Switching Behavior in Optimal Communication Strategies for Team Jamming Games under Resource Constraints

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Abstract—In this paper, we address the issue of malicious intrusion in the communication network present in a team of autonomous vehicles. In our current scenario, we consider the special case of two teams with each team consisting of two mobile agents. Agents belonging to the same team communicate over wireless ad hoc networks, and they try to split their available power between the tasks of communication and jamming the nodes of the other team. Contrary to our earlier work, this paper addresses the scenario in which each player has an omni-directional antenna for jamming the communication between the members of the other team. The agents have constraints on their total energy and instantaneous power usage. The cost function adopted is the difference between the rates of erroneously transmitted bits of each team.

We model the problem as a zero-sum differential game between the two teams and use *Isaacs'* approach to obtain the necessary conditions for the optimal trajectories. We model the adaptive modulation problem as a zero-sum matrix game which in turn gives rise to a continuous kernel game to handle power control. Based on the communications model, we present sufficient conditions on the physical parameters of the agents for the existence of a pure strategy saddle-point equilibrium (PSSPE). This leads to a switching behavior in the optimal communication strategy within a team, over the time horizon of the entire game. This behavior is illustrated in our simulations for the case when the agents are holonomic.

I. INTRODUCTION

The decentralized nature of wireless ad hoc networks makes them vulnerable to security threats. A prominent example of such threats is jamming: a malicious attack whose objective is to disrupt the communication of the victim network intentionally, causing interference or collision at the receiver side. The problem under consideration in this paper is inspired from many sources. In the past year, there have been reports of predator drones being hacked [10], resulting in intruders gaining access to classified data being transmitted from an aircraft. Motivated from such incidents, we have analyzed various scenarios of evading jamming attacks among autonomous agents. In the case of a *single* jammer trying to intrude the communication link between a transmitter and a receiver, the problem can be formulated as multi-player (specifically, three player) pursuit-evasion game [2], [12]. In [5], we investigated the problem of finding motion strategies for two unmanned autonomous vehicles (UAVs) to evade jamming in the presence of an

aerial intruder. We considered a differential game theoretic approach to compute optimal strategies by a team of UAVs. We formulated the problem as a zero-sum pursuit-evasion game. The cost function was picked as the termination time of the game. We used *Isaacs'* approach to derive the necessary conditions to arrive at the equations governing the saddle-point strategies of the players. In [7], we extended the previous analysis to a team of heterogeneous vehicles containing UAVs and autonomous ground vehicles (AGVs). In [4], we analyzed the problem of multiple jammers intruding the communication network in a formation of UAVs. In [6], we analyzed the problem of connectivity maintenance in multi-agent systems in the presence of a jammer. In this current work, we study a scenario where a *team* of malicious nodes launch a jamming attack on another team, which is capable of jamming as well.

Antenna design can facilitate intentional jamming. Reconfigurable antennas are of particular interest in jamming applications as they allow for increased degrees of freedom in adapting the operating frequency and radiation patterns. In [3], [16], [18], the authors propose various antenna designs based on changing the operating frequency while maintaining the radiation patterns. In [11], [19], the authors present reconfigurable antennas that are capable of adapting both the frequency and radiation patterns. Radiation patterns reconfigurability is key to our previous work [8], [13] where we have considered a jamming scheme in which each team member is capable of generating multiple jamming signals, of varying powers, each specific to a given agent of the opposing team. In this work, we consider a simplified design and assume that omnidirectional antennas are employed. In other words, each agent exploits the broadcast nature of the wireless channels and generates a single jamming signal to be injected into the channels of the other team. Our main aim is to study the performance-complexity trade-off associated with our new approach.

The rest of the document is organized as follows. In Section II, we provide our problem formulation. In Section III, we present the key results associated with the optimal control problem. The saddle-point equilibrium properties of the team power control problem are studied in Section IV for the specific example of systems employing uncoded M-quadrature amplitude modulations (QAM). In Section VI, we present our results on the adaptive modulation schemes

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that leads to a switching behavior in optimal communication within a team. We conclude the paper and provide future directions in Section VII. An Appendix at the end includes explicit expressions for some of the variables introduced in the paper.

II. PROBLEM FORMULATION

Consider two teams of mobile agents. Each agent is communicating with members of the team it belongs to, and at the same time- jamming the communication between members of the other team. We consider a scenario where each team has two members, though at a conceptual level our development applies to higher number of team members as well. Team A is comprised of the two players $\{1^a, 2^a\}$ and Team B is comprised of the two players $\{1^b, 2^b\}$. The agents move on a plane and therefore, have two degrees of freedom (x, y) . The dynamics of the players are given by the following equations:

- Team A:

$$\left. \begin{aligned} \dot{x}_i^a &= f_{x_i}^a(\mathbf{x}_i^a, \mathbf{u}_i^a, t) \\ \dot{y}_i^a &= f_{y_i}^a(\mathbf{x}_i^a, \mathbf{u}_i^a, t) \end{aligned} \right\} i \in \{1, 2\} \quad (1)$$

- Team B:

$$\left. \begin{aligned} \dot{x}_i^b &= f_{x_i}^b(\mathbf{x}_i^b, \mathbf{u}_i^b, t) \\ \dot{y}_i^b &= f_{y_i}^b(\mathbf{x}_i^b, \mathbf{u}_i^b, t) \end{aligned} \right\} i \in \{1, 2\} \quad (2)$$

In the above equations, $\mathbf{x}_i$ and $\mathbf{u}_i$ denote vectors representing the state and control input of agent i , with the superscript (a or b) identifying the corresponding team. The state space of the entire system is represented by $\mathbf{X} \simeq \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$. Moreover, $\mathbf{u}_i \in \mathcal{U}_i \simeq \{\phi : [0, t] \rightarrow \mathcal{A}_i \mid \phi(\cdot) \text{ is measurable}\}$, where $\mathcal{A}_i \subset \mathbb{R}^{p_i}$. $f : \mathbb{R}^2 \times \mathcal{A}_i \times \mathbb{R} \rightarrow \mathbb{R}$ is uniformly continuous, bounded and Lipschitz continuous in $\mathbf{x}_i$ for fixed $\mathbf{u}_i$. Consequently, given a fixed $\mathbf{u}_i(\cdot)$ and an initial point, there exists a unique trajectory solving (1) and (2) [1].

Now, we describe the physical layer communications model in the presence of a jammer which is motivated by [17] and has also been discussed in [13]. For each transmitter and receiver pair, we adopt the following communications model. Given that the transmitter and the receiver are separated by a distance d , and the transmitter transmits with constant power P_T , the received signal power P_R is given by

$$P_R = \rho P_T (1 + d)^{-\alpha}, \quad (3)$$

where ρ depends on the antennas' gains and the wavelength of the transmitted signal.

The signal-to-interference ratio (SINR) s is given by

$$s = \frac{P_R}{I + \sigma}, \quad (4)$$

where σ is the ambient Gaussian noise level and I is the total received Gaussian interference power due to jamming. The Bit Error Rate (BER) is given by the following expression:

$$p(t) = g(s), \quad (5)$$

where $g(\cdot)$ is a decreasing function of s . The instantaneous BER depends on the SINR, the modulation scheme, and the error control coding scheme utilized. Communications literature contains closed-form expressions and tight bounds that can be used to calculate the BER when the noise and interference are assumed to be Gaussian [9]. For uncoded M-QAM, where Gray encoding is used to map the bits into the symbols of the constellation, the BER can be approximated by [15]

$$p(t) = g(s) \approx \frac{\zeta}{r} \mathcal{Q}(\sqrt{\beta s}), \quad (6)$$

where $r = \log(M)$, $\zeta = 4(1 - 1/\sqrt{M})$, $\beta = 3/(M - 1)$, and $\mathcal{Q}(\cdot)$ is the tail probability of the standard Gaussian distribution. We also assume that players in each team have access to different M-QAM modulation schemes. We denote the set of available modulation sizes to the players in Team A by $\mathcal{M}^a$ and that available to players of Team B by $\mathcal{M}^b$. The sizes of the employed QAM modulation by the teams are $M^a \in \mathcal{M}^a$ and $M^b \in \mathcal{M}^b$. We assume that Team A can choose among n different modulation schemes, and Team B chooses from a set of m different schemes, i.e., $|\mathcal{M}^a| = n$, $|\mathcal{M}^b| = m$.

To ensure a non-zero communication rate between the agents of each team, we impose a minimum rate constraint for each agent: $R_i^a(t), R_i^b(t) \geq \tilde{R}$, where $\tilde{R} > 0$ is a threshold design rate, which we assume is the same for all agents. The results can be readily extended to networks of players having a different value of the minimum design rate.

For an initial position $\mathbf{x}_0 \in \mathbf{X}$, the outcome of the game π , is given by the following expression:

$$\pi(\mathbf{x}_0, \mathbf{u}_1^a, \mathbf{u}_2^a, \mathbf{u}_1^b, \mathbf{u}_2^b) = N \underbrace{\int_0^T \sum_{i=1}^2 [p_i^a(t) - p_i^b(t)] dt}_L \quad (7)$$

where $p_i^a(t)$ and $p_j^b(t)$ are the BERs of agent i in Team A and agent j in Team B, respectively; $\mathbf{u}_i^a$ and $\mathbf{u}_j^b$ are likewise the control inputs of agents i and j in teams A and B, respectively; N is the total number of transmitted bits which remains constant throughout the game; and T is the time of termination of the game. p_i depends in s_i , i.e., the SINR perceived by agent i . From (3), s_i depends on the mutual distances between the players. Therefore, we can see that the outcome functional, π , depends on the state of the players and hence, their control inputs. The outcome functional models the difference in the erroneous communication packets exchanged between the members of the same team during the entire course of the game. The objective of team A is to minimize π and the objective of team B is to maximize it.

Let $P_i^a(t)$ and $P_j^b(t)$ denote the instantaneous power levels for communication used by player i in Team A and player j in Team B, respectively. Since the agents are mobile, there are limitations on the amount of energy available to each agent that is dictated by the capacity of the power source carried by each agent. We model this restriction as

the following integral constraint for each agent

$$\int_0^T P_i(t) dt \leq E. \quad (8)$$

The game is said to terminate when any one agent runs out of power, that is 8 is violated.

In addition to the energy constraints, there are limitations on the maximum power level of the devices that are used on-board each agent for the purpose of communication. For each player, this constraint is modeled by the following set of inequalities:

$$0 \leq P_i^a(t), P_i^b(t) \leq P_{max}. \quad (9)$$

At every instant, each agent has to decide on the fraction of the power that needs to be allocated for communication and jamming. Each player uses its power for the following purposes: (1) Communicating with the team-mate, and (2) Jamming the communication of the other team. We assume that team members communicate using full-duplexing, e.g., using frequency division multiplexing (FDD). Also we assume that the channels used for internal communications for Team A and Team B are orthogonal. In order to jam, a team needs to hop to the frequency band used by the other team to inject noise. Unlike our previous analysis, the players do not have the freedom to separately allocate the jamming power for the members of the opponent team. The SINR received by the member of the other team due to the jamming signal solely depends on the mutual distance between the players in addition to some constant parameters.

Table I provides a list of the decision variables for the players that models this allocation. Each decision variable is a non-negative real number and lies in the interval $[0, 1]$. The fraction of the total power allocated by the player in row i to the player in column j is given by the first entry in the cell (i, j) . This allocated power is used for jamming if the player in column j belongs to the other team; otherwise, it is used to communicate with the agent in the same team. Similarly, the distance between the agent in row i and the agent in column j is given by the second entry in cell (i, j) . Since distance is a symmetric quantity, $d^{ij} = d^{ji}$ and $d_{ij} = d_{ji}$.

In the above game, each agent has to compute the following variables at every instant:

- 1) The instantaneous control, $\mathbf{u}_i(t)$.
- 2) The instantaneous power level, $P_i(t)$.
- 3) All the decision variables present in the row corresponding to the agent in Table I.
- 4) the size of the QAM schemes, M^a or M^b

In the next section, we analyze the problem of computing the optimal controls for each agent.

III. OPTIMAL CONTROL PROBLEM

From the problem formulation presented in the previous section, we can conclude that the objective functions of the two teams are in conflict. The tuple $(\mathbf{u}_1^{a*}, \mathbf{u}_2^{a*}, \mathbf{u}_1^{b*}, \mathbf{u}_2^{b*})$ is said to be optimal (or, in pair-wise saddle-point equilibrium) for the players if it satisfies the following conditions:

TABLE I
DECISION VARIABLES AND DISTANCES AMONG AGENTS.

	1^b	2^b	1^a	2^a
1^a	$(1 - \gamma^{12}), d_1^1$	$(1 - \gamma^{12}), d_2^1$	—	γ^{12}, d^{12}
2^a	$(1 - \gamma^{21}), d_1^2$	$(1 - \gamma^{21}), d_2^2$	γ^{21}, d^{21}	—
1^b	—	δ_{12}, d_{12}	$(1 - \delta_{12}), d_1^1$	$(1 - \delta_{12}), d_2^1$
2^b	δ_{21}, d_{21}	—	$(1 - \delta_{21}), d_2^1$	$(1 - \delta_{21}), d_2^2$

$$\begin{aligned} \pi[\mathbf{x}_0, \mathbf{u}_1^{a*}, \mathbf{u}_2^{a*}, \mathbf{u}_1^b, \mathbf{u}_2^b] &\leq \pi[\mathbf{x}_0, \mathbf{u}_1^{a*}, \mathbf{u}_2^{a*}, \mathbf{u}_1^{b*}, \mathbf{u}_2^{b*}] \\ \pi[\mathbf{x}_0, \mathbf{u}_1^{a*}, \mathbf{u}_2^{a*}, \mathbf{u}_1^{b*}, \mathbf{u}_2^{b*}] &\leq \pi[\mathbf{x}_0, \mathbf{u}_1^a, \mathbf{u}_2^a, \mathbf{u}_1^{b*}, \mathbf{u}_2^{b*}] \end{aligned}$$

In simple terms, the above equations imply that agents in Team A are solving a joint optimization problem of minimizing the outcome. Similarly, agents in Team B are solving a joint optimization problem of maximizing the outcome. Moreover, the two teams are playing a zero-sum game against one another. In this case, the value of the game, denoted by the function $J : \mathbf{X} \rightarrow \mathbb{R}$, can be defined as follows:

$$J(\mathbf{x}) = \pi[\mathbf{x}_0, \mathbf{u}_1^{a*}, \mathbf{u}_2^{a*}, \mathbf{u}_1^{b*}, \mathbf{u}_2^{b*}] \quad (10)$$

In order to compute the optimal control of the players, we will use the *Isaacs'* conditions [12]. These provide the necessary conditions that need to be satisfied by the Hamiltonian of the system when the players follow their optimal strategies. The Hamiltonian of the system is given by the following expression:

$$\begin{aligned} H &= L + \nabla J \cdot f(\mathbf{x}) \\ &= \underbrace{p_1^a(t) + p_2^a(t) - p_1^b(t) - p_2^b(t)}_L + \nabla J \cdot f(\mathbf{x}) \end{aligned} \quad (11)$$

From the analysis in [8], we obtain the following equations that govern the optimal control of the players

$$\left. \begin{aligned} \mathbf{u}_i^{a*} &= \min_{\mathbf{u}_i^a} \frac{\partial J}{\partial \mathbf{x}_i^a} \cdot f_i^a(\mathbf{x}_i^a, \mathbf{u}_i^a, t) \\ \mathbf{u}_i^{b*} &= \max_{\mathbf{u}_i^b} \frac{\partial J}{\partial \mathbf{x}_i^b} \cdot f_i^b(\mathbf{x}_i^b, \mathbf{u}_i^b, t) \end{aligned} \right\} \quad i = 1, 2$$

Additionally, the gradient of the value function satisfies the *retrogressive path equations* (RPE) given by the following

partial differential equation for the players of Team A.

$$\begin{aligned}\dot{J}_{x_i^a} &= \nabla J \cdot \frac{\partial f_{x_i}^a(\mathbf{x})}{\partial x_i^a} + \alpha \sum_{k=1,2} \left[\frac{s_k^a g'(s_k^a)(x_j^a - x_i^a)}{(d^{12})^2} + \right. \\ &\quad \left. \frac{s_k^b g'(s_k^b) P_{max} \gamma_k^i (x_i^a - x_k^b)}{(d_k^i)^{-(\alpha+2)}} \right] \\ \dot{J}_{y_i^a} &= \nabla J \cdot \frac{\partial f_{y_i}^a(\mathbf{x})}{\partial y_i^a} - \alpha \sum_{k=1,2} \left[\frac{s_k^a g'(s_k^a)(y_j^a - y_i^a)}{(d^{12})^2} + \right. \\ &\quad \left. \frac{s_k^b g'(s_k^b) P_{max} \gamma_k^i (y_i^a - y_k^b)}{(d_k^i)^{-(\alpha+2)}} \right]\end{aligned}$$

Here, $(\cdot)$ denotes derivative with respect to retrograde time. Since termination is only a function of the power of each player, J is independent of the position of the players on the terminal manifold. Therefore, $\nabla J = 0$ at termination. This forms the boundary condition for the RPE. A similar relation holds for the players in Team B, and can be obtained from the RPE.

In the next section, we address the problem of power allocation.

IV. POWER ALLOCATION

From (4), the SINR received by each agent in terms of the power levels of the other agents as well as their mutual distances is given by the following expressions:

$$\begin{aligned}s_1^a &= \frac{P_2^a(t) \gamma^{21} (d^{12})^{-\alpha}}{\frac{\sigma}{\rho} + P_1^b(t)(1 - \delta_{12})(d_1^1)^{-\alpha} + P_2^b(t)(1 - \delta_{21})(d_2^1)^{-\alpha}} \\ s_2^a &= \frac{P_1^a(t) \gamma^{12} (d^{12})^{-\alpha}}{\frac{\sigma}{\rho} + P_1^b(t)(1 - \delta_{12})(d_1^1)^{-\alpha} + P_2^b(t)(1 - \delta_{21})(d_2^1)^{-\alpha}} \\ s_1^b &= \frac{P_2^b(t) \delta_{21} (d_{12})^{-\alpha}}{\frac{\sigma}{\rho} + P_1^a(t)(1 - \gamma^{12})(d_1^1)^{-\alpha} + P_2^a(t)(1 - \gamma^{21})(d_1^1)^{-\alpha}} \\ s_2^b &= \frac{P_1^b(t) \delta_{12} (d_{12})^{-\alpha}}{\frac{\sigma}{\rho} + P_1^a(t)(1 - \gamma^{12})(d_2^1)^{-\alpha} + P_2^a(t)(1 - \gamma^{21})(d_2^1)^{-\alpha}}\end{aligned}\quad (12)$$

Since the players do not communicate, they possess information only about their own decision variables. Let γ^{ij} denote the fraction of power that agent i allocates for communicating with agent j in Team A. Similarly, let δ_{ij} denote the fraction of power that agent i allocates for communicating with agent j in Team B.

From (11), we can conclude that the power allocation among the agents only affects the term L in the Hamiltonian. This makes the power allocation problem a continuous kernel zero-sum game between the teams. The objective of each player in the team is given as follows:

Team A: The objective of each agent is to minimize L .

$$\min_{P_i^a \gamma^{ij}} L(M^a, M^b) \Rightarrow \min_{P_i^a, \gamma^{ij}} \underbrace{(p_j^a - p_1^b - p_2^b)}_{L_i^a(M^a, M^b)} \quad (13)$$

subject to:

$$\begin{aligned}0 &\leq P_i^a(t) \leq P_{max}, \quad R_i^a \geq \tilde{R} \\ \frac{1}{a_i} (2\tilde{R} - 1) &\leq \gamma^{ij} \leq 1 \quad j \neq i\end{aligned}$$

Team B: The objective of each agent is to maximize L .

$$\max_{P_i^b, \delta_{ij}} L(M^a, M^b) \Rightarrow \max_{P_i^b, \delta_{ij}} \underbrace{(p_1^a + p_2^a - p_j^b)}_{L_i^b(M^a, M^b)} \quad (14)$$

subject to:

$$\begin{aligned}0 &\leq P_i^b(t) \leq P_{max}, \quad R_i^b \geq \tilde{R} \\ \frac{1}{k_i} (2\tilde{R} - 1) &\leq \delta_{ij} \leq 1 \quad j \neq i\end{aligned}$$

In the above expressions a_i is defined in 15. k_i can be defined in a similar manner. Now we consider the problem of allocating the instantaneous power level for each player. From the expression of the SINR, we can conclude that the optimal value of the power consumption for each player is P_{max} . Therefore, the following corollary regarding the time horizon of the differential game follows immediately.

Corollary 1: The entire game terminates in a fixed time $T = \frac{E}{P_{max}}$ irrespective of the initial position of the agents.

Now we consider the problem of computing the optimal value of the decision variables for the players. In order to do so, we use preexisting results from continuous kernel games.

From [2, p. 178], we can conclude that the power allocation game has a Nash equilibrium in mixed strategies since the decision variables of each player lie on a simplex which is compact. Moreover, L is a continuous function of the decision variables of all the players. Therefore, the game admits a MSSPE. Although, the MSSPE has been computed for some games by exploiting some special characteristics in the cost functions, there are no standard techniques to compute MSSPE for general continuous-kernel games [2], [14]. Therefore, we search for the conditions under which the power allocation game admits a pure strategy saddle-point equilibrium (PSSPE).

The conditions under which we can guarantee existence of a PSSPE for the power allocation game are provided in [2]. Let us consider the case of agent 1^a . The expressions for SINR provided in (12) relevant to the optimization problem being solved by 1^a can be written in a concise form as shown below:

$$s_2^a = a_1 \gamma^{12}, \quad s_1^b = \frac{b_1}{c_1 - \gamma^{12}}, \quad s_2^b = \frac{d_1}{e_1 - \gamma^{12}} \quad (15)$$

where the expression for the constants can be easily obtained from the expression of the SINR.

Note that a_1, b_1, c_1, d_1 and e_1 are independent of the decision of 1^a .

Theorem 1: The power allocation team game has a unique saddle-point equilibrium in pure strategies if the following conditions hold for Team A:

$$\begin{aligned}(a_i)^2 g''(s_j^a) &- \frac{(s_1^b)^2 g''(s_1^b)}{b_i} - \frac{(s_2^b)^2 g''(s_2^b)}{d_i} - \\ &2 \left[\frac{(s_1^b)^3 g''(s_1^b)}{b_i} + \frac{(s_2^b)^3 g''(s_2^b)}{d_i} \right] > 0,\end{aligned}$$

where $i, j \in \{1, 2\}$ and $j \neq i$. The constants $b_2, c_2, d_2, e_i, l_i, m_i, n_i$, and o_i are obtained by re-writing the SINR

$$\frac{\xi_a \sqrt{\beta_a} \log(M_b)}{\xi_b \sqrt{\beta_b} \log(M_a)} \sqrt{\frac{a_i}{(\gamma^{12})^3}} (1 + \beta_a s_j^a) \exp(-\frac{\beta_a}{2} s_2^a) > \frac{2(b_i)^{\frac{1}{2}} + (b_i)^{-\frac{1}{2}}(c_i - \gamma^{12})}{\sqrt{(c_i - \gamma^{12})^3}} (1 + \beta_b s_1^b) \exp(-\frac{\beta_b}{2} (s_1^b)) \\ + \frac{2(d_i)^{\frac{1}{2}} + (d_i)^{-\frac{1}{2}}(e_i - \gamma^{12})}{\sqrt{(e_i - \gamma^{12})^3}} (1 + \beta_b s_2^b) \exp(-\frac{\beta_b}{2} (s_2^b)) \quad (16)$$

expressions as done above. An equivalent condition can be obtained for players in Team B:

The proof of the above theorem follows on the same lines as in [8, Theorem 4]. In the next section, we evaluate the sufficient conditions for PSSPE for the special case of M -QAM modulation schemes.

V. EXISTENCE OF PSSPE UNDER M -QAM MODULATION SCHEMES

The BER depends on the SINR, the modulation scheme, and the error control coding scheme utilized. Communications literature contains closed-form expressions and tight bounds that can be used to calculate $g(s)$ when the noise and interference are assumed to be Gaussian [9]. For example, using uncoded M-QAM, where Gray encoding is used to map the bits into the symbols of the constellation, the BER can be approximated by [15]

$$g(s) \approx \frac{\zeta}{\log(M)} \mathcal{Q}(\sqrt{\beta s}), \quad (17)$$

where $\zeta = 4(1 - 1/\sqrt{M})$, $\beta = 3/(M - 1)$, and $\mathcal{Q}(\cdot)$ is the tail probability of the standard Gaussian distribution. The conditions of Theorem 1 depend primarily on the employed modulation scheme.

Theorem 2: When all players employ uncoded M-QAM modulation schemes, the power allocation team game has a unique PSSPE solution. (16) provides the conditions for the existence of the unique PSSPE for players in Team A. A similar expression can be obtained for the players in Team B.

Proof: The conditions are obtained by using (17) in the conditions of Theorem 1. ■

VI. ADAPTIVE MODULATION

Adaptive modulation is a widely used technique as it allows for choosing the design parameters of a communications system to better match the physical characteristics of the channels. In this work, we model the adaptive modulation as a matrix zero-sum game between the two teams. The saddle-point equilibrium of the game dictates the modulations to be adopted by the teams at each time instant. The competitive nature of the jamming teams makes our approach to the problem most practical by the very definition of equilibrium.

The matrix game is given in (18). The rows are the possible actions for players of Team A, and the columns are the different options available for Team B. The (i, j) -th element of the matrix is the value of the objective function L when Team A employs $M^a = \mathcal{M}^a(i)$, and Team B employs $M^b = \mathcal{M}^b(j)$. The condition for the existence

of a PSSPE is $\min_i \max_j \mathbf{A}_{ij} = \max_j \min_i \mathbf{A}_{ij}$ [2]. The *ordered interchangeability property* removes any ambiguity associated with the existence of multiple equilibria, as the teams do not need to communicate to each other which equilibrium solution they will be adopting.

In case a PSSPE does not exist for the adaptive modulation game, we seek a solution in the larger class of mixed-strategies. A pair of strategies $\{M^{a*}, M^{b*}\}$ is said to be a mixed-strategy saddle point equilibrium (MSSPE) if $(M^{a*})^T \mathbf{A} M^b \leq (M^{a*})^T \mathbf{A} M^{b*} \leq (M^a)^T \mathbf{A} M^{b*}$. The *minimax theorem* guarantees the existence of an MSSPE for the adaptive modulation game [2]. Literature contains different efficient low-complexity algorithms that computes MSSPEs for matrix games. We refer the interested reader to [2] for a discussion of some of these approaches.

Finally, note that in the case of a MSSPE solution or when multiple PSSPE solutions exist, teammates should inform each other of the modulation scheme they will be using in order to ensure correct demodulation of the received signals. This information can be exchanged using the overhead of the transmitted packets.

From numerous simulations, we conclude the following results. Although, the trajectories of the players differ in our current setting from the scenario in [8] and [13], the gain in terms of the BER of each team is negligible. Moreover, the modulation schemes have a similar switching behavior in both scenarios. Figure 1 shows the simulation results for a given initial condition of the players. The parameters of the problems have been set to the following values in these simulations:

- $P_{\max} = 1, \sigma = 0.01$
- $f_1 = 300$ MHz and $f_2 = 100$ MHz
- The values of M are 16, 20 and 24.
- $u_1^a = u_2^a = u_1^b = u_2^b = 1$

The plots on the left are the trajectories of the players, modulation schemes, and BER of each team for the current scenario in which players are equipped with an omnidirectional antenna. The plots on the right depict the trajectories of the players, modulation schemes and BER for each team for our previous scenario in which each agent can configure the radiation pattern so as to allocate different power levels for jamming the members of the other team. This suggests that each agent might be able to guarantee a similar performance using an omnidirectional antenna rather than using more sophisticated antenna design techniques for reconfiguring antenna radiation patterns.

VII. CONCLUSION

This paper has studied the power allocation problem for jamming teams. The motion of the teams was modeled using

$\mathbf{A} =$

$$\mathbf{A} = \begin{matrix} & \mathcal{M}^b(1) & \mathcal{M}^b(2) & \dots & \mathcal{M}^b(m-1) & \mathcal{M}^b(m) \\ \begin{matrix} \mathcal{M}^a(1) \\ \mathcal{M}^a(2) \\ \vdots \\ \mathcal{M}^a(n) \end{matrix} & \begin{matrix} L(\mathcal{M}^a(1), \mathcal{M}^b(1)) \\ L(\mathcal{M}^a(2), \mathcal{M}^b(1)) \\ \vdots \\ L(\mathcal{M}^a(n), \mathcal{M}^b(1)) \end{matrix} & \begin{matrix} L(\mathcal{M}^a(1), \mathcal{M}^b(2)) \\ L(\mathcal{M}^a(2), \mathcal{M}^b(2)) \\ \vdots \\ L(\mathcal{M}^a(n), \mathcal{M}^b(2)) \end{matrix} & \begin{matrix} \cdot \\ \cdot \\ \vdots \\ \cdot \end{matrix} & \begin{matrix} L(\mathcal{M}^a(1), \mathcal{M}^b(m-1)) \\ L(\mathcal{M}^a(2), \mathcal{M}^b(m-1)) \\ \vdots \\ L(\mathcal{M}^a(n), \mathcal{M}^b(m-1)) \end{matrix} & \begin{matrix} L(\mathcal{M}^a(1), \mathcal{M}^b(m)) \\ L(\mathcal{M}^a(2), \mathcal{M}^b(m)) \\ \vdots \\ L(\mathcal{M}^a(n), \mathcal{M}^b(m)) \end{matrix} \end{matrix} \quad (18)$$

Fig. 1. Trajectories, modulation schemes, and BER of the players.

the framework of pursuit-evasion games and the optimal strategies were derived. An underlying static game was used to obtain the optimal power allocation, where the power budget of each user is split between communication and jamming powers. A separate matrix game was utilized in order to arrive at the optimal modulation schemes for each team. Since the value of the matrix game was the function of the mutual distances of the players, we observed a switching behavior in the optimal communication scheme among members of each team.

This work focused on the analysis of teams consisting of two players only. Moreover, in contradistinction with our earlier work, each member of the team was equipped with an omnidirectional antenna for jamming. From the simulations, we concluded that the physical modification in the jamming strategy had negligible effect on the BER incurred by each team and the variations in the modulation scheme. This suggests that each team member might as well choose a simpler jamming architecture from the ones suggested in our previous setup [8], [13]. Potential future directions include constructing the globally optimal trajectories by computing the singular surfaces arising in the game and extending the analysis for the case when the two teams have unequal number of players.

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