TIME-DOMAIN RADIATION FROM AN APERTURE

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I. Introduction

Switching on and off an electrical source generates radiation. As suggested by Wu [1], such radiation could behave strangely. In this paper, time domain fields from a uniform current distribution over a circular aperture are analyzed. It is found that, at any instant after the current is switched on, there is a region the size of the aperture near its axis in which the electric and the magnetic fields maintain constant strength. The energy carried by the pulse in this region decreases as the inverse of the distance from the aperture.

II. Integral Representation of the Time Domain Fields

An aperture of radius a located at the $z=0$ plane is considered. The coordinate system is scaled so that the aperture is described by $z=0$ and $\rho < 1$. Assume a uniform current distribution over the aperture:

$$J(r, t) = \begin{cases} \hat{z} \delta(z) & t > 0, \rho < 1 \\ 0 & \text{otherwise} \end{cases}$$

There is no field for $|z| > ct$. For $|z| < ct$, the non-vanishing field components are shown to be given by:

$$E_x(r, t) = -\frac{1}{2} \epsilon_0 \int_{-i\infty+C}^{i\infty+C} \frac{ds}{2\pi i} \left(\frac{a^2}{c^2 t^2 - z^2} \right)^{s+1} g(\rho, s)$$

$$H_y(r, t) = -\frac{z}{2ct} \int_{-i\infty+C}^{i\infty+C} \frac{ds}{2\pi i} \left(\frac{a^2}{c^2 t^2 - z^2} \right)^{s+1} F(1, \frac{1}{2}; -s; 1 - \frac{z^2}{c^2 t^2}) g(\rho, s)$$

where $-\frac{1}{2} < C < -\frac{1}{4}$, F is the hypergeometric function and

$$g(\rho, s) = \begin{cases} \frac{1}{s+1} F(-s, -s-1; 1; \rho^2) & \rho < 1 \\ \rho^{2s} F(-s, -s; 2, \rho^{-2}) & \rho > 1 \end{cases}$$

III. Specific Results

From the integral representation, it can be deduced that:

(a) In the region $c^2 t^2 - z^2 < (1 - \rho)^2 a^2$

$$E_x(r, t) = \begin{cases} 0 & \rho > 1 \\ -\zeta_0/2 & \rho < 1 \end{cases}$$

$$H_y(r, t) = \begin{cases} 0 & \rho > 1 \\ -(\operatorname{sgn} z)/2 & \rho < 1 \end{cases}$$

where

$$\operatorname{sgn} z = \begin{cases} 1 & z > 0 \\ -1 & z < 0 \end{cases}$$

(b) In the region $(1+\rho)^2 a^2 < c^2 t^2 - z^2$

$$E_x(r, t) = 0$$

$$H_y(r, t) = -\frac{a^2 z}{4r^3} \sum_{m=0}^{\infty} \frac{(-1)^m \Gamma(m + \frac{3}{2})}{\Gamma(m+2) \Gamma(\frac{3}{2})} \left(\frac{a}{r}\right)^{2m} F(-m, m + \frac{3}{2}; 1; \frac{a^2 \rho^2}{r^2})$$

which is the static field established behind the transient. Note that for $r \gg a$,

$$H_y(r, t) \approx -\frac{a^2 z}{4r^3}.$$

(c) The fields in the region

$$(1-\rho)^2 a^2 < c^2 t^2 - z^2 < (1+\rho)^2 a^2$$

cannot be evaluated easily. But for $\rho=1$,

$$E_x(r, t) = -\frac{1}{2\pi} \zeta_0 \cos^{-1} u$$

$$\begin{aligned} H_y(r, t) &= -\frac{1}{4}(\operatorname{sgn} z) + \frac{zu}{4ct} (\pi)^{3/2} \sum_{m=0}^{\infty} \frac{\Gamma(m + \frac{1}{2})}{(\Gamma(m + \frac{1}{2}) \Gamma(m+1))} u^{2m} F(1, \frac{1}{2}; m + \frac{3}{2}; 1 - \frac{z^2}{c^2 t^2}) \\ &\approx -\frac{1}{2\pi} \cos^{-1} u \quad \text{for } ct \gg 2a, \end{aligned}$$

where $u^2 = (c^2 t^2 - z^2)/(4a^2)$.

IV. Conclusions

The results obtained thus far can be summarized with the aid of the accompanying figures. The fields in region A are those of an infinite plate. The presence of the edge causes these fields to change in region B. Behind region B, the magnetostatic field is established in region C.

The radiation into the cylindrical region $\rho < 1$ is the most interesting. The pulse, which consists of regions A and B, propagates along the z-axis without reducing its electromagnetic field strengths. Since the volume of region A decreases as $1/z$ as the pulse propagates outwards, the energy carried by the pulse in this region decreases only as $1/z$. The electric and the magnetic fields of the pulse in region B decrease from fixed magnitudes at the boundary between regions A and B to their values at the boundary between regions B and C. In particular, the energy flux at $\rho=1$ also decreases only as $1/z$ as z increases. It is speculated that the energy carried by the pulse in the $\rho < 1$ portion of region B decreases only as $1/z$.

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VI. Reference

- (1) Tai Tsun Wu, "Electromagnetic Missiles," J. Appl. Phys., 57 (7), 2370-2373, April 1985.

Radiated fields from a uniform current distribution at two instances, $t_2 > t_1$:

Region A: outward going EM fields of constant strengths.

Region B: outward going EM fields of varying strengths.

Region C: magnetostatic field.