corresponds to

$$BW = M \cdot (\Delta f)_c \quad (1)$$

where BW is the bandwidth of the MC-CDMA transmission and $(\Delta f)_c$ is the coherence bandwidth of the frequency selective fading channel.

To exploit the M fold frequency diversity, it is unnecessary to spread a user's information symbol over all N ($N \gg M$) subcarriers. The total frequency diversity can be exploited when each user's information is sent over only M subcarriers separated by the coherence bandwidth $(\Delta f)_c$. In FD-MC-CDMA, each user's information bit is only transmitted over a group of M subcarriers, as opposed to all N subcarriers. By having a sufficiently large frequency separation between each group (larger than or equal to the coherence bandwidth of the channel), the frequency diversity gain is comparable to that of MC-CDMA. Since each group supports only $K \leq M$ users, the MAI at the receiver side is much lower than that of the MC-CDMA system.

As an example, the FD-MC-CDMA frequency decomposition scheme is illustrated in Figure 1 where M subcarriers exist in $\frac{N}{M}$ subcarrier-groups. In this example, the total number of subcarriers is $N = 32$, the coherence bandwidth is $(\Delta f)_c = 8\Delta f$. Hence, the 32 subcarriers are decomposed into $\frac{N}{M} = 8$ sets, each only consisting of $M = 4$ non-contiguous subcarriers.

Since all subcarriers are orthogonal to one another, each subcarrier set forms a virtual MC-CDMA system consisting of M subcarriers supporting up to M orthogonal users, without any interference from other subcarrier sets.

An important benefit of FD-MC-CDMA is the low complexity of its optimum *maximum likelihood* (ML) *multi user detection* (MUD) receiver. Since there are only up to M users

in each subcarrier set, the complexity of the ML MUD receiver is proportional to Q^M where Q is the modulation order (i.e. 2 for BPSK, 4 for QPSK, 16 for 16-QAM, etc). Compared to the complexity of the ML MUD for a fully loaded MC-CDMA system which is proportional to Q^N , it becomes realistic to build a ML MUD for the FD-MC-CDMA system. With the MUD gain, FD-MC-CDMA system significantly outperforms its MC-CDMA cousin at normal operating signal to noise ratios (SNRs).

It has been recognized in [8] and [9] that due to the nature of the spreading codes being used in conventional MC-CDMA systems, the BER performance of the commonly used Hadamard transform is asymptotically bad. A phase rotated spreading transform was proposed in [8] to achieve better asymptotic performance. However, in previous works the spreading code length is assumed to be quite small (e.g., the length is 8 in [8] and 4 or 8 in [9]). Therefore, either a optimum maximum likelihood detection receiver or a sub-optimum linear equalization detection receiver can be exploited. In conventional MC-CDMA systems, the spreading code length is usually large (e.g., 32 or 64), hence the performance gain of using such phase rotated codes is not significant.

Therefore, it is a natural win-win situation to combine FD-MC-CDMA system with the phase rotated spreading code design. The subcarrier decomposition nature of FD-MC-CDMA creates multiple parallel virtual MC-CDMA systems with small spreading code length. Hence, the benefit of phase rotated spreading codes can be fully exploited in FD-MC-CDMA systems and provides significant performance gain.

The remaining part of this paper is organized as follows. In Section II, we introduce the phase rotated spreading code design and prove that it maintains the orthogonality of the code matrix and how full diversity is always exploited. In Section III, we provide numerical results over different conditions to show the benefit of the proposed system. Conclusion follows.

II. UNEVEN POWER DISTRIBUTION ACROSS SUBCARRIERS IN FD-MC-CDMA SYSTEM

Due to the binary nature of the Hadamard-Walsh spreading codes used in MC-CDMA systems, it has been shown that the power distribution of MC-CDMA is uneven across all subcarriers. Often times the power is condensed in a small number of subcarriers, sometimes even on one subcarrier. This leads to less frequency diversity exploited in the MC-CDMA system.

The FD-MC-CDMA system in its current setting suffers the same problem and does not always exploit the full diversity in the channel due to the binary nature of the Hadamard-Walsh spreading codes. In the example we have discussed earlier, since there are $M = 4$ subcarriers in each subcarrier set, the Hadamard-Walsh spreading code matrix is

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (2)$$

where each row is a spreading code for one user.

Let's assume that there are 2 active users on this subcarrier set: user 1 is using spreading code 1 (1, 1, 1, 1) and user 2 is using spreading code 3 (1, 1, -1, -1). If user 1 is transmitting data 1 and user 2 is also transmitting data 1, given the binary nature of the spreading code, the accumulated transmitted signal actually becomes (2, 2, 0, 0). In other words, only two subcarriers have power while the other two actually are not transmitting anything. Therefore, only half of the diversity available in the channel is exploited in this scenario. Figure 2 illustrates this scenario.

Fig. 2. Diversity Loss Case 1

More significantly, there are scenarios where only one subcarrier is carrying all the transmission power. Figure 3 shows this worst case scenario. Assume all four users are transmitting where user 1 is using spreading code (1, 1, 1, 1), user 2 is using spreading code (1, -1, 1, -1), user 3's spreading code is (1, 1, -1, -1) and user 4's code is (1, -1, -1, 1). When all four users are transmitting data 1, the total transmitted signal only has power on the first subcarrier while the other three are not transmitting anything. In this case, no diversity is exploited at all: if the subcarrier carrying all the power is under deep fading, all four users' data symbols are lost.

To solve this problem, we adopt previously developed phase rotated code design and integrate it with FD-MC-CDMA [8] [9].

III. PHASE ROTATED CODE

The transmitted signal of a downlink FD-MC-CDMA system on one subcarrier set can be described as

$$s(t) = Re \left\{ \sum_{k=0}^{K-1} b^{(k)} A \sum_{n=0}^{M-1} \beta_n^{(k)} e^{j2\pi f_n t} p(t) \right\} \quad (3)$$

where K is the total number of active users ($K \leq M$), $b^{(k)}$ is the k^{th} user's data symbol ($b^{(k)} \in \{1, -1\}$ if BPSK modulation is used), $A = \sqrt{\frac{2E_s}{N \cdot T_s}}$ is the amplitude, E_s is the symbol energy, T_s is the symbol duration, M is the number

Fig. 3. Diversity Loss Case 2

of subcarriers in this subcarrier set, $\beta_n^{(k)}$ is the n^{th} component of user k 's spreading code, f_n is the frequency of the n^{th} subcarrier, and $p(t)$ is the rectangular pulse shape.

Normally, a binary code matrix $\mathbf{C}$ is employed to assign spreading codes to all users. The most commonly used spreading code is the Hadamard-Walsh code. In the code matrix $\mathbf{C}$, every row represents a spreading code for one user:

$$\mathbf{C} = \begin{bmatrix} \vec{\beta}^{(0)} \\ \vec{\beta}^{(1)} \\ \vdots \\ \vec{\beta}^{(M-1)} \end{bmatrix} = \begin{bmatrix} \beta_0^{(0)} & \beta_1^{(0)} & \cdots & \beta_{M-1}^{(0)} \\ \beta_0^{(1)} & \beta_1^{(1)} & \cdots & \beta_{M-1}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_0^{(M-1)} & \beta_1^{(M-1)} & \cdots & \beta_{M-1}^{(M-1)} \end{bmatrix} \quad (4)$$

where $\vec{\beta}^{(k)}$ represents the spreading code of the k^{th} user and $\vec{\beta}^{(k)} = (\beta_0^{(k)} \beta_1^{(k)} \cdots \beta_{M-1}^{(k)})$.

Using the example of $M = 4$ discussed earlier, let's take a closer look at how the binary spreading code is causing loss of diversity. Assuming there are $K = 4$ users on one of the subcarrier set in FD-MC-CDMA system and BPSK modulation is used, there are total of $2^K = 16$ possible data combinations $\mathbf{B}$ of these 4 users. After multiplying with the spreading code matrix $\mathbf{C}$, the 16 possible transmitted signals over the four subcarriers are:

$$\vec{y} = \mathbf{B}\mathbf{C}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (5)$$

$$= \begin{bmatrix} 4 & 0 & 0 & 0 \\ 2 & -2 & -2 & -2 \\ 2 & 2 & -2 & 2 \\ 2 & -2 & 2 & 2 \\ 2 & 2 & 2 & -2 \\ 0 & 0 & -4 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 4 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ -2 & -2 & -2 & 2 \\ -2 & 2 & -2 & -2 \\ -2 & -2 & 2 & -2 \\ -2 & 2 & 2 & 2 \\ -4 & 0 & 0 & 0 \end{bmatrix}$$

As can be seen from equation (5), there are 8 cases that all transmission power are concentrated on only one subcarrier. In other words, no diversity is exploited half of the time. This leads to significant performance loss.

Figure 4(a) explains the problem of zero power accumulation due to the binary nature of the code. Since user k is transmitting the product of data symbol and spreading code $b^{(k)}\beta_n^{(k)}$ on the n^{th} subcarrier, user l is transmitting $b^{(l)}\beta_n^{(l)}$, due to the binary nature of the code $\beta_n^{(k)}$ (and $\beta_n^{(l)}$) and the data symbol $b^{(k)}$ (and $b^{(l)}$), the code/data combination $b^{(k)}\beta_n^{(k)}$ is either +1 or -1. Therefore, it is inevitable that sometimes one user's code/data combination will be +1 and another user's code/data combination will be -1 and when they transmit the signal results in a zero, as shown in Figure 4(a).

Now, we adopt the phase rotated spreading code in developed in [8] [9] to avoid this problem. Specifically, the new code matrix $\mathbf{C}_{\text{New}}$ is designed by introducing an unique phase offset to each and every row of the original Hadamard-Walsh code matrix:

Fig. 4. Binary Code and Phase Rotated Code

$$\begin{aligned}
 \mathbf{C}_{\text{New}} &= \begin{bmatrix} P^{(0)} \cdot \vec{\beta}^{(0)} \\ P^{(1)} \cdot \vec{\beta}^{(1)} \\ \vdots \\ P^{(M-1)} \cdot \vec{\beta}^{(M-1)} \end{bmatrix} \\
 &= \begin{bmatrix} e^{j \frac{\pi}{M} 0} \cdot \vec{\beta}^{(0)} \\ e^{j \frac{\pi}{M} 1} \cdot \vec{\beta}^{(1)} \\ \vdots \\ e^{j \frac{\pi}{M} (M-1)} \cdot \vec{\beta}^{(M-1)} \end{bmatrix}
 \end{aligned} \quad (6)$$

As shown in equation (6), a phase rotator $P^{(k)}$ is multiplied to the k^{th} row of the original Hadamard-Walsh code matrix where $P^{(k)} = e^{j \frac{\pi}{M} k}$. This way, every row is rotated by a different amount in the phase space. It is easy to show that the introduction of the phase rotation does not change the orthogonality of the spreading code matrix, i.e., $\mathbf{C}_{\text{New}}$ is still an orthogonal matrix. The inner product of the k^{th} row and the l^{th} row of the new code matrix $\mathbf{C}_{\text{New}}$ is:

$$\langle P^{(k)} \cdot \vec{\beta}^{(k)}, P^{(l)} \cdot \vec{\beta}^{(l)} \rangle = P^{(k)} \cdot P^{*(l)} \langle \vec{\beta}^{(k)}, \vec{\beta}^{(l)} \rangle \quad (7)$$

Since $\mathbf{C}$ is an orthogonal matrix, $\langle \vec{\beta}^{(k)}, \vec{\beta}^{(l)} \rangle = 0, \forall k \neq l$. Therefore, $\langle P^{(k)} \cdot \vec{\beta}^{(k)}, P^{(l)} \cdot \vec{\beta}^{(l)} \rangle$ is also 0 for any two different rows in $\mathbf{C}_{\text{New}}$.

Therefore, no matter what the code/data combination of every user is, it is guaranteed that they will not accumulate to zero. This concept is shown in Figure 4(b). In Figure 4(b), 4 different phases separated by $\frac{\pi}{4}$ are introduced to the 4 different users' spreading codes. As shown in Figure 4(b), it is guaranteed that the sum of different user's signal will not be zero. In this way, we allow the FD-MC-CDMA receiver to always exploit the full diversity available in the channel.

It is important to note that when higher modulation such as QPSK is used, the phase rotation needs to be adjusted accordingly. When BPSK modulation is employed, the phase rotation is $P^{(k)} = e^{j \frac{\pi}{M} k}$. When QPSK modulation is employed, the phase rotation becomes $P^{(k)} = e^{j \frac{\pi}{2M} k}$.

IV. NUMERICAL RESULTS

In this section, we simulate the BER performance of FD-MC-CDMA system with phase rotated spreading codes (and MC-CDMA system) in multi-path fading channel to validate its benefits. We use the example discussed earlier where there exists $M = 4$ -fold diversity in a $N = 32$ subcarrier system. Generation of such correlated fades, for purposes of simulation, has been discussed in [10].

We first show the simulation results of BER versus SNR for a half loaded FD-MC-CDMA system using Hadamard-Walsh codes and a half loaded MC-CDMA system also using Hadamard-Walsh codes in Figure 5. In MC-CDMA receiver, a linear minimized mean square error combining (MMSEC) receiver is used. In FD-MC-CDMA system, a ML MUD receiver is employed. In Figure 5, the curve marked with circles represents the performance of FD-MC-CDMA, while the curve marked with stars represents the performance of MC-CDMA system. As can be seen from Figure 5, when SNR is lower than 20dB, FD-MC-CDMA system outperforms MC-CDMA system due to its capability of exploiting both diversity gain and MUD gain. However, since FD-MC-CDMA cannot exploit full diversity as often as MC-CDMA system, when SNR is higher than 20dB, its performance becomes worse than that of MC-CDMA system.

We now show the simulation results of the same half loaded systems with the phase rotated codes in Figure 6. Similarly, the curve marked with circles represents the performance of FD-MC-CDMA, while the curve marked with stars represents the

Fig. 5. Simulation 1

performance of MC-CDMA system. We show the performance of using both the Hadamard-Walsh codes and the new codes in this figure. It is evident that the new codes help both the MC-CDMA system and the FD-MC-CDMA system, due to its capability of exploiting full diversity at all times. However, it helps FD-MC-CDMA system much more than MC-CDMA system. With the new codes, FD-MC-CDMA system significantly outperforms MC-CDMA at all SNRs and avoids the crossing point at 20dB that we observed in Figure 5. In fact, as SNR grows, the gain of FD-MC-CDMA over MC-CDMA becomes larger. By introducing the new code, FD-MC-CDMA has about 2dB gain over MC-CDMA at $BER = 10^{-4}$ and about 3dB gain at $BER = 10^{-5}$.

Fig. 6. Simulation 2

Next, we show the case of fully loaded system with $K = N = 32$ users in Figure 7. Again, we see that there was a crossing point at 15dB between the curve of FD-MC-CDMA with Hadamard-Walsh code and the curve of MC-CDMA with Hadamard-Walsh code. However, when new

codes are introduced, both systems get better but FD-MC-CDMA significantly outperforms its MC-CDMA counterpart. At $BER = 10^{-4}$ FD-MC-CDMA has about 3dB gain over MC-CDMA and almost 4dB gain at $BER = 10^{-5}$.

Fig. 7. Simulation 3

We now show the simulation results of BER versus number of active users for both FD-MC-CDMA and MC-CDMA systems, with Hadamard-Walsh codes and new codes. Figure 8 shows the case of SNR=10dB, Figure 9 shows the case of SNR=15dB and Figure 10 shows the case of SNR=20dB. As can be seen from all three figures, BER of both systems becomes worse when the number of users grows due to the growth of multiple access interference power. By introducing the new codes, both FD-MC-CDMA system and MC-CDMA system enjoy performance enhancement. However, FD-MC-CDMA system enjoys a much bigger performance gain than MC-CDMA system. Particularly, as shown by Figure 10, when SNR is high, FD-MC-CDMA system with Hadamard-Walsh codes actually performs worse than MC-CDMA because of the diversity loss due to the binary nature of the spreading codes. When new codes are introduced, FD-MC-CDMA system outperforms MC-CDMA system significantly across all scenarios.

V. CONCLUSION

In this paper, we have combined previously developed phase rotated spreading codes with FD-MC-CDMA system to exploit full diversity in the channel. We show that although the phase rotated spreading codes help both conventional MC-CDMA system and FD-MC-CDMA system, the gain achieved with FD-MC-CDMA system is much more significant. The phase rotated spreading code allows FD-MC-CDMA system to truly shine its color by taking advantage of full diversity gain and low complexity MUD gain, leading to a system with superb performance in multi-path fading channels.

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Fig. 8. Simulation 4

Fig. 10. Simulation 6

Fig. 9. Simulation 5

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