

An Enhanced Circuit-Based Model for Single-Cell Battery

Jiucui Zhang, Song Ci, Hamid Sharif
 Department of Computer and Electronics Engineering
 University of Nebraska-Lincoln
 NE 68182, USA
 Email: jczhang@huskers.unl.edu, {sci, hsharif}@unl.edu

Mahmoud Alahmad
 Department of Architecture Engineering
 University of Nebraska-Lincoln
 NE 68182, USA
 Email: malahmad2@unl.edu

Abstract—Battery performance prediction is crucial for battery-aware power management, battery maintenance, and multi-cell battery design. However, the existing battery models cannot capture the circuit characteristics and nonlinear battery effects, especially recovery effect. This paper aims to fill this gap by developing an enhanced circuit-based model for single-cell battery. The proposed model is validated by comparing simulation results with experimental data collected through battery testbed. The comparison shows that the proposed model can accurately characterize and predict the single-cell battery performance with considerations of various nonlinear battery effects under both constant and variable loads.

I. INTRODUCTION

Battery has been widely used in various mobile devices such as PDA, laptop, battery-powered electric vehicle, and battery energy storage system. An accurate battery model, which can capture complicated and dynamic battery circuit features and nonlinear capacity effects, is very crucial for circuit simulation, multi-cell battery analysis, battery performance prediction and optimization, and battery maintenance.

So far, many battery models have been proposed in literature [1]. In general, existing battery models can be divided into physical models, analytical models, and circuit-based models. In physical models, differential equations have been used to capture the complex electrical-chemical process in a battery [2]. Therefore, physical models are accurate and generic, which can be used to characterize battery behaviors. However, physical models require intensive computations to solve the interdependent partial differential equations. In addition, due to lack of battery model parameters such as battery structure and chemical composition, physical models are difficult to be configured and used [1], [3], [4]. To reduce the computational complexity, analytical models have been developed, where an equivalent mathematical representation is used to approximate the battery performance [5]–[8]. Analytical models are accurate and simple enough for power management, but they ignore circuit features such as voltage and internal resistance, making them infeasible for multi-cell battery design and analysis as well as circuit simulation. In circuit-based models, battery nonlinear circuit behaviors can be emulated by using capacitors, voltage and current resources, and resistors from the circuit analysis point of view. Circuit-based models can capture the complicated battery properties, which can be

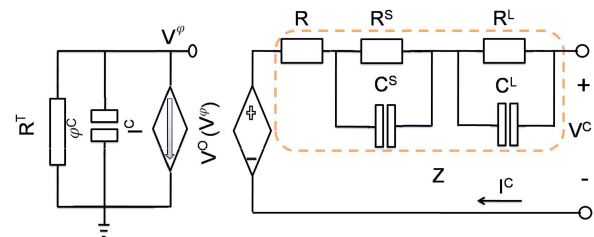


Fig. 1. Existing battery model [9]

TABLE I
SUMMARY OF NOTATIONS

| | |
|------------|--|
| V^o | Open-circuit voltage |
| V_i^C | Output voltage |
| V_i^F | Cutoff voltage of the single-cell battery |
| R^T | Self discharge resistance |
| R | Internal resistance |
| R^S | Short-transient resistance |
| R^L | Long-transient resistance |
| C^S | Short-transient capacitance |
| C^L | Long-transient capacitance |
| φ | State of charge |
| α^f | Full capacity of a single cell |
| α^A | Consumed capacity of a single-cell battery |
| I^C | Discharge current rate |
| μ | Recoverable capacity |

easily implemented in electronic design automation (EDA) tools at different levels of abstraction [9]–[12]. However, current circuit-based models cannot estimate the impact of nonlinear behaviors on battery available capacity, leading to an inaccurate prediction of remaining battery capacity [8].

In this paper, we propose a new circuit-based battery model to capture the battery circuit features and nonlinear battery capacity effects, especially recovery effect. The model can accurately capture the battery performance both at constant and variable loads. We have validated the proposed battery model with experimental data collected through the ARBIN battery testing equipment.

The rest of this paper is organized as follows. Section II presents the related work. The battery model is proposed in Section III. The proposed battery model is validated in Section IV. We conclude the paper in Section V.

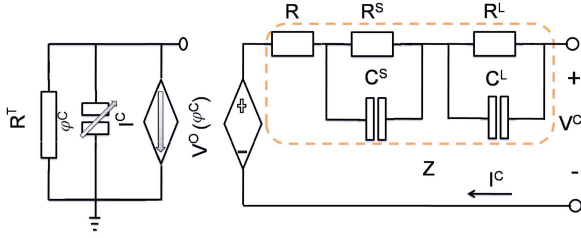


Fig. 2. The proposed battery model

II. RELATED WORK

Figure 1 illustrates the existing circuit-based battery model [9]. Here, the voltage-controlled voltage source is used to represent State of Charge (SOC) and open-circuit voltage. A current-controlled current source is used to represent battery capacity and SOC. The RC network emulates the transient voltage response. All model parameters, such as open-circuit voltage, resistors, and capacitors, can be approximated by mathematic equations listed as follows.

$$\begin{cases} \alpha^A(I^C, t_s, t_e) = I^C(t_e - t_s) \\ \varphi^C = 1 - \frac{\alpha^A}{c^f} \\ V^o(\varphi^C) = a_1 e^{a_2 \varphi^C} + a_3 \varphi^C - a_4 \varphi^{C^2} + a_5 \varphi^{C^3} + a_6 \\ R(\varphi^C) = b_1 e^{b_2 \varphi^C} + b_3 \varphi^C - b_4 \varphi^{C^2} + b_5 \varphi^{C^3} + b_6 \\ R^S(\varphi^C) = d_1 e^{-d_2 \varphi^C} + d_3 \\ C^S(\varphi^C) = f_1 e^{f_2 \varphi^C} + f_3 \\ R^L(\varphi^C) = g_1 e^{g_2 \varphi^C} + g_3 \\ C^L(\varphi^C) = l_1 e^{l_2 \varphi^C} + l_3 \end{cases} \quad (1)$$

where, α^A is the accumulated capacity during time period $[t_s, t_e]$ at rate of I^C ; R , V^o , c^f , and φ^C are battery internal resistance, open-circuit voltage, the full capacity, and SOC, respectively; R^S , R^L , C^S , and C^L are resistances and capacitors to capture the transient response of battery voltage. $a_1 \sim a_6$, $b_1 \sim b_6$, $d_1 \sim d_3$, $f_1 \sim f_3$, $g_1 \sim g_3$, and $l_1 \sim l_3$ are coefficients of the model.

In general, the circuit-based model can accurately capture the dynamic circuit characteristics of a battery such as non-linear open-circuit voltage, temperature, cycle number, and self-discharge. However, the existing circuit-based model uses constant capacitor to model battery capacity, meaning that it is unable to capture and model the capacity relaxation process such as battery recovery effect. In addition, the accuracy of the existing model is very sensitive to the load variation rate, making it unable to handle dynamic battery load.

III. PROPOSED BATTERY MODEL

A. The Proposed Circuit-based Model

We propose an enhanced circuit-based model by replacing the consistent capacitor by a variable capacitor, as shown in Figure 2. The proposed model enables us to capture both battery circuit features and nonlinear battery capacity effects, making it a comprehensive and accurate model. The proposed

battery model can be denoted as:

$$\begin{cases} \alpha^A(I^C, \beta, L, t_s, t_e) = I^C F(L, t_s, t_e, \beta) \\ F(L, t_s, t_e, \beta) = t_s - t_e \\ \quad + 2 \sum_{m=1}^{\infty} \frac{e^{-\beta^2 m^2 (L-t_s)} - e^{-\beta^2 m^2 (L-t_e)}}{\beta^2 m^2} \\ \varphi^C = 1 - \frac{\alpha^A}{c^f} \\ V^o(\varphi^C) = a_1 e^{a_2 \varphi^C} + a_3 \varphi^C - a_4 \varphi^{C^2} + a_5 \varphi^{C^3} + a_6 \\ R(\varphi^C) = b_1 e^{b_2 \varphi^C} + b_3 \varphi^C - b_4 \varphi^{C^2} + b_5 \varphi^{C^3} + b_6 \\ R^S(\varphi^C) = d_1 e^{-d_2 \varphi^C} + d_3 \\ C^S(\varphi^C) = f_1 e^{f_2 \varphi^C} + f_3 \\ R^L(\varphi^C) = g_1 e^{g_2 \varphi^C} + g_3 \\ C^L(\varphi^C) = l_1 e^{l_2 \varphi^C} + l_3 \\ V^C(\varphi^C) = V_i^o(\varphi^C) - R(\varphi^C) I^C - \frac{R^S(\varphi^C)}{R^S(\varphi^C) \cdot j\omega \cdot C^S(\varphi^C) + 1} I^C \\ \quad - \frac{R^L(\varphi^C)}{R^L(\varphi^C) \cdot j\omega \cdot C^L(\varphi^C) + 1} I^C \end{cases} \quad (2)$$

where, $V^C(\varphi^C)$ denotes battery output voltage; ω means the current variation rate.

B. Remaining Capacity

In the proposed model, the accumulated capacity is denoted by an analytical expression [6] to capture the battery recovery effect. The consumed capacity $\alpha^C(I, \beta, L, t_s, t_e)$, which is dissipated during the load period $[t_s, t_e]$ at the discharge current I^C , can be written as [6]:

$$\begin{cases} \alpha^A(I^C, \beta, L, t_s, t_e) = I^C F(L, t_s, t_e, \beta) \\ F(L, t_s, t_e, \beta) = t_s - t_e \\ \quad + 2 \sum_{m=1}^{\infty} \frac{e^{-\beta^2 m^2 (L-t_s)} - e^{-\beta^2 m^2 (L-t_e)}}{\beta^2 m^2} \end{cases} \quad (3)$$

In this equation, the first term $I^C(t_s - t_e)$ is the consumed capacity by the load I^C during the load period $[t_s, t_e]$. The second term $2I^C \sum_{m=1}^{\infty} \frac{e^{-\beta^2 m^2 (L-t_s)} - e^{-\beta^2 m^2 (L-t_e)}}{\beta^2 m^2}$ is the amount of discharging loss due to the current effect, which is the maximum recoverable battery capacity at t_e . It can be observed that the discharge loss will increase as the discharge current increases. β^2 is a constant related to the diffusion rate within battery. The larger the β^2 , the faster the battery diffusion rate is, thus the less the discharging loss. L is the total operating time of the battery. m determines the computational complexity and accuracy of the model.

When a fully charged battery is discharged over time $\tau = \{t_0, t_1, \dots, t_N\}$, the remaining capacity can be denote as:

$$\alpha^C = \alpha^f - \sum_{i=1}^N \alpha^A(I_i^C, \beta, L, t_{i-1}, t_i) \quad (4)$$

where, α^f is the full capacity of the battery.

As seen from Eq. 4, α^C will change with current variation, which is accurately capture the battery current effect. When the output voltage of the battery reaches cutoff voltage, the battery state of charge gets to 0.

The capacity loss could be recovered. Considering a constant load with profile defined as:

$$i(t) = \begin{cases} I & t \in [0, T) \\ 0 & T \end{cases} \quad (5)$$

The maximum recoverable capacity occurs at $t = T$ can be denoted as:

$$\begin{aligned} \mu_{\max}(L, I, \beta, T, T) &= 2I \sum_{i=1}^{\infty} \frac{e^{-\beta^2 i^2 L_1} - e^{-\beta^2 i^2 (L_1 - T)}}{\beta^2 i^2} \\ &= 2I \sum_{i=1}^{\infty} \frac{e^{-\beta^2 i^2 L_1} (1 - e^{-\beta^2 i^2 T})}{\beta^2 i^2} \end{aligned} \quad (6)$$

The recovery rate of a battery ε with a constant profile over time Δt is:

$$\begin{aligned} \varepsilon(T, \Delta t, \beta) &= \frac{\mu(L, 0, \beta, T, T + \Delta t)}{\mu_{\max}(L, 0, \beta, T, T)} \\ &= \frac{2I \sum_{i=1}^{\infty} \frac{e^{-\beta^2 i^2 (L_1 - T)} (1 - e^{-\beta^2 i^2 \Delta T})}{\beta^2 i^2}}{2I \sum_{i=1}^{\infty} \frac{e^{-\beta^2 i^2 L_1} (1 - e^{-\beta^2 i^2 T})}{\beta^2 i^2}} \\ &= \frac{\sum_{i=1}^{\infty} \frac{e^{-\beta^2 i^2 T} (1 - e^{-\beta^2 i^2 \Delta T})}{i^2}}{\sum_{i=1}^{\infty} \frac{1 - e^{-\beta^2 i^2 T}}{i^2}} \end{aligned} \quad (7)$$

Therefore, the recovered capacity of the battery over time Δt is:

$$\begin{aligned} \mu(L, I, \beta, T, \Delta t) &= \mu_{\max} \times \varepsilon(T, \Delta t, \beta) \\ &= 2I \sum_{i=1}^{\infty} \frac{e^{-\beta^2 i^2 L_1} (1 - e^{-\beta^2 i^2 T})}{\beta^2 i^2} \times \frac{\sum_{i=1}^{\infty} \frac{e^{-\beta^2 i^2 T} (1 - e^{-\beta^2 i^2 \Delta T})}{i^2}}{\sum_{i=1}^{\infty} \frac{1 - e^{-\beta^2 i^2 T}}{i^2}} \end{aligned} \quad (8)$$

which not only relies on the discharging current, but only determined by the discharging time T , rest time Δt , and battery parameter β .

Self-discharge resistor R_T is used to characterize the self-discharge energy loss when battery are stored for a long time, which is a function of SOC, temperature, and cycle number. The usable capacity decreases slowly with time when no load is connected to the battery. So, in this paper we ignore the self-discharge.

IV. MODEL VALIDATION

A. Experiment Setup

We have validated the proposed battery model under both constant currents and variable currents by using HE18650 battery whose full capacity, nominal voltage, and cutoff voltage are 2600mAh, 3.7V and 3V, respectively. All parameters of the proposed battery model, as shown in Table II, can be obtained by using the standard least-square estimator [8], [12]. The simulation results of the battery are obtained by using MATLAB, and the experimental data are collected through the ARBIN battery testing instrument BT2000 as shown in Figure 3 [13]. The battery is first charged to its full capacity through Constant Current Constant Voltage (CCCV), and then it will be rested for 30 minutes [14]. Then, the battery will be discharged under different predefined profiles, respectively.

B. Simulation and Experiment Results

Figure 4 shows the battery performance at constant discharge current rate of 0.25A and 1A, respectively. When the battery output voltage goes from full capacity voltage to cutoff voltage, the state of charge of battery drops from a certain value to 0. The SOC of the full charged battery varies with discharge current rate, which reflects the current effect. Experiment results match experimental data well.

Both simulation and experiment results for a four-phase dynamic load profile at discharge current rate of 1A, 2A, 0.2A,



Fig. 3. ARBIN battery testing instrument BT2000

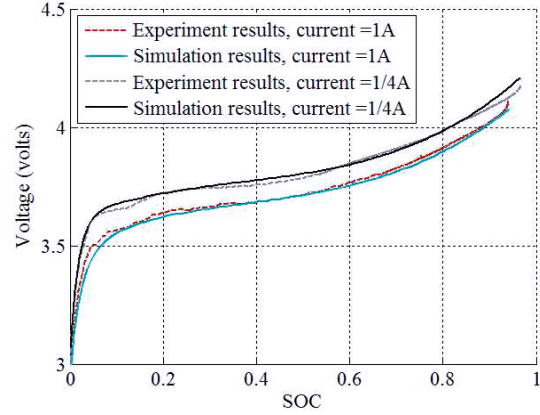


Fig. 4. Battery model validation at constant loads

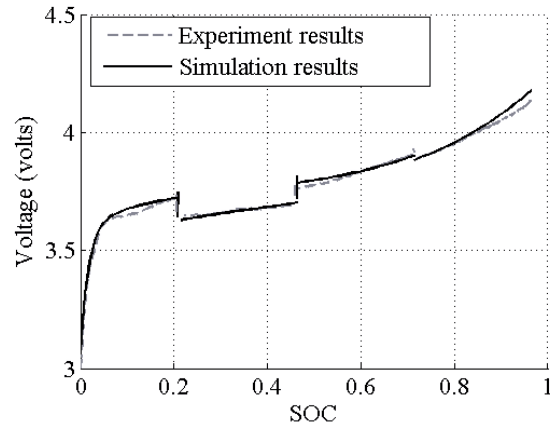


Fig. 5. Battery model validation at variable loads

TABLE II
BATTERY MODEL PARAMETERS

| | | | | | | | | | |
|-------|--------|-------|---------|-------|---------|-------|---------|---------|---------|
| a_1 | -0.402 | a_2 | -50.58 | a_3 | 0.8849 | a_4 | -1.662 | a_5 | 1.482 |
| a_6 | 3.574 | b_1 | -0.1726 | b_2 | -20.07 | b_3 | 0.0944 | b_4 | -0.2301 |
| b_5 | 0.1772 | b_6 | 0.06105 | f_1 | -752.9 | f_2 | -13.51 | f_3 | 703.6 |
| g_1 | 6.603 | g_2 | -155.2 | g_3 | 0.04984 | l_1 | -6056 | l_2 | -27.12 |
| l_3 | 4475 | d_1 | 0.3208 | d_2 | -29.14 | d_3 | 0.04669 | β | 0.35 |

and 1A are shown in Figure 5. As the battery discharged over time, the battery voltage will reach the cutoff voltage, and battery SOC will be 0. This means that the proposed battery model can accurately quantify the battery characteristic.

From both figures, we can observe that the proposed model generate voltage response less than 20mV. Therefore, we can conclude that the simulation results of the proposed battery model matches well with the experiment data. The close agreement between simulation results and experimental data indicates that the battery parameters have been accurately extracted to predict run-time battery behaviors in both steady state and transient state voltage responses.

V. CONCLUSION

In this work, an accurate and comprehensive circuit-based battery model has been proposed to capture circuit features and nonlinear battery effects such as current effect and recovery effect. Both simulation results and experiment results show that the proposed models can be used to accurately model and predict battery performance. The computational complexity of the proposed model could be controlled, which provides a way to tradeoff computational complexity and model accuracy. Therefore, the proposed model will greatly help research on circuit simulation, multi-cell battery analysis, battery performance prediction and optimization, and battery maintenance.

ACKNOWLEDGMENT

This research was supported in part by NSF ECCS Grant # 0801736.

REFERENCES

- [1] R. Rao, S. Vrudhula, and D. Rakhmatov, "Battery modeling for energy aware system design," *Computer*, vol. 36, no. 12, pp. 77–87, Dec. 2003.
- [2] M. Doyle, T. Fuller, and J. Newman, "Modeling of Galvanostatic Charge and Discharge of the Lithium PolymerInsertion Cell," *Journal Electrochemical Society*, vol. 140, pp. 1526–1533, 6 1993.
- [3] W. Gu and C. Wang, "Thermal-electrochemical modeling of battery systems," *Journal of Electrochemical Society*, vol. 147, no. 8, pp. 2910–2922, 2000.
- [4] M. Doyle, J. Newman, A. S. Gozdz, C. N. Schmutz, and J.-M. Tarascon, *Journal Electrochemical Society*, vol. 143, pp. 18–90, 1996.
- [5] P. Rong and M. Pedram, "An Analytical Model for Predicting the Remaining Battery Capacity of Lithium-Ion Batteries," *IEEE Journal Very Large Scale Integrated*, vol. 14, pp. 441–451, May 2006.
- [6] D. Rakhmatov, S. Vrudhula, and D. A. WallachAn, "Analytical High-Level Battery Model for use in energy Management of Portable Electronics Systems," *IEEE Journal Very Large Scale Integrated*, vol. 11, Dec. 2003.
- [7] D. N. Rakhmatov and S. B. K. Vrudhula, "An analytical high-level battery model for use in energy management of portable electronic systems," in *IEEE/ACM international conference on Computer-aided design*, Piscataway, NJ, USA, 2001, pp. 488–493.

- [8] P. Rong and M. Pedram, "An analytical model for predicting the remaining battery capacity of lithium-ion batteries," *IEEE Transactions on Very Large Scale Integration Systems*, vol. 14, no. 5, pp. 441–451, May 2006.
- [9] M. Chen and G. A. Rinc'on-Mora, "Accurate electrical battery model capable of predicting runtime and icv performance," *IEEE Transactions on Energy Conversion*, vol. 21, no. 2, pp. 504–511, 2006.
- [10] L. Benini, G. Castelli, A. Macci, E. Macci, M. Poncino, and R. Scarsi, "Discrete-time battery models for system-level low-power design," *IEEE Transactions on Very Large Scale Integrated System*, vol. 9, no. 5, pp. 630–640, 2001.
- [11] L. Gao, S. Liu, and R. A. Dougal, "Dynamic lithium-ion battery model for system simulation," *IEEE Transactions on Component Package Technology*, vol. 25, no. 3, pp. 495–505, 2002.
- [12] W. X. Shen, C. C. Chan, E. W. C. Lo, and K. T. Chau, "Estimation of battery available capacity under variable discharge currents," *Journal of Power Sources*, vol. 103, no. 2, pp. 180 – 187, 2002.
- [13] "Bt2000 battery testing system," Arbin Inc., Tech. Rep., 2009.
- [14] M. Alahmad and H. Hess, "Evaluation and analysis of a new solid-state rechargeable micro-scale lithium battery," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 9, pp. 3391 – 3401, 2008.