THE USE OF NONLINEAR PROGRAMMING TECHNIQUES FOR PHASE-ONLY NULL SYNTHESIS

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Introduction

Current interest in the subject of phase-only control of array weights for adaptive cancellation of interference^{1,2} and for null synthesis³ is the result of the widespread use of phased arrays. The restriction that the perturbations of the array weights be of phases only, results in complexities in null synthesis not present when the array weights can be controlled in both phase and amplitude. The complexities stem from the nonlinearity of the equations for imposing pattern nulls with phase-only control, whereas the null equations for combined phase and amplitude control are linear.

In null synthesis it is typically the case that the synthesized pattern, with nulls at a set of specified locations, should satisfy criteria such as sidelobes not exceeding a given level. One way of meeting these criteria is to start with a pattern that is acceptable in all respects except that of having nulls at the desired locations, and to then perturb the array weights as little as possible so as to impose pattern nulls at the specified locations. It is then to be expected that the resulting pattern change will be minimized and that the perturbed pattern will bear a close resemblance to the original pattern.

In this paper we describe the use of nonlinear programming techniques to solve the problem of phase-only null synthesis with minimized weight perturbations. From the large literature on analytic and numerical methods for solving the nonlinear programming problem -- minimizing a nonlinear objective function of several variables subject to nonlinear equality/inequality constraints -- we have selected two particular methods. These methods give impressive results in general although convergence problems are encountered in some situations.

Formulation of the Nonlinear Programming Problem

We consider a linear array of equispaced isotropic elements with spacing d and phase reference at the center of the array. Let a_n , $n = 1, \dots, N$, be a given amplitude taper, assumed symmetric with respect to the array center, of the element weights. The unperturbed field pattern is then

$$p_0(u) = \sum_{n=1}^N a_n e^{j d_n u} = \sum_{n=1}^N a_n \cos d_n u$$

where $d_n = (N-1)/2 - (n-1)$, $u = 2\pi/\lambda d \sin \theta$, and θ = angle measured from broadside. Let ϕ_n , $n = 1, \dots, N$, be the set of phase perturbations that will impose nulls in the pattern at the locations $u = u_k$, $k = 1, \dots, M$, and minimize the sum of the squares of the weight perturbations

$$\sum_{n=1}^N |a_n (e^{j\phi_n} - 1)|^2 = 4 \sum_{n=1}^N [a_n \sin(\phi_n/2)]^2 \quad (1)$$

The ϕ_n are assumed to be odd-symmetric, i.e., $\phi_{N-n+1} = -\phi_n$, so that the perturbed pattern is real. The null constraint equations are then

$$\sum_{n=1}^N a_n \cos(d_n u_k + \phi_n) = 0, \quad k = 1, 2, \dots, M \quad (2)$$

The minimized weight perturbation, phase-only null synthesis problem is thus expressed as the nonlinear programming problem of minimizing (1) subject to Eqs. (2).

Numerical Results

Two nonlinear programming methods with readily available computer codes, LPNLP⁴ and VMCON⁵, were selected for use on the phase-only null synthesis problem. The algorithms were tested on the problem of imposing nulls in the pattern of a 41 element array with uniform amplitude and $\lambda/2$ spacing. A series of sets of imposed null locations was used starting with 1 null at 4.0° , then two nulls at 4.0° and 4.6° , up through 5 nulls at 4.0 , 4.6 , 5.2 , 5.8 , and 6.4 degrees. Both algorithms were run in double precision on a CDC 6600 computer with the unknown phase perturbations set initially to zero. Null depths and computation time are shown in Table 1. For up to 3 nulls LPNLP was more efficient, while for 4 nulls VMCON performed more efficiently. For the 5-null case LPNLP converged extremely slowly while VMCON terminated without convergence. When the algorithms both converged they did so to the same solution to within a very small difference attributable to the fact that different termination criteria are employed.

Some measure of the numerical difficulty of phase-only nulling is the size of the phase perturbations required to impose nulls. For small phase perturbations, the phase-only null synthesis problem approaches linearity and so is much easier to solve numerically than is the case when large phase perturbations are required and the nulling problem is essentially nonlinear. The test problem was intentionally chosen to be a severe test of the algorithms, requiring nulls to be imposed at closely spaced locations in a near-in sidelobe region of a uniform array. The average absolute phase perturbation and the objective function (1) are

shown in Table 2. Note the sharp increase in these values, especially for the 4- and 5-null cases.

As an indication of the performance of LPNLP and VMCON on a less difficult null synthesis problem, the algorithms were tested on the problem of imposing nulls in the pattern of a 41 element array with uniform amplitude and $\lambda/2$ spacing at the set of locations 25, 35, 45, 55, and 65 degrees. For one null at 25°, LPNLP generated the phase perturbations for a null depth of -276 dB in 2.3 seconds while VMCON took 12.1 seconds to form a null of -283 dB. For the 5-null case, LPNLP required 7.1 seconds to form nulls < -282, and VMCON took 16.0 seconds to form nulls < -241 dB. The values of the average absolute phase perturbation and objective function were 0.044 radians and 0.095 for the 1-null case, and 0.063 radians and 0.242 for the 5-null case. These values are much smaller than the corresponding values for the first test problem.

As one further test, LPNLP and VMCON were run on the difficult problem of imposing nulls at the 4 locations 14.70°, 15.28°, 15.86°, and 16.44° in the pattern of a 41 element array with $\lambda/2$ spacing and a 20 dB Chebyshev amplitude taper⁶. VMCON solved this problem in about 40 seconds CP time with null depths < -250 dB, but LPNLP failed to converge to a solution. Further work is needed to understand clearly the reasons for this failure and for the difficulties encountered in the 5-null case of the first test problem. In general, however, both LPNLP and VMCON provide effective means for calculating the phases required for minimized weight perturbation, phase-only null synthesis.

References

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NUMBER OF NULLS

	1		2		3		4		5	
	NULL DEPTH (dB)	CP TIME (SEC)	NULL DEPTH (dB)	CP TIME (SEC)	NULL DEPTH (dB)	CP TIME (SEC)	NULL DEPTH (dB)	CP TIME (SEC)	NULL DEPTH (dB)	CP TIME (SEC)
LPNLP	-255	2.9	<-289	5.7	<-265	12.0	<-249	68.9	<-186	1007
VMCON	-266	25.4	<-254	32.5	<-275	39.5	<-283	34.4	-	-

TABLE 1. Comparison of performance of LPNLP and VMCON on phase-only nulling in pattern of 41 element, uniform amplitude array with $\lambda/2$ spacing. Nulls imposed at the series of locations 4.0, 4.6, 5.2, 5.8, and 6.4 degrees.

NUMBER OF NULLS

	1		2		3		4		5	
	AVERAGE ABSOLUTE PHASE PERTURBATION (RADIAN)									
AVERAGE ABSOLUTE PHASE PERTURBATION (RADIAN)	0.2838		0.3073		0.3321		0.5365		0.8803	
OBJECTIVE FUNCTION	3.7887		4.5743		7.5406		19.993		36.665	

TABLE 2. Average absolute phase perturbation and the objective function $4 \int \sin^2 (\phi_n/2)$ for phase-only nulling in the pattern of 41 element uniform amplitude array with $\lambda/2$ spacing. Nulls imposed at the series of locations 4.0, 4.6, 5.2, 5.8, and 6.4 degrees using LPNLP and VMCON.