Exact and Asymptotic Analysis of an Infinite Grid of Metal Strips using G1DMULT

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ABSTRACT

In this paper the scattering properties of an infinite grid of conducting strips in a multi-layer structure is studied by a spectral domain technique. To calculate the scattered field we Fourier expand the induced currents on the strips and use the numerical algorithm G1DMULT to evaluate the spectral domain Green's functions. The results are compared with data obtained using the asymptotic strip boundary condition (ASBC). We have shown that the exact numerical results approach the ASBC results when the strip period decreases for all relative strip widths.

1. Introduction

Periodic structures have for a long time been a subject of great interest due to their many applications. They can be used as filters for microwave and optical signals and are then referred to as frequency selective surfaces (FSSs) [1]. Periodic arrays of metal strips on dielectric substrates can be used as polarization selective surfaces [2] and, when placed on top of a grounded dielectric substrate, they can be used to obtain soft or hard surfaces [3]. The latter can be applied in horn antennas to obtain a symmetrical beam with low cross-polarization in the diagonal planes [4], and to reduce field blockage from cylindrical objects [5].

We formulate the problem of electromagnetic scattering by relating the scattered field from the conducting strips to the surface currents on the strips induced by an incident plane wave, and we solve the problem using the moment method. We consider y -directed strips in the xy -plane, as shown in Fig.1 where w is the width of the strips and p_x the period. We analyze the problem in two ways. First by assuming that the strip period is infinitely small, in which case we may handle it simply as an anisotropic boundary condition, referred to as the asymptotic strip boundary condition (ASBC) [6]. Second, by analyzing the strip-loaded multilayer structure in a rigorous way by expanding the fields in Floquet modes. We compare the two methods in order to study the region of validity of the ASBC.

By using a numerical algorithm called G1DMULT, we calculate the Green's functions of the multilayer structure in the spectral domain [7]. In this algorithm the problem is divided into equivalent subproblems, one for each layer. The algorithm calculates the field in each layer by using equivalent electric and magnetic currents on the boundaries immediately above and below the layer. These spectral currents can be interpreted, in the spatial domain, as infinite current sheets with harmonic variations in the x - and y -directions. The algorithm has the advantage of analytic simplicity and allows us to include infinitely thin anisotropic layers [8]. Thereby, we were able to easily implement the ASBC in G1DMULT. The ASBC is general and can be applied, for any kind of incident wave, to surfaces of arbitrary shape.

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2. Formulation of the procedure

Let us consider the structure infinite or at least large compared to the wavelength λ . We solve simultaneously for both TE_z and TM_z polarizations of the incident plane wave. We set up one integral equation for each polarization, in terms of the unknown currents induced on the strips. We solve the problem by enforcing the boundary condition that the total tangential electric fields must be zero on the metal strips, i. e.

$$\mathbf{z} \times (\mathbf{E}^i + \mathbf{E}_m^r + \mathbf{E}_s^r) = 0 \quad (1)$$

where $\mathbf{E}^i$ is the incident plane wave, $\mathbf{E}_m^r$ is the reflected field from the multilayer structure and $\mathbf{E}_s^r$ is the scattered field from the strips in the presence of the multilayer structure.

To calculate the scattered fields we run the algorithm G1DMULT twice. First to calculate the reflected field from the structure without the periodic element and second to calculate the scattered field from the induced currents on the strips for each Floquet mode. Then we use superposition to evaluate the total field, including the incident plane wave.

We expand the currents on the strips in basis functions which have the same harmonic variation along the strips as the incident wave has, and which are sinusoidal in the transverse x -direction for the x -directed current and uniform plus cosines for the y -directed current (the strips are small compared to the wavelength). Expressing the currents in the spectral domain, we get an infinite summation of harmonic currents times a Green's function. Each term of the summation corresponds to different values of the spectral variable k_x which take into account the angles of incidence and the associated phase difference between the elements. These discrete values are associated with the Floquet modes of the infinite array of strips.

We use Galerkin's formulation, and the moment method, to find the unknown coefficients of the currents. Finally, by calling G1DMULT with the known currents, we can calculate the reflection coefficient for different angles of incidence.

3. Asymptotic boundary conditions for ideal strip-loaded slab.

This approach has been presented in [6] and it is valid only in an asymptotic way, i.e. when the period p_x is small compared to the wavelength λ . To derive the Green's function using the ASBC we use the following boundary conditions at the plane of the strips (see Fig.1a):

$$E_y|^{up} = 0, \quad E_y|^{low} = 0, \quad E_x|^{low} = E_x|^{up}, \quad H_y|^{low} = H_y|^{up} \quad (2)$$

where *up* and *low* refer to the fields above and below the plane where the strips are located, respectively. The four boundary conditions are enough for determining the Green's functions.

4. Results

The absolute value of the reflection coefficient Γ is calculated for an infinite array of conducting strips in free space (see Fig.1b). The results presented are obtained using one basis function for each polarization, which was tested to be sufficient. Results are presented for different angles of incidence in the xz -plane ($\theta=0^\circ, 30^\circ, 60^\circ$). In Fig. 2 we vary the periodicity of the strips when the ratio between the width and the period is equal to $w/p_x=0.25$. The reflection coefficient Γ is shown both for the TE ($E_z = 0$) and TM ($H_z = 0$) cases, respectively. Comparing the results with the ASBC calculations, we see that the accuracy of the latter is reasonable when the period is smaller than about 0.2 wavelengths (λ), for the TE case. The TM results also improve when the period decreases. The accuracy is good when the period is much that about 0.4λ . The same behaviour is in principle present also for larger w/p_x . The accuracy of the ASBC is best for TE polarization when w/p_x is large, and it is best for TM polarization for small w/p_x .

In Fig. 3, we vary the strip width and fix the strip period to 0.1λ . When the strip width approaches the period, all TE results converge to the ASBC case, which is a reflection coefficient of -1. This corresponds to the case of an infinite perfect electric conductor. For TM polarization the curves converge to the ASBC results when the width of the strips decreases, since in that case the ASBC models the plane of the strips as a transparent plane with a reflection coefficient equal to zero.

All results show that the ASBC has better accuracy for large angles of incidence.

In the oral presentation, results will be presented also when a multilayer structure is included and for other planes of incidence.

5. Conclusions

We have applied a rigorous Floquet mode expansion technique in the spectral domain in order to analyze the properties of an infinite planar array of conducting strips over a multilayer dielectric slab. The results are compared with those obtained using a simple asymptotic boundary condition ASBC for the strips. In both cases a multilayer structure is included using the G1DMULT subroutine. The results show that the ASBC is accurate when the strip period is small compared to the wavelength, and they converge to the ASBC solution for small strip periods, independent of the strip width to period ratio. The ASBC is advantageous because it requires much less computer time than the Floquet mode technique. The ASBC is general and easily applied to more complex problems, such as non-planar structures and arbitrary types of incident fields.

References

- [1] R. Mittra, C. H. Chan, and T. Cwik, "Techniques for analyzing frequency selective surfaces-a review," *Proc. IEEE*, vol. 76, no. 12, pp.1593-1615, Dec. 1988.
- [2] R. C. Johnson and H. Jasik (eds.), *Antenna Engineering Handbook*, 2nd ed., New York: McGraw-Hill, 1984, chapter 23.
- [3] P-S. Kildal, "Definition of artificially soft and hard surfaces for electromagnetic waves," *Electron. Lett.*, vol. 24, no.3, pp. 168-170, Feb. 1988.
- [4] E. Lier and T. Schaug-Pettersen, "The strip-loaded hybrid-mode feed horn," *IEEE Trans. Antennas Propagat.*, vol. 35, pp. 1086-1089, Sept. 1987.
- [5] P-S. Kildal, A. A. Kishk, and A. Tengs, "Reduction of forward scattering from cylindrical object using hard surfaces," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1509-1520, Nov. 1996.
- [6] A. A. Kishk and P-S. Kildal, "Asymptotic boundary conditions for strip-loaded scatterers applied to circular dielectric cylinders under oblique incidence," to appear in *IEEE Trans. Antennas Propagat.*.
- [7] P-S. Kildal, Z. Sipus, and M. Johansson, "G1DMULT - a numerical algorithm for computing Green's functions," to be presented at 1997 ACES Symposium, Monterey, CA, March 1997.
- [8] Z. Sipus, H. Merkel, and P-S. Kildal, "Green's functions for planar soft and hard surfaces derived by asymptotic boundary conditions," submitted to *Proc. IEE Part H*.

Fig. 1. Infinite grid of metal strips on a multilayer structure. Geometry and coordinate system (left) and Cross section (right).

Fig. 2. Absolute value of reflection coefficient for transverse incidence for $w/p_x=0.25$. TE polarization (left) and TM polarization (right).

Fig. 3. Absolute value of reflection coefficient for transverse incidence for $p_x=0.1\lambda$. TE polarization (left) and TM polarization (right).