MULTI-OBJECTIVE CONTROL WITH MODAL NOTCHING VIA IMSC

Smith, M.J. and Berg, J.L. *

USAF Phillips Laboratory
Edwards AFB, CA 93523-5000

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Abstract

This paper develops a control methodology that allocates control authority for specific modes of a linear system via the IMSC technique while projecting excess control authority onto an orthogonal space which independently satisfies other control objectives. The orthogonal control authority used in this paper is LQG-based, and the resulting control system turns out to be the solution of an output feedback problem.

1 Introduction

Future spacecraft such as the envisioned Space-Based Radar (SBR) will be composed of several substructures which can be classified as Large Space Structures (LSS). Examples of LSS include reflector antennas, radar membranes, radar arrays and solar arrays. It's common to have LSS attached to orbiting vehicles (e.g. the satellites deployed by the National Geological Survey).

Orbiting vehicles are commonly exposed to internal excitation such as the onboard rotating machinery mentioned in reference to cryogenic coolers [1], as well as exogenous disturbances due to slewing, temperature gradients, pulse excitations, etc. A control technique that distinguishes frequency dependent excitations from random disturbances and controls both using orthogonal control authority becomes beneficial

under such conditions. This paper develops a control scheme that orthogonally notches out excitation from frequency dependent sources such as critical resonant modes and simultaneously dampens the excitation from random disturbances. Control simulations using this technique are shown to be convincing.

2 IMSC Review

It is important to review the IMSC control technique developed by Meirovitch in the early 80's [2]. The IMSC technique seeks modal forces, which can be transformed to the configuration space to acquire actuator forces. When the number of independent actuators equals the number of controlled modes, a matrix inversion of the modal participation matrix provides the transformation to the configuration space, otherwise a pseudo-inversion is carried out which yields a coupled control law referred to as pseudo-inverse IMSC. The following derivation is a synopsis of the work done by Meirovitch, et al. ([3] through [5]).

The mathematical description of a flexible, viscously damped structure can be modelled in modal coordinates as:

$$\ddot{\eta}(t) + \Xi \dot{\eta}(t) + \Lambda \eta(t) = \Phi^T B_m \bar{u}(t) + \Phi^T D_m \bar{w}(t) \quad (1)$$

$$\bar{y}(t) \doteq C \begin{Bmatrix} \eta(t) \\ \dot{\eta}(t) \end{Bmatrix}$$

where $\eta(t)$ is the vector of modal amplitudes, $[\Xi]$ and $[\Lambda]$ are diagonal matrices which represent the system damping and stiffness, respectively ($[\Xi] = \text{diag}(2\zeta_r \omega_r)$, $[\Lambda] = \text{diag}(\omega_r^2)$). $[\Phi]$ is the modal matrix. $\bar{u}(t)$ is the control input, $\bar{y}(t)$ is the system's

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output and $\tilde{w}(t)$ is the external disturbance. For conciseness, the following matrices defined as

$$[B \ D] \doteq \Phi^T [B_m \ D_m]$$

will be used in the remaining portion of this paper.

Assuming the modal state is obtainable, we may choose diagonal gain matrices in order to maintain independence for the controlled modes. The modal control law becomes:

$$\begin{aligned} B\tilde{u}(t) &= [G_1 \ G_2] \begin{Bmatrix} \hat{\eta}(t) \\ \dot{\hat{\eta}}(t) \end{Bmatrix} \\ G_1 &\doteq \text{diag}(g_{11} \ g_{12} \ \dots \ g_{1n}) \\ G_2 &\doteq \text{diag}(g_{21} \ g_{22} \ \dots \ g_{2n}) \end{aligned} \quad (2)$$

where $\hat{\eta}(t)$ and $\dot{\hat{\eta}}(t)$ represent the estimated modal position and velocity coordinates of the structure, provided by modal filters [6]. The control vector in the configuration space, $\tilde{u}(t)$, can be uniquely determined, provided there are as many independent actuators as there are modes to be controlled.

$$\tilde{u}(t) = B^{-1} [G_1 \ G_2] \begin{Bmatrix} \hat{\eta}(t) \\ \dot{\hat{\eta}}(t) \end{Bmatrix} \quad (3)$$

The existence of the true inverse of the transpose of the modal matrix permits the controller to maintain independence between the modes for the closed-loop system, thereby allowing one to control the modes individually. This yields higher computational efficiency during the solution of the Riccati Equations, performed during the determination of the optimal gains.

Extended work in IMSC includes that of Sinha [7], in which a sliding mode approach to account for system nonlinearities has been proposed. Lindberg and Longman [8] studied the questions of actuator quantity and placement using the IMSC technique. An approach for using fewer actuators than the number of controlled modes without resorting to pseudo-inversion is addressed by Baz and Poh [9]. Silverberg and Meirovitch [10] proposed a solution to dynamic limitations of actuators by targeting specified modes to be controlled by blocks of actuators. Nguyen [11] also addressed actuator issues using IMSC. More recently, an experimental comparison of IMSC and Linear Quadratic control is performed by Berg [12], in which the pseudo-inverse IMSC controller was found to be more robust with respect to uncertainties in the system dynamics with the penalty of slightly higher

energy requirements. It's also important to mention here that numerous papers have been published on partial and regional pole assignment with varying control objectives, and recent papers by Haddad and Bernstein [13] and Lu, etc. [14], contain a well documented list of sources.

3 Main Result

It is expected that space structures will experience excitation sources that are both frequency dependent and random. It turns out that IMSC can be used to dampen key resonant modes while simultaneously controlling, in an orthogonal manner, random disturbances. This allows one to control Line-of-Sight (LOS) errors by satisfying multiple control objectives. We begin by introducing the following theorem:

Theorem 1 *When more independent actuators than designated modes are used in the IMSC method, multiple control objectives can be obtained by constraining excess control authority to the null space of the spacial rows of the modal forces.*

Proof 1 lemma 1 (Rao and Mitra, [15]) *The equation $LXR = Y$ can be solved for X iff L, R , and Y have the property $LL'YR'R = Y$ where L' and R' are any matrices satisfying*

$$\begin{aligned} LL'L &= L \\ RR'R &= R \end{aligned}$$

and if there exists any solutions, then all of them are given by

$$X = L'YR' - L'LZRR' + Z$$

where Z is an arbitrary matrix of correct dimension.

Recall that the IMSC controller is given by Equation (3), but when more independent actuators are used than the number modes one wishes to control, application of the lemma to Equation (2), choosing $[L] \doteq [B]$ and $[R] \doteq [I]$, leaves us with the solution:¹

$$\tilde{u}(t) = B^+ [G_1 \ G_2] \begin{Bmatrix} \hat{\eta}(t) \\ \dot{\hat{\eta}}(t) \end{Bmatrix} + [I - B^+ B] Z \quad (4)$$

¹ $[L]^+ \doteq$ pseudo-inverse of $[L]$

where $[Z]$ is a free matrix of correct dimension. Substitution of

$$Z = G_z \begin{Bmatrix} \dot{\eta}(t) \\ \dot{\eta}(t) \end{Bmatrix}$$

into Equation (4) gives an alternate set of solutions

$$\bar{u}(t) = [B^+[G_1 \ G_2] + [I - B^+B] G_z] \begin{Bmatrix} \dot{\eta}(t) \\ \dot{\eta}(t) \end{Bmatrix}$$

where the control gains G_1 and G_2 are devoted to the control of specific modes (modal notching) and G_z provides freedom for additional control objectives.

3.1 Comment

One can see that minimizing the norm on the gain equates to minimizing the function:

$$\begin{aligned} f(G_z) &\doteq \|B^+[G_1 \ G_2] + [I - B^+B] G_z\| \\ &\geq \|B^+[G_1 \ G_2]\| \quad \forall G_z \end{aligned}$$

This verifies our suspicion that any control objective beyond control of specific modes using the modal notching technique will result in additional control effort.

4 Modal Notch Control with Generalized Measurement

In this section, we construct the first-order, state space form which contains the previously dedicated control for a subset of r plant modes, and the remaining control authority will be allocated for various objectives.

4.1 State Space Construction

In flexible space structures, inertial positions, rates, and accelerations are typically provided by onboard instrumentation. The measurement vector is generalized as:

$$\bar{z}(t) = H_1 \bar{\eta}(t) + H_2 \dot{\bar{\eta}}(t) + H_3 \ddot{\bar{\eta}}(t) + \bar{v}(t)$$

or equivalently (by substitution of Equation (1))

$$\begin{aligned} \bar{z}(t) &= [[H_1 \ H_2] - H_3[\Lambda \ \Xi]] \begin{Bmatrix} \bar{\eta}(t) \\ \dot{\bar{\eta}}(t) \end{Bmatrix} \\ &\quad + H_3 \{B\bar{u}(t) + D\bar{w}(t)\} + \bar{v}(t) \end{aligned}$$

We define the state vector as:

$$\bar{q} \doteq \begin{Bmatrix} \bar{\eta}(t) \\ \dot{\bar{\eta}}(t) \end{Bmatrix}$$

and the matrices:

$$\begin{aligned} \hat{A} &\doteq \begin{bmatrix} 0 & I \\ -\Lambda & -\Xi \end{bmatrix} & \hat{B} &\doteq \begin{bmatrix} 0 \\ B \end{bmatrix} & \hat{E} &\doteq \begin{bmatrix} 0 \\ D \end{bmatrix} \\ \hat{C} &\doteq [[H_1 \ H_2] - H_3[\Lambda \ \Xi]] \\ \hat{D} &\doteq H_3B & \hat{F} &\doteq H_3D \end{aligned}$$

The plant and measurement vectors are:

$$\begin{aligned} \dot{\bar{q}}(t) &= \hat{A}\bar{q}(t) + \hat{B}\bar{u}(t) + \hat{E}\bar{w}(t) \\ \bar{z}(t) &= \hat{C}\bar{q}(t) + \hat{D}\bar{u}(t) + \hat{F}\bar{w}(t) + \bar{v}(t) \end{aligned} \quad (5)$$

with the estimated state $\hat{\eta}$, the modal notch controller is:

$$\bar{u}(t) = [B_r^+[G_1 \ G_2]H_m + [I - B_r^+B_r] G_z] \hat{q}(t).$$

where the matrix $B_r \doteq \Phi^T B_m$ constrains r modes for the notch controller. It is clear from the previous development that r modes are guaranteed to have the damping characteristics by the IMSC methodology, and the remaining control freedom (gain matrix $[G_z]$) can be dedicated to other control purposes. The matrix H_m is a rectangular matrix whose rows consist of standard unit vectors. H_m is used to factor out the modes of interest for the IMSC controller from a more generalized set of modes. The larger set of modes provides a more observable system for the orthogonal controller (i.e., the LQG control in this paper).

Control objectives are problem dependent which makes the ability of the Modal Notch Control to accommodate various control schemes, applicable to many different practical problems. In this paper, we will show that important modes can be controlled while an LQG based control scheme simultaneously achieves LOS pointing accuracy for a flexible space structure. Simulations using this control technique are applied to the FEM model of ASTREX (a test facility at the Phillips Laboratory, Edwards AFB, CA) with the results presented in Section 6.

5 Optimal Estimation & Control with Modal Notching

The issue of finding the optimal estimator with correlated plant and measurement noise has been resolved

in [16] and has been used in an application concerning an actuator and sensor study [17]. The system described by the Equations (5) clearly has correlated noise, and the optimal estimator matrix can be found by solving a standard Riccati equation as described below.

By defining the expected noise values:

$$W \doteq E_{\infty}(\bar{w}(t)\bar{w}^T(t)) \quad V \doteq E_{\infty}(\bar{v}(t)\bar{v}^T(t))$$

and the matrix

$$\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \doteq \begin{bmatrix} \hat{E}W\hat{E}^T & \hat{E}W\hat{F}^T \\ \hat{F}W\hat{E}^T & \hat{F}W\hat{F}^T + V \end{bmatrix},$$

one can find the optimal estimator matrix $[L]$ by solving the Riccati equation:

$$\begin{aligned} X_{11}[\hat{A} - V_{12}V_{22}^{-1}\hat{C}]^T + [\hat{A} - V_{12}V_{22}^{-1}\hat{C}]X_{11} \\ - X_{11}[\hat{C}^TV_{22}^{-1}\hat{C}]X_{11} + [V_{11} - V_{12}V_{22}^{-1}V_{12}^T] = 0. \end{aligned}$$

Using the solution $[X_{11}]$ to determine the estimator matrix:

$$L = (X_{11}\hat{C}^T + V_{12})V_{22}^{-1},$$

the resulting optimal state estimator equation is

$$\dot{\hat{q}}(t) = [\hat{A} + \hat{B}G - L\hat{C} - L\hat{D}G]\hat{q}(t) + L[\bar{z}(t)],$$

where the total feedback gain matrix is defined as:

$$G \doteq [B_r^+[G_1 \ G_2]H_m + [I - B_r^+B_r]G_z].$$

Since a portion of the gain matrix must satisfy the conditions for modal space control, an output feedback problem can be constructed by yet another augmentation of the states to guarantee that r modes will be notched in an orthogonal manner.

Define the new state and noise vectors as:

$$\bar{x}(t) \doteq \begin{bmatrix} \bar{q}(t) \\ \hat{q}(t) \end{bmatrix} \quad \bar{\mu}(t) \doteq \begin{bmatrix} \bar{w}(t) \\ \bar{v}(t) \end{bmatrix}$$

and the matrices:

$$\begin{aligned} G_m &\doteq B_r^+[G_1 \ G_2]H_m \\ M_z &\doteq [I - B_r^+B_r] \\ \bar{A} &\doteq \begin{bmatrix} \hat{A} & \hat{B}G_m \\ L\hat{C} & [\hat{A} + \hat{B}G_m - L\hat{C}] \end{bmatrix} \\ \bar{B} &\doteq \begin{bmatrix} \hat{B}M_z \\ \hat{B}M_z \end{bmatrix} \\ \bar{D} &\doteq \begin{bmatrix} \hat{E} & 0 \\ L\hat{D}\hat{F} & L \end{bmatrix}, \end{aligned}$$

then the equivalent output feedback problem is:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}u_z(t) + \bar{D}\bar{\mu}(t) \\ u_z(t) &\doteq G_z[0 \ I]\bar{x}(t) \\ y(t) &= [C \ 0]\bar{x}(t). \end{aligned}$$

It is clear that $[G_z]$ is the control gain matrix which must be utilized for additional control objectives beyond modal space control. $[G_z]$ can be found optimally by satisfying constraint equations for a typical output feedback problem as shown in [18].

6 Numerical Example

Simulations of the Notch Control concept are performed on the ASTREX FEM model. Six modes are retained using the modal cost analysis technique [19]. Notice in Figure 1 that the ASTREX test article has two linear thrusters in the primary structure which have control authority in the z -direction, one disturbance source (vibrating cryocooler) at the secondary placed at a 45 degree angle in the x - y plane, and twelve accelerometers in the x - and y -directions at six nodes of the secondary supports.

Figure 1: ASTREX Test Article

Figure 2 shows a superposition of a continuous 11.2 Hz sinusoidal disturbance, which is close to the third modal frequency (11.4 Hz), and an impulse that occurs at $t = 20$ seconds, both emanating from the cryocooler. Figure 3 is the resulting open loop response (displacement time history) in the x direction for one of the accelerometer locations. The control objective is to attenuate the third mode while controlling the

Fig 2. Disturbance History

Fig 4. Closed Loop Disp. OVC

Fig 6. Open, Notch-OVC, & OVC Freq. Resp.

Fig 8. Notch-OVC Closed Loop Impulse Resp.

Fig 3. Open Loop Disp.

Fig 5. Closed Loop Disp. Notch-OVC

Fig 7. Open Loop Impulse Resp.

Fig 9. OVC Closed Loop Impulse Resp.

output variances due to a pulse excitation. To show the effectiveness of this control scheme, an OVC (Output Variance Constraint, [20]) controller was first synthesized with position and velocity constraints at the accelerometer location. The closed loop response to the disturbance displayed in Figure 2 is shown in Figure 4. In our example, the third mode is significantly effected by excitation from the cryocooler, which justifies increasing the damping ratio of that mode by modal notching. By choosing the damping ratio of the third mode to be 0.6, the disturbance rejection using modal notch control is presented in Figure 5. The comparison of frequency responses of the OVC and modal notch controlled systems with the uncontrolled open loop system is shown in Figure 6, and clearly shows the significance of using modal notch control to eliminate key modes while simultaneously meeting output variance requirements. Figures 7, 8,

and 9 are plots of the open loop, modal notch using OVC, and the OVC impulse responses with which the latter two have identical output variances using the two control techniques.

7 Conclusions

An analytical technique was presented to control the response of a dynamical system exposed to frequency dependent and random disturbances. The method is advantageous because multiple control schemes can be combined to achieve disturbance rejection which results in a useful approach for solving practical controls problems. By supplementing a control scheme with an orthogonal, IMSC-based control scheme, one can readily devote energy to the damping of particularly troublesome modes to overcome frequency dependent disturbances. This is a practical approach

when one considers that a structure's frequency dependent disturbances are not always predictable, and due to the addition of hardware, will most likely change during the service life of the structure.

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