FUNDAMENTAL PROPERTIES OF CHIRAL TYPE MEDIA

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1. INTRODUCTION

Chiral media [1-7] and ferrite media [8, 9] have been studied over the last 15 and 30 years respectively for many applications. Chiral media have been examined as coatings for reducing radar cross-section, for antennas and arrays, for antenna radomes, in waveguides and for microstrip substrates. Ferrites have seen use in various nonreciprocal devices like phase shifters, isolators, and circulators.

It is our interest here to examine chiral media by itself and then a composite medium having both chiral and ferrite properties. The chiral part of the medium by itself will be chosen to have complex activity coefficient and this will be shown to suggest, by itself, nonreciprocity in Section 2. Of course, by itself, the ferrite part of the medium is nonreciprocal.

2. THEORETICAL FOUNDATIONS

The constitutive relations for chiral media are [6]

$$\mathbf{D} = \epsilon \mathbf{E} + \xi_{c1} \mathbf{H}, \quad \xi_{c1} = \alpha_{cr1} + i\alpha_{ci1} \quad (1a)$$

$$\mathbf{B} = \mu \mathbf{H} + \xi_{c2} \mathbf{E}, \quad \xi_{c2} = \alpha_{cr2} + i\alpha_{ci2} \quad (1b)$$

Here i is the imaginary number $\sqrt{-1}$. Permittivity ϵ and permeability μ are scalars, as are $\alpha_c(r, i)(1, 2)$. For lossless chiral media $\alpha_{cr1} = \alpha_{cr2} = 0$ and $\alpha_{ci1} = -\alpha_{ci2}$.

Ferrite media constitutive relations are [8, 9]

$$\begin{aligned} \mathbf{D} &= \epsilon \mathbf{E}, \quad \mathbf{B} = \bar{\mu} \mathbf{H} \\ \bar{\mu} &= \begin{bmatrix} \mu & ik_m & 0 \\ -ik_m & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}, \quad \begin{aligned} \mu &= \mu' - i\mu'' \\ k_m &= k_m' - ik_m'' \end{aligned} \end{aligned} \quad (4ab)$$

and the dc magnetic field B_0 is in the z -direction.

The composite medium is hypothesized to have the following constitutive relations

$$\mathbf{D} = \epsilon \mathbf{E} + \xi_{c1} \mathbf{H}; \quad \mathbf{B} = \bar{\mu} \mathbf{H} + \xi_{c2} \mathbf{E} \quad (6ab)$$

Maxwell's equations in the frequency domain with electric $\mathbf{J}$ and magnetic $\mathbf{M}$ volume currents are

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B} - \mathbf{M}, \quad \nabla \times \mathbf{H} = i\omega \mathbf{D} + \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{D} = 0 \quad (7ab)$$

assuming electric charge neutral problems.

In the search for new media which display nonreciprocal properties, it is essential to establish fundamentally this behavior. This is done here by applying the anisotropic reaction theorem [10, 11]

$$(\mathbf{b}, \mathbf{a}) = (\tilde{\mathbf{a}}, \mathbf{b}) \quad (8)$$

or its integral form

$$\iiint (\mathbf{E}_b \cdot \mathbf{J}_a - \mathbf{H}_b \cdot \mathbf{M}_a) dv = \iiint (\tilde{\mathbf{E}}_a \cdot \mathbf{J}_b - \tilde{\mathbf{H}}_a \cdot \mathbf{M}_b) dv \quad (9)$$

In (9) source currents $(\mathbf{J}_a, \mathbf{M}_a)$ and $(\mathbf{J}_b, \mathbf{M}_b)$ produce respectively fields $(\mathbf{E}_a, \mathbf{H}_a)$ and $(\mathbf{E}_b, \mathbf{H}_b)$ and the tilde over the fields indicates a new medium altered from the original medium with respectively 6×6 constitutive tensors

$$\tilde{\mathbf{C}} = \begin{bmatrix} \hat{\epsilon}^T & -\hat{\rho}^T \\ -\hat{\rho}^T & \hat{\mu}^T \end{bmatrix}, \quad \hat{\mathbf{C}} = \begin{bmatrix} \hat{\epsilon} & \hat{\rho} \\ \hat{\rho} & \hat{\mu} \end{bmatrix} \quad (10ab)$$

with $\hat{\rho}$ and $\hat{\rho}'$ being the optical activity 3×3 tensors. Reciprocity only occurs if

$$\langle b, a \rangle = \langle a, b \rangle \quad (11)$$

that is, if (8) requires $\hat{C} = \hat{C}$ (12)

For chiral media (1) and (2) hold, leading to

$$\hat{\epsilon} = \epsilon I, \hat{\rho} = \xi_{c1} I, \hat{\rho}' = \xi_{c2} I, \hat{\mu} = \mu I \quad (13)$$

defining the component tensors making up $\hat{C}$ in (10b). To have reciprocity (12) imposes $\epsilon I^T = \epsilon I, -\xi_{c2} I^T = \xi_{c1} I, -\xi_{c1} I^T = \xi_{c2} I, \mu I^T = \mu I$ or $\xi_{c1} = -\xi_{c2}$. Thus

$$\alpha_{cr1} = -\alpha_{cr2}, \alpha_{ci1} = -\alpha_{ci2} \quad (14a)$$

Condition (14b) is the usual one satisfied for lossless chiral media. The existence of $\alpha_{cr1} \neq 0, \alpha_{cr2} \neq 0$ suggests a different condition in the chiral media, and (14a) not being fulfilled. Therefore

$$\alpha_{cr1} \neq 0, \alpha_{cr2} \neq 0 \quad (15)$$

may make the chiral media nonreciprocal. Issues of physical consistency and realizability must still be addressed.

For ferrite media equations (5) hold, leading to

$$\hat{\epsilon} = \epsilon I, \hat{\rho} = \hat{\rho}' = 0, \hat{\mu} = \bar{\mu} \quad (16)$$

To have reciprocity, (12) imposes

$$\bar{\mu} = \bar{\mu} \quad (17)$$

which by (5) requires $-ik_m = ik_m$ or $k_m = 0$, the nonferrite case. Thus we see that the ferrite medium, as well known, is nonreciprocal and satisfies the anisotropic reaction theorem (8).

3. VECTOR HELMHOLTZ EQUATIONS

The H-field vector Helmholtz equation is desired by inserting constitutive relations (7) into Maxwell's equation (8)

$$\nabla \times \mathbf{E} = -i\omega \bar{\mu} \mathbf{H} - i\omega \xi_{c2} \mathbf{E} - \mathbf{M} \quad (18a)$$

$$\nabla \times \mathbf{H} = i\omega \epsilon \mathbf{E} + i\omega \xi_{c1} \mathbf{H} + \mathbf{J} \quad (18b)$$

Solving for $\mathbf{E}$ in (18b) gives

$$\mathbf{E} = -[i(\nabla \times \mathbf{H} - \mathbf{J})/\omega + \xi_{c1} \mathbf{H}]/\epsilon \quad (19)$$

and putting this into (18a) provides the source H-field vector Helmholtz equation

$$\begin{aligned} \nabla \times \nabla \times \mathbf{H} + i\omega(\xi_{c2} - \xi_{c1})\nabla \times \mathbf{H} - \omega^2 \epsilon \mu_0 (\bar{\mu}/\mu_0 - \xi_{c1} \xi_{c2}/\mu_0 \epsilon) \mathbf{H} \\ = \nabla \times \mathbf{J} + i\omega \xi_{c2} \mathbf{J} - i\omega \epsilon \mathbf{M} \end{aligned} \quad (20)$$

E-field vector Helmholtz equation is found by solving for $\mathbf{H}$ in (18a)

$$\mathbf{H} = \frac{i}{\omega} [\bar{\mu}^{-1} \nabla \times \mathbf{E} + i\omega \xi_{c2} \bar{\mu}^{-1} \mathbf{E} + \bar{\mu}^{-1} \mathbf{M}] \quad (21)$$

and place (21) into (18b) giving the source E-field vector Helmholtz equation

$$\begin{aligned} \nabla \times \bar{\mu}^{-1} \nabla \times \mathbf{E} + i\omega(\xi_{c2} \nabla \times \bar{\mu}^{-1} \mathbf{E} - \xi_{c1} \bar{\mu}^{-1} \nabla \times \mathbf{E}) - \omega^2 \epsilon (I - \xi_{c1} \xi_{c2} \bar{\mu}^{-1}/\epsilon) \mathbf{E} \\ = -\nabla \times \bar{\mu}^{-1} \mathbf{M} + i\omega \xi_{c1} \bar{\mu}^{-1} \mathbf{M} - i\omega \mathbf{J} \end{aligned} \quad (22)$$

Sourceless vector Helmholtz's equations are found by setting $\mathbf{J} = 0, \mathbf{M} = 0$ in (20) and (22). The inverse permeability tensor $\bar{\mu}$ is using (5)

$$\bar{\mu}^{-1} = \begin{bmatrix} \mu & -ik_m & 0 \\ ik_m & \mu & 0 \\ 0 & 0 & (\mu^2 - k_m^2)/\mu_0 \end{bmatrix} \frac{1}{(\mu^2 - k_m^2)} \quad (23)$$

4. DYADIC GREEN'S FUNCTIONS

Derivation of the H-field dyadic in unbounded composite chiral-ferrite media follows. Start from (20), $\nabla \times \nabla \times = \nabla(\nabla \cdot) - \nabla^2$ and define

$$U_h(r) = \nabla \times J + i\omega \xi_{c2} J - i\omega \epsilon M, \quad (24)$$

$$[\nabla \nabla - \nabla^2 I - \omega^2 \epsilon \mu_0 (\bar{\mu}/\mu_0 - \xi_{c1} \xi_{c2}/(\mu_0 \epsilon) I) + i\omega(\xi_{c2} - \xi_{c1}) \nabla \times I] \cdot H = U_h(r) \quad (25)$$

The direct space dyadic for H is defined by

$$H(r) = \int_V dv' G_h(r, r') \cdot U_h(r') ; U_h(r) = \int_V dv' U_h(r') \delta(r - r') \quad (26ab)$$

into (25) gives

$$\begin{aligned} & \int_V \left\{ [\nabla \nabla - \nabla^2 I - \omega^2 \epsilon \mu_0 (\bar{\mu}/\mu_0 - \xi_{c1} \xi_{c2}/(\mu_0 \epsilon) I) + i\omega(\xi_{c2} - \xi_{c1}) \nabla \times I] \cdot G_h(r, r') \right\} \cdot U_h(r') \\ & = \int_V dv' \{ \delta(r - r') I \} \cdot U_h(r') \end{aligned} \quad (27)$$

Since $U_h(r')$ is arbitrary,

$$\begin{aligned} & [\nabla \nabla - \nabla^2 I - \omega^2 \epsilon \mu_0 (\bar{\mu}/\mu_0 - \xi_{c1} \xi_{c2}/(\mu_0 \epsilon) I) + i\omega(\xi_{c2} - \xi_{c1}) \nabla \times I] \cdot G_h(r, r') \\ & = I \delta(r - r') \end{aligned} \quad (28)$$

Reciprocal space, Fourier transformed variable p and Green's function dyadic $g_h(p)$ are

$$G_h(r, r') = (2\pi)^{-3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 p g_h(p) e^{ip \cdot (r - r')} \quad (29a)$$

$$\delta(r - r') = (2\pi)^{-3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 p e^{ip \cdot (r - r')} \quad (29b)$$

$$g_h(p) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 (r - r') G_h(r, r') e^{-ip \cdot (r - r')} \quad (29c)$$

Translation invariance assumed in the use of $r - r'$ comes from the unbounded nature of the problem although uniformly anisotropic. Using operator properties on $\exp(ip \cdot r)$, $\nabla = ip$, $\nabla \nabla = -pp$, $-\nabla^2 = p^2$, $\nabla \times I = ip \times I$, we find

$$\begin{aligned} & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 p [-pp + p^2 I - \omega^2 \epsilon \mu_0 (\bar{\mu}/\mu_0 - \xi_{c1} \xi_{c2}/(\mu_0 \epsilon) I) + i\omega(\xi_{c2} - \xi_{c1}) ip \times I] e^{ip \cdot (r - r')} g_h(p) \\ & = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 p I e^{ip \cdot (r - r')} \end{aligned} \quad (30)$$

Because of the common $\exp[ip \cdot (r - r')]$ coefficient in (30)

$$g_h(p) = [-pp + p^2 I - \omega^2 \epsilon \mu_0 (\bar{\mu}/\mu_0 - \xi_{c1} \xi_{c2}/(\mu_0 \epsilon) I) + i\omega(\xi_{c2} - \xi_{c1}) ip \times I]^{-1} \quad (31)$$

The direct space dyadic is found from (31) by doing the inverse operation implied by (31) and applying the transform (29a) on $g(p)$.

Derivation of the E-field dyadic follows the H-field steps, starting from (22) and defining

$$U_e(\mathbf{r}) = -\nabla \times \bar{\mu}^{-1} \mathbf{M} + i\omega \xi_{e1} \bar{\mu}^{-1} \mathbf{M} - i\omega \mathbf{J}, \quad (32)$$

$$\begin{aligned} & [-\nabla \times \bar{\mu}^{-1} \nabla \times \mathbf{I} + \omega^2 \epsilon (1 - \xi_{e1} \xi_{e2} \bar{\mu}^{-1} / \epsilon) \mathbf{I} + i\omega (-\xi_{e2} \nabla \times \bar{\mu}^{-1} + \xi_{e1} \bar{\mu}^{-1} \nabla \times) \mathbf{I}] G_e(\mathbf{r}, \mathbf{r}') \\ & = \mathbf{I} \delta(\mathbf{r} - \mathbf{r}') \end{aligned} \quad (33)$$

The reciprocal space dyadic $g_e(\mathbf{p})$

$$g_e(\mathbf{p}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3(\mathbf{r} - \mathbf{r}') G_e(\mathbf{r} - \mathbf{r}') e^{-i\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')} \quad (34)$$

is found using $\nabla \times \bar{\mu}^{-1} = i\mathbf{p} \times \bar{\mu}^{-1}$, $\nabla \times \bar{\mu}^{-1} \nabla \times \mathbf{I} = -\mathbf{p} \times \bar{\mu}^{-1} \mathbf{p} \times \mathbf{I}$, yielding

$$g_e(\mathbf{p}) = [-\mathbf{p} \times \bar{\mu}^{-1} \mathbf{p} \times \mathbf{I} + \omega^2 \epsilon (1 - \xi_{e1} \xi_{e2} \bar{\mu}^{-1} / \epsilon) \mathbf{I} - \omega (-\xi_{e2} \mathbf{p} \times \bar{\mu}^{-1} + \xi_{e1} \bar{\mu}^{-1} \mathbf{p} \times) \mathbf{I}]^{-1} \quad (35)$$

Direct space dyadic is then determined by using the same form of transform as (29a). Equation (35) can be compared to less general form in [12].

5. CONCLUSIONS

Electromagnetic properties of chiral and nonreciprocal composite chiral-ferrite media have been delineated in this paper in terms of reaction theorems, vector Helmholtz equations, and dyadic functions. Exploring new media for use in microwave and millimeter wave devices is of great interest today. This is especially true in view of the trend toward miniaturization, including the current emphasis on monolithic microwave circuits using Si, GaAs, and other substrates operating in the 1 - 100 GHz frequency spectrum.

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