The Scattering of a Partially-Coherent Electromagnetic Field from a Bianisotropic Object

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Abstract—The scattering of a spatially partially-coherent electromagnetic field from a bianisotropic object is investigated. Assuming a Schell-model form for the incident-field cross-spectral-density (CSD) matrix, the far-zone scattered-field CSD matrix is derived via the far-zone vector potentials and the bianisotropic Born approximations. The mathematical form for the scattered-field CSD matrix is very physical and is thoroughly discussed. The derived scattered-field CSD matrix is used to predict the scattered Stokes parameters for two anisotropic cubes (a dielectric biaxial cube and a bianisotropic cube) illuminated by incident fields with varying polarization states and spatial correlation radii. The polarization and coherence effects in the results are highlighted and physically interpreted.

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1. INTRODUCTION

Recent advances in manufacturing technology have made materials that control light in novel ways a reality. Constructed from metallic split-ring resonators (SRR), Ω particles, wire helices, dielectric inclusions, etc. [1–9], these new materials, broadly termed metamaterials, promise new and exciting applications such as hyperlenses and cloaking [10–14]. Because of the required manufacturing tolerances, a majority of the published metamaterial structures have been constructed for use in the microwave and terahertz ranges of the electromagnetic spectrum [1, 10–12]; however, these structures are beginning to be fabricated for use in infrared and visible light applications [2, 4, 8, 13, 15–18].

Occurring in parallel with the rapid advances in metamaterials theory and technology, light control via coherence manipulation has also been a very active area of optics research. Beginning with Wolf’s unified theory of coherence and polarization [19], techniques have been developed and demonstrated that produce light with practically any desired shape, coherence, and polarization properties [20–22]. Much research has been performed predicting how these partially-coherent fields propagate through random media [20, 23–27], scatter from deterministic and random objects [19, 20, 28–35], and propagate through anisotropic media [36–39]. To the

Figure 1. Geometry for a partially-coherent electromagnetic field scattered from a bianisotropic object.

author’s knowledge, no work has been performed predicting how partially-coherent fields interact with or scatter from metamaterials, i.e., bianisotropic media [8, 40–43].

In this paper, the scattering of a partially-coherent electromagnetic field from a bianisotropic object is investigated. Section 2 presents the theoretical analysis. Applying the volume equivalence principle for bianisotropic media, expressions for the equivalent electric and magnetic volume currents are derived. These currents are approximated using the bianisotropic Born approximations, from which, the far-zone scattered electric field is derived via the far-zone vector potentials. Assuming a Schell-model form for the incident-field cross-spectral-density (CSD) matrix [19, 20], the scattered-field CSD matrix is then derived and physically discussed. Section 3 presents and discusses the scattered Stokes parameters (obtained from the scattered-field CSD matrix) for two anisotropic objects illuminated by incident fields with varying coherence and polarization properties. Lastly, this paper is concluded with a summary of the work presented.

2. THEORY

Problem description

Consider the geometry shown in Fig. 1. The figure depicts a bianisotropic object illuminated by a partially-coherent electromagnetic field. The partially-coherent incident field originates from the source plane (the u - w plane) whose origin is located at $(x_s, -y_s, z_s)$.

The spatial domain vectors defined from the source plane origin, i.e., $\mathbf{t} = -\hat{x}x_s + \hat{y}y_s - \hat{z}z_s$ and $\mathbf{s} = \mathbf{t} + \mathbf{r}$, point to the center of the object and to any location in space $v \geq 0$,

respectively. The wavenumber vector $\mathbf{k}^i = \hat{\mathbf{x}}k_u^i + \hat{\mathbf{y}}k_v^i + \hat{\mathbf{z}}k_w^i$ denotes the direction of a plane wave emanating from the source plane origin. The vector $\mathbf{k}^i$ is defined spherically by the angles α and β as depicted.

The incident field is assumed to be a wide-sense stationary random process. Thus, the theoretical development presented below is performed in the space-frequency domain, i.e.,

$$\begin{aligned} \underline{\mathbf{W}}^i(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \omega) &= \left\langle \mathbf{E}^i(\boldsymbol{\eta}_1, \omega) \mathbf{E}^{i\dagger}(\boldsymbol{\eta}_2, \omega) \right\rangle \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \underline{\boldsymbol{\Gamma}}^i(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \tau) e^{j\omega\tau} d\tau \end{aligned} \quad (1)$$

where $\mathbf{E}^i = \hat{\mathbf{x}}E_u^i + \hat{\mathbf{z}}E_w^i$, ω is the radian frequency, $\dagger$ is the Hermitian transpose, $\underline{\boldsymbol{\Gamma}}^i$ is the incident-field coherence matrix [44], and $\underline{\mathbf{W}}^i$ is the incident-field CSD matrix [19,20].

It is further assumed that the elements of the incident-field CSD matrix take the common Schell-model form:

$$\begin{aligned} W_{ab}^i(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \omega) &= [S_a(\boldsymbol{\eta}_1, \omega) S_b(\boldsymbol{\eta}_2, \omega)]^{1/2} \\ &\quad \gamma_{ab}(\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2, \omega) \end{aligned} \quad (2)$$

where S_a and S_b are the spectral densities of the a and b field components, respectively, γ_{ab} is correlation function between the a and b field components, and $a, b = x, z$ [19, 20].

Bianisotropic volume equivalence

The incident field must exist in the absence of the object; therefore, the incident electromagnetic field is a solution of Maxwell's equations in the following form [45,46]:

$$\begin{aligned} \nabla \times \mathbf{E}^i &= -\mathcal{M}^i - j\omega\mu_0 \mathbf{H}^i \\ \nabla \times \mathbf{H}^i &= \mathbf{J}^i + j\omega\varepsilon_0 \mathbf{E}^i \end{aligned} \quad (3)$$

where $\mathbf{E}^i$ and $\mathbf{H}^i$ are the incident electric and magnetic fields, respectively, $\mathbf{J}^i$ and $\mathcal{M}^i$ are the electric and equivalent magnetic sources (included for generality) that maintain the incident field, respectively, and ε_0 and μ_0 are the permittivity and permeability of vacuum, respectively. Note that $\mathbf{E}^i$, $\mathbf{H}^i$, $\mathbf{J}^i$, and $\mathcal{M}^i$ are functions of position and wavelength omitted here for brevity.

The total electromagnetic field with the obstacle present is a solution of

$$\begin{aligned} \nabla \times \mathbf{E} &= -\mathcal{M}^i - j\omega\underline{\boldsymbol{\mu}} \cdot \mathbf{H} - j\omega\underline{\boldsymbol{\zeta}} \cdot \mathbf{E} \\ \nabla \times \mathbf{H} &= \mathbf{J}^i + j\omega\underline{\boldsymbol{\varepsilon}} \cdot \mathbf{E} + j\omega\underline{\boldsymbol{\xi}} \cdot \mathbf{H} \end{aligned} \quad (4)$$

where $\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s$ and $\mathbf{H} = \mathbf{H}^i + \mathbf{H}^s$ [46, 47]. Here, $\mathbf{E}^s$ and $\mathbf{H}^s$ are the scattered electric and magnetic fields, respectively. The electromagnetic material properties of the object are represented by $\underline{\boldsymbol{\varepsilon}}$, $\underline{\boldsymbol{\mu}}$, $\underline{\boldsymbol{\xi}}$, and $\underline{\boldsymbol{\zeta}}$. These

symbols corresponds to the permittivity, permeability, and magnetoelectric tensors of the object, respectively. They take the general form

$$\underline{\boldsymbol{\varepsilon}}, \underline{\boldsymbol{\mu}}, \underline{\boldsymbol{\xi}}, \underline{\boldsymbol{\zeta}} = \begin{pmatrix} \kappa_{xx} & \kappa_{xy} & \kappa_{xz} \\ \kappa_{yx} & \kappa_{yy} & \kappa_{yz} \\ \kappa_{zx} & \kappa_{zy} & \kappa_{zz} \end{pmatrix} \quad (5)$$

and are, in general, functions of position and wavelength. Note that the underlying geometrical structure/symmetry of the medium plays a key role in determining the forms of these tensors. A very thorough explanation of this can be found in [5].

Subtracting (3) from (4) and simplifying yields

$$\begin{aligned} \nabla \times \mathbf{E}^s &= -\mathcal{M}^{\text{eq}} - j\omega\mu_0 \mathbf{H}^s \\ \nabla \times \mathbf{H}^s &= \mathbf{J}^{\text{eq}} + j\omega\varepsilon_0 \mathbf{E}^s \\ \mathbf{J}^{\text{eq}} &= j\omega \left(\underline{\boldsymbol{\varepsilon}} - \varepsilon_0 \underline{\mathbf{I}} \right) \cdot \mathbf{E} + j\omega\underline{\boldsymbol{\xi}} \cdot \mathbf{H} \quad , \\ \mathcal{M}^{\text{eq}} &= j\omega \left(\underline{\boldsymbol{\mu}} - \mu_0 \underline{\mathbf{I}} \right) \cdot \mathbf{H} + j\omega\underline{\boldsymbol{\zeta}} \cdot \mathbf{E} \end{aligned} \quad (6)$$

where $\underline{\mathbf{I}}$ is the identity dyadic [46–48].

Equation (6) is convenient in the sense that Maxwell's equations are now expressed in terms of the desired scattered fields. In addition, the object has been effectively removed, replaced with equivalent electric and magnetic volume currents that maintain the scattered fields in free space. However, the equivalent volume currents, required to determine the scattered fields, are expressed in terms of the unknown total electric and magnetic fields and, for a general object, cannot be analytically found.

Traditionally, this problem has been solved by formulating volume integral equations from the relations $\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s$ and $\mathbf{H} = \mathbf{H}^i + \mathbf{H}^s$, where the scattered fields are expressed in terms of the unknown total fields via the electric and magnetic vector potentials [45, 46, 48]. The object is then discretized into volumetric cells, the unknown total fields are then expanded in a suitable set of vector basis functions, and the resulting equations are then tested yielding a matrix equation which is inverted to find the basis function weights and ultimately the unknown fields [45, 48].

Bianisotropic Born approximations

While simple in theory, because the resulting matrix equation is dense and the edge lengths of the volumetric cells are typically less than $\lambda/10$, this approach is applicable to electrically small objects and is not commonly used in optics. However, if the object's material properties do not differ substantially from the surrounding vacuum (i.e., the object is a low-contrast scatterer), then

$$\begin{aligned} \mathbf{J}^{\text{eq}} &\approx j\omega\varepsilon_0 \left(\underline{\boldsymbol{\varepsilon}}_r - \underline{\mathbf{I}} \right) \cdot \mathbf{E}^i + j\omega\underline{\boldsymbol{\xi}} \cdot \mathbf{H}^i \\ \mathcal{M}^{\text{eq}} &\approx j\omega\mu_0 \left(\underline{\boldsymbol{\mu}}_r - \underline{\mathbf{I}} \right) \cdot \mathbf{H}^i + j\omega\underline{\boldsymbol{\zeta}} \cdot \mathbf{E}^i \end{aligned} \quad (7)$$

making approximate analytical expressions for the scattered fields possible. These approximations for the equivalent electric and magnetic volume currents are the Born approximations for a bianisotropic object and are used in the theoretical analysis to follow. Note that while these relations are derived in the exact same manner as the traditional Born approximation relations found throughout the literature [19, 20, 45, 49], to the author's knowledge, the bianisotropic Born approximation relations have not been reported elsewhere.

Far-zone scattered electric field

The incident electric field in the source plane takes the form

$$\mathbf{E}^i(\boldsymbol{\eta}, v=0) = \hat{\mathbf{x}}E_u^i(\boldsymbol{\eta}) + \hat{\mathbf{z}}E_w^i(\boldsymbol{\eta}). \quad (8)$$

Utilizing the plane-wave spectrum representation of electromagnetic fields [50], the incident electric and magnetic fields at any point in space $v \geq 0$ are

$$\begin{aligned} \mathbf{E}^i(\mathbf{s}) &= \frac{1}{(2\pi)^2} \iint_{\Gamma} \tilde{\mathbf{E}}^i(k_u^i, k_w^i) e^{-j\mathbf{k}^i \cdot \mathbf{s}} dk_u^i dk_w^i \\ \mathbf{H}^i(\mathbf{s}) &= \frac{1}{(2\pi)^2} \iint_{\Gamma} \frac{\mathbf{k}^i \times \tilde{\mathbf{E}}^i}{\omega\mu_0} e^{-j\mathbf{k}^i \cdot \mathbf{s}} dk_u^i dk_w^i \\ \tilde{\mathbf{E}}^i &= \left(\hat{\mathbf{x}} - \hat{\mathbf{y}} \frac{k_u^i}{k_v^i} \right) \tilde{E}_u^i + \left(\hat{\mathbf{z}} - \hat{\mathbf{y}} \frac{k_w^i}{k_v^i} \right) \tilde{E}_w^i \\ \tilde{E}_{u,w}^i &= \int_{-\infty}^{\infty} E_{u,w}^i(u, w) e^{jk_u^i u} e^{jk_w^i w} du dw \end{aligned} \quad (9)$$

where Γ represents the region $(k_u^i)^2 + (k_w^i)^2 \leq k^2$ ($k = 2\pi/\lambda$ is the wavenumber). Note that by integrating over this finite region, as opposed to $-\infty$ to ∞ , any evanescent wave contributions to the scattered fields are neglected. Thus, the source plane is assumed to be far enough away from the object such that this approximation is valid. This condition is typically met in practice.

The scattered electric field in the far zone is found by utilizing the far-zone vector potentials $\mathbf{N}$ and $\mathbf{L}$ [51]:

$$\begin{aligned} \mathbf{E}^s(\mathbf{r}) &\approx \frac{-j k e^{-jk r}}{4\pi r} \\ &\left[\left(\hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}} + \hat{\boldsymbol{\phi}}\hat{\boldsymbol{\phi}} \right) \cdot \sqrt{\frac{\mu_0}{\epsilon_0}} \mathbf{N} + \left(\hat{\boldsymbol{\theta}}\hat{\boldsymbol{\phi}} - \hat{\boldsymbol{\phi}}\hat{\boldsymbol{\theta}} \right) \cdot \mathbf{L} \right] \\ \mathbf{N}(\mathbf{r}) &= \iiint_{-\infty}^{\infty} \mathbf{J}^{\text{eq}}(\mathbf{r}') V(\mathbf{r}') e^{jk\hat{\mathbf{r}} \cdot \mathbf{r}'} d^3 r' \\ \mathbf{L}(\mathbf{r}) &= \iiint_{-\infty}^{\infty} \mathbf{M}^{\text{eq}}(\mathbf{r}') V(\mathbf{r}') e^{jk\hat{\mathbf{r}} \cdot \mathbf{r}'} d^3 r' \end{aligned} \quad (10)$$

where $\mathbf{J}^{\text{eq}}$ and $\mathbf{M}^{\text{eq}}$ are given in (7) and V is a positive function ($\max V = 1$ and $\min V = 0$) which represents the volume of the object and captures the spatial dependence of $\underline{\epsilon}$, $\underline{\mu}$, $\underline{\xi}$, and $\underline{\zeta}$.

Substituting (7) and (9) into (10), changing to spherical

coordinates, and simplifying yields

$$\begin{aligned} \mathbf{E}^s(\mathbf{r}) &= \mathbf{E}_e^s(\mathbf{r}) + \mathbf{E}_h^s(\mathbf{r}) \\ \mathbf{E}_e^s &= \frac{k^2}{(2\pi)^2} \frac{e^{-jk r}}{4\pi r} \mathbf{P}_e(\theta, \phi) \\ &\cdot \int_0^\pi \int_0^\pi \tilde{\mathbf{E}}^i(\alpha, \beta) \sin^2 \alpha \sin \beta e^{jk f(\alpha, \beta)} \\ &\mathcal{I}(\alpha, \beta, \theta, \phi) d\alpha d\beta \\ \mathbf{E}_h^s &= \frac{k^2}{(2\pi)^2} \frac{e^{-jk r}}{4\pi r} \mathbf{P}_h(\theta, \phi) \\ &\cdot \int_0^\pi \int_0^\pi \frac{\mathbf{k}^i \times \tilde{\mathbf{E}}^i(\alpha, \beta)}{k} \sin^2 \alpha \sin \beta e^{jk f(\alpha, \beta)} \\ &\mathcal{I}(\alpha, \beta, \theta, \phi) d\alpha d\beta \end{aligned} \quad (11)$$

where f , $\mathcal{I}$, $\mathbf{P}_e$, and $\mathbf{P}_h$ are

$$\begin{aligned} f &= \sin \alpha (x_s \cos \beta - y_s \sin \beta) + z_s \cos \alpha \\ \mathcal{I} &= \iiint_{-\infty}^{\infty} V(\mathbf{r}') e^{j(k\hat{\mathbf{r}} - \mathbf{k}^i) \cdot \mathbf{r}'} d^3 r' \\ \mathbf{P}_e &= k^2 \left(\hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}} + \hat{\boldsymbol{\phi}}\hat{\boldsymbol{\phi}} \right) \cdot \left(\underline{\epsilon}_r - \underline{\mathbf{I}} \right) \\ &+ \omega k \left(\hat{\boldsymbol{\theta}}\hat{\boldsymbol{\phi}} - \hat{\boldsymbol{\phi}}\hat{\boldsymbol{\theta}} \right) \cdot \underline{\zeta} \\ \mathbf{P}_h &= \omega k \left(\hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}} + \hat{\boldsymbol{\phi}}\hat{\boldsymbol{\phi}} \right) \cdot \underline{\xi} \\ &+ k^2 \left(\hat{\boldsymbol{\theta}}\hat{\boldsymbol{\phi}} - \hat{\boldsymbol{\phi}}\hat{\boldsymbol{\theta}} \right) \cdot \left(\underline{\mu}_r - \underline{\mathbf{I}} \right) \end{aligned} \quad (12)$$

Although complicated, (11) is quite physical. Because of the superposition principle, the scattered electric field can be decomposed into two contributions—one arising from the incident electric field, $\mathbf{E}_e^s$, and the other stemming from the incident magnetic field, $\mathbf{E}_h^s$. These scattered field vectors are the dot products of the e and h polarization dyadics ($\mathbf{P}_e$ and $\mathbf{P}_h$, respectively) with the incident electric and magnetic fields (expressed as integrals over their plane-wave spectra) evaluated over the volume of the object. Note that if the object is naturally anisotropic ($\underline{\xi} = \underline{\zeta} = 0$ and $\underline{\mu} = \mu_0 \underline{\mathbf{I}}$, e.g., mica, calcite, quartz, etc.) rather than bianisotropic, the scattered electric field arises only from the incident electric field as one would physically expect for a nonmagnetic object.

Far-zone scattered-field CSD matrix

Taking the autocorrelation of the scattered electric field in (11) and simplifying yields

$$\begin{aligned}
\mathbf{W}^s(\mathbf{r}_1, \mathbf{r}_2, \omega) &= \langle \mathbf{E}_1^s \mathbf{E}_2^{s\dagger} \rangle \\
&= \frac{k^4}{(2\pi)^4} \frac{e^{-jk(r_1-r_2)}}{(4\pi)^2 r_1 r_2} \\
&\quad \left(\mathbf{P}_{e,1} \cdot \mathbf{S}_{ee} \cdot \mathbf{P}_{e,2}^\dagger + \mathbf{P}_{e,1} \cdot \mathbf{S}_{eh} \cdot \mathbf{P}_{h,2}^\dagger \right. \\
&\quad \left. + \mathbf{P}_{h,1} \cdot \mathbf{S}_{he} \cdot \mathbf{P}_{e,2}^\dagger + \mathbf{P}_{h,1} \cdot \mathbf{S}_{hh} \cdot \mathbf{P}_{h,2}^\dagger \right) \\
&\quad \mathbf{S}_{ee}(\theta_1, \phi_1, \theta_2, \phi_2) \\
&= \int_0^\pi \int_0^\pi \int_0^\pi \langle \tilde{\mathbf{E}}_1^i \tilde{\mathbf{E}}_2^{i\dagger} \rangle \sin^2 \alpha_1 \sin^2 \alpha_2 \\
&\quad \sin \beta_1 \sin \beta_2 e^{jk(f_1-f_2)} \mathcal{I}_1 \mathcal{I}_2^* d\alpha_1 d\beta_1 d\alpha_2 d\beta_2 \\
&\quad \mathbf{S}_{eh}(\theta_1, \phi_1, \theta_2, \phi_2) \\
&= \int_0^\pi \int_0^\pi \int_0^\pi \left\langle \tilde{\mathbf{E}}_1^i \left(\frac{\mathbf{k}_2^i \times \tilde{\mathbf{E}}_2^i}{k} \right)^\dagger \right\rangle \sin^2 \alpha_1 \sin^2 \alpha_2 \\
&\quad \sin \beta_1 \sin \beta_2 e^{jk(f_1-f_2)} \mathcal{I}_1 \mathcal{I}_2^* d\alpha_1 d\beta_1 d\alpha_2 d\beta_2
\end{aligned} \tag{13}$$

where the subscripts “1” and “2” on the functions f and $\mathcal{I}$ (and on the electric field vectors and polarization dyadics) denote that those expressions are evaluated at (α_1, β_1) , (α_2, β_2) , (θ_1, ϕ_1) , or (θ_2, ϕ_2) whichever is/are applicable. The expressions for $\mathbf{S}_{he}$ and $\mathbf{S}_{hh}$ are similar to those for $\mathbf{S}_{ee}$ and $\mathbf{S}_{eh}$ and therefore omitted for brevity. Note that the exact form of the scattered-field CSD matrix or dyadic is

$$\begin{aligned}
\mathbf{W}^s &= \hat{\boldsymbol{\theta}}_1 W_{\theta\theta}^s \hat{\boldsymbol{\theta}}_2 + \hat{\boldsymbol{\theta}}_1 W_{\theta\phi}^s \hat{\boldsymbol{\phi}}_2 \\
&\quad + \hat{\boldsymbol{\phi}}_1 W_{\phi\theta}^s \hat{\boldsymbol{\theta}}_2 + \hat{\boldsymbol{\phi}}_1 W_{\phi\phi}^s \hat{\boldsymbol{\phi}}_2
\end{aligned} \tag{14}$$

In the far zone, θ and ϕ polarizations are essentially equivalent to vertical and horizontal polarizations, respectively. Therefore, $W_{\theta\theta}^s \approx W_{VV}^s$ and $W_{\phi\phi}^s \approx W_{HH}^s$ are the traditional co-polarization terms and $W_{\theta\phi}^s \approx W_{VH}^s$ and $W_{\phi\theta}^s \approx W_{HV}^s$ are the cross-polarization terms.

As expected from the form of (11), the scattered-field CSD matrix can be expressed as the sum of four terms—each term is the dot product of two polarization dyadics and one autocorrelation dyadic. These terms are in the form of coordinate transformations. For instance, the $\mathbf{P}_{e,1} \cdot \mathbf{S}_{eh} \cdot \mathbf{P}_{h,2}^\dagger$ term

transforms the eh -autocorrelation dyadic from h -polarization to e -polarization. The other terms are interpreted in a similar manner. The polarization changes due to the bianisotropic object are completely captured in the polarization dyadics.

The autocorrelation dyadics, $\mathbf{S}_{ee}$, $\mathbf{S}_{eh}$, $\mathbf{S}_{he}$, and $\mathbf{S}_{hh}$, contain different forms of the plane-wave-spectrum incident-field CSD matrix [related to (1) and (2) via (9)] and are expressed as integrals over the plane-wave spherical angles α and β . When one considers the form of the volume integral

$\mathcal{I}$ given in (12) (the far-zone diffraction pattern of the object), the integrals comprising the autocorrelation terms are very similar to convolution integrals, where the Fourier transform of the product of the incident-field spectral density S and correlation function γ are convolved with $\mathcal{I}_1$ and $\mathcal{I}_2$. If the Fourier transform of the S - γ product is “narrower” than $\mathcal{I}_1 \mathcal{I}_2^*$, the lobing structure of the far-zone diffraction pattern [common examples are $\text{sinc}(x)$ and $\text{jinc}(x)$] is conspicuous. This condition occurs when the incident field is spatially coherent and fully illuminates the object. On the other hand, if the Fourier transform of the S - γ product is “broader” than $\mathcal{I}_1 \mathcal{I}_2^*$, the far-zone diffraction pattern is smoothed out. This condition occurs when the incident field is spatially incoherent or does not fully illuminate the object.

Approximations and simplifications

The far-zone scattered-field CSD matrix shown in (13) is very general. The only approximation made thus far has been that the Born approximations for the volume currents hold. To further simplify and more easily interpret (13), forms for the volume function V and incident-field CSD matrix elements W_{ab}^i must be chosen.

Volume function V —Very few choices for V yield closed-form expressions for $\mathcal{I}$. One V that does is a rectangular prism, i.e.,

$$\begin{aligned}
V(\mathbf{r}') &= \text{rect}\left(\frac{x'}{W_x}\right) \text{rect}\left(\frac{y'}{W_y}\right) \text{rect}\left(\frac{z'}{W_z}\right) \\
\text{rect}(x) &= \begin{cases} 1 & |x| < 1/2 \\ 1/2 & |x| = 1/2 \\ 0 & \text{otherwise} \end{cases}, \tag{15}
\end{aligned}$$

where W_x , W_y , and W_z are the rectangular prism lengths in the x , y , and z directions, respectively [52]. Substituting (15) into the expression for $\mathcal{I}$ (12) and evaluating the integral yields

$$\begin{aligned}
\mathcal{I}(\alpha, \beta, \theta, \phi) &= \prod_{m=x,y,z} W_m \\
&\quad \text{sinc}\left[\frac{W_m}{2} \hat{\mathbf{m}} \cdot (k\hat{\mathbf{r}} - \mathbf{k}^i)\right].
\end{aligned} \tag{16}$$

This $\mathcal{I}$ is used in the analysis to follow.

It should be noted that recently, Korotkova *et al.* introduced volume functions expressed as the weighted sums of three-dimensional Gaussian functions (termed multi-Gaussian distributions) [53]. This permits one to compute closed-form expressions for $\mathcal{I}$ (in the form of summations) for a wide variety of object shapes, e.g., cylinders, spheres, and ellipsoids. This approach does have the drawback that it yields objects with semi-hard boundaries and thus, is approximate. Of course, the semi-hard-edged objects better approximate the true hard-edged objects as more terms in the summations are retained.

Incident-field CSD matrix elements W_{ab}^i —The author knows of no choice for W_{ab}^i that yields closed-form expressions for the autocorrelation dyadics. Therefore, to further simplify (13), a statistically homogeneous and isotropic form for the incident-field CSD matrix elements is assumed:

$$W_{ab}^i(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2) = A_a A_b B_{ab} \exp\left(-\frac{|\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2|^2}{2\delta_{ab}^2}\right), \tag{17}$$

where $A_{a,b}$ is the magnitude of the a, b field component, B_{ab} is the complex correlation coefficient between the a and

b field components, δ_{ab} is, in general, the cross-correlation radius of the Gaussian-shaped cross-correlation function, and $a, b = x, z$. The values of these parameters are subject to the realizability constraints detailed in [19, 20, 54].

Equation (17) implies that the incident field is a partially-coherent plane wave. Its choice allows two of the four plane-wave spectrum integrals comprising the autocorrelation dyadics to be evaluated trivially. It also permits one to examine the effects incident-field coherence has on far-zone scattered irradiance and polarization without having to consider incident-field shape, i.e., beam shape. Note that equivalent expressions for the autocorrelation dyadics, to those obtained by using (17), would be derived by utilizing the popular Gaussian Schell-model form for W_{ab}^i [19, 20] and then evaluating two of the plane-wave spectrum integrals asymptotically assuming that the incident-field spectral density widths become very large. This approach was utilized by Fischer *et al.* in analyzing the effects of field coherence on the far-zone scattering from spheres [32, 33].

In general, the CSD matrices comprising $\underline{\mathcal{S}}_{ee}$, $\underline{\mathcal{S}}_{eh}$, $\underline{\mathcal{S}}_{he}$, and $\underline{\mathcal{S}}_{hh}$ are full 3×3 matrices because the incident-field plane-wave spectrum (9) has a y component (i.e., the mean propagation direction). Physical intuition dictates that this longitudinal component should be much smaller than the transverse x and z spectrum components. The mathematical form of the incident-field plane-wave spectrum lends credence to this intuition considering that the y component of the spectrum is divided by k_v^i , which is the wavenumber in the mean propagation direction. It, therefore, is assumed that all autocorrelation dyadic terms possessing a y component are negligible. This has the physical implication that the incident field is TEM.

This approximation was verified by utilizing (17) to compute the full 3×3 plane-wave-spectrum incident-field CSD matrices which appear in (13) and then comparing the magnitudes of the elements possessing a y component to those possessing only x and z components. This analysis was performed assuming a “worst-case” incident field in terms of coherence, i.e., $\delta = \lambda$ [19], as this required the broadest spectrum of wavenumbers. The results showed that even in this “worst case,” CSD matrix elements containing only transverse components were approximately six times greater than elements containing a y component. Thus, the approximation that the y component of the incident-field plane-wave spectrum is negligible is justified and henceforth applied.

Simplified far-zone scattered-field CSD matrix—Utilizing the assumptions and approximations discussed above, the simplified

field $\underline{W}^s$ becomes

$$\begin{aligned} \underline{W}^s(\mathbf{r}_1, \mathbf{r}_2, \omega) &\approx \frac{k^2}{(2\pi)^2} \frac{e^{-jk(r_1-r_2)}}{(4\pi)^2} \\ &\left(\underline{P}_{e,1} \cdot \underline{\mathcal{S}}_{ee} \cdot \underline{P}_{e,2}^\dagger + \underline{P}_{e,1} \cdot \underline{\mathcal{S}}_{eh} \cdot \underline{P}_{h,2}^\dagger \right. \\ &\quad \left. + \underline{P}_{h,1} \cdot \underline{\mathcal{S}}_{he} \cdot \underline{P}_{e,2}^\dagger + \underline{P}_{h,1} \cdot \underline{\mathcal{S}}_{hh} \cdot \underline{P}_{h,2}^\dagger \right) \\ \underline{\mathcal{S}}_{ee} &\approx \hat{x}\mathcal{S}_{ee,xx}\hat{x} + \hat{x}\mathcal{S}_{ee,xz}\hat{z} \\ &\quad + \hat{z}\mathcal{S}_{ee,zx}\hat{x} + \hat{z}\mathcal{S}_{ee,zz}\hat{z} \\ \underline{\mathcal{S}}_{eh} &\approx \hat{x}\mathcal{S}_{eh,xz}\hat{x} - \hat{x}\mathcal{S}_{eh,xx}\hat{z} \\ &\quad + \hat{z}\mathcal{S}_{eh,zz}\hat{x} - \hat{z}\mathcal{S}_{eh,xz}\hat{z} \\ \underline{\mathcal{S}}_{he} &\approx \hat{x}\mathcal{S}_{eh,xz}\hat{x} + \hat{x}\mathcal{S}_{eh,zz}\hat{z} \\ &\quad - \hat{z}\mathcal{S}_{eh,xx}\hat{x} - \hat{z}\mathcal{S}_{eh,xz}\hat{z} \\ \underline{\mathcal{S}}_{hh} &\approx \hat{x}\mathcal{S}_{hh,zz}\hat{x} - \hat{x}\mathcal{S}_{hh,xz}\hat{z} \\ &\quad - \hat{z}\mathcal{S}_{hh,xz}\hat{x} + \hat{z}\mathcal{S}_{hh,xx}\hat{z} \\ \mathcal{S}_{ee,ab} &= \delta_{ab}^2 A_a A_b B_{ab} \int_0^\pi \int \sin^4 \alpha \sin^2 \beta \mathcal{I}_1 \mathcal{I}_2^* \\ &\quad \exp \left[-\frac{1}{2} k^2 \delta_{ab}^2 (\sin^2 \alpha \cos^2 \beta + \cos^2 \alpha) \right] d\alpha d\beta \\ \mathcal{S}_{eh,ab} &= \delta_{ab}^2 A_a A_b B_{ab} \int_0^\pi \int \sin^5 \alpha \sin^3 \beta \mathcal{I}_1 \mathcal{I}_2^* \\ &\quad \exp \left[-\frac{1}{2} k^2 \delta_{ab}^2 (\sin^2 \alpha \cos^2 \beta + \cos^2 \alpha) \right] d\alpha d\beta \\ \mathcal{S}_{hh,ab} &= \delta_{ab}^2 A_a A_b B_{ab} \int_0^\pi \int \sin^6 \alpha \sin^4 \beta \mathcal{I}_1 \mathcal{I}_2^* \\ &\quad \exp \left[-\frac{1}{2} k^2 \delta_{ab}^2 (\sin^2 \alpha \cos^2 \beta + \cos^2 \alpha) \right] d\alpha d\beta \end{aligned} \quad (18)$$

where $\mathcal{I}_{1,2} = \mathcal{I}(\alpha, \beta, \theta_{1,2}, \phi_{1,2})$ and is given in (16) and $\mathcal{S}_{eh,ab} = \mathcal{S}_{he,ab}$. All other symbols have been previously defined. This form for $\underline{W}^s$ will be utilized in the following section.

3. RESULTS

Preliminaries

In this section, the scattered Stokes parameters for two anisotropic cubes are presented. The Stokes parameters in terms of the scattered-field CSD matrix elements are [19, 20]

$$\begin{aligned} S_0(\mathbf{r}, \omega) &= W_{HH}^s(\mathbf{r}, \mathbf{r}, \omega) + W_{VV}^s(\mathbf{r}, \mathbf{r}, \omega) \\ S_1(\mathbf{r}, \omega) &= W_{HH}^s(\mathbf{r}, \mathbf{r}, \omega) - W_{VV}^s(\mathbf{r}, \mathbf{r}, \omega) \\ S_2(\mathbf{r}, \omega) &= W_{HV}^s(\mathbf{r}, \mathbf{r}, \omega) + W_{VH}^s(\mathbf{r}, \mathbf{r}, \omega) \\ S_3(\mathbf{r}, \omega) &= j[W_{VH}^s(\mathbf{r}, \mathbf{r}, \omega) - W_{HV}^s(\mathbf{r}, \mathbf{r}, \omega)] \end{aligned} \quad (19)$$

The α and β integrals comprising the autocorrelation dyadics in (18) were computed using numerical quadrature.

The first anisotropic cube was a dielectric biaxial cube with

Table 1. Incident-Field Stokes Parameters and Correlation Lengths

Full Linear (1 1 0 0) ^T			
Case	δ_{xx}	δ_{zz}	δ_{xz}
I	0.25L	0.25L	N/A
II	0.50L	0.50L	N/A
III	1.00L	1.00L	N/A

Unpolarized (1 0 0 0) ^T			
Case	δ_{xx}	δ_{zz}	δ_{xz}
I	0.25L	0.38L	N/A
II	0.50L	0.75L	N/A
III	1.00L	1.50L	N/A

Partial Elliptical (1 0.26 0.24 0.42) ^T			
Case	δ_{xx}	δ_{zz}	δ_{xz}
I	0.25L	0.38L	0.39L
II	0.50L	0.75L	0.78L
III	1.00L	1.50L	1.56L

relative permittivity tensor

$$\underline{\epsilon}_r = \begin{pmatrix} 1.20 & 0 & 0 \\ 0 & 1.35 & 0 \\ 0 & 0 & 1.50 \end{pmatrix}. \quad (20)$$

This relatively simple material was selected so that the scattered Stokes parameters could be easily compared with physical intuition. The second cube was a bianisotropic cube with the following material tensors characteristic of a SRR metamaterial [8]:

$$\begin{aligned} \underline{\epsilon}_r &= \begin{pmatrix} 1.50 & 0 & 0 \\ 0 & 1.35 & 0 \\ 0 & 0 & 1.20 \end{pmatrix} \\ \underline{\mu}_r &= \begin{pmatrix} 1.20 & 0 & 0 \\ 0 & 1.10 & 0 \\ 0 & 0 & 1.15 \end{pmatrix} \\ \underline{\xi} &= \sqrt{\epsilon_0 \mu_0} \begin{pmatrix} 0 & 0 & -j0.50 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \underline{\zeta} &= \underline{\xi}^\dagger \end{aligned} \quad (21)$$

Again, the simple forms of these tensors were chosen so that the results could be easily compared with intuition. In the scattered-field CSD matrix expressions reported in (13) and (18), $\underline{\epsilon}_r$, $\underline{\mu}_r$, $\underline{\xi}$, and $\underline{\zeta}$ can take any form.

Each $L \times L \times L$ ($L = 10\lambda$) anisotropic cube was illuminated by nine incident fields with different polarization and coherence properties. The nine incident fields were organized by polarization state into three groups. In each polarization group, three different values for δ_{xx} , δ_{zz} , and δ_{xz} were chosen—in the results to follow, cases I, II, and III, respectively. Table 1 reports the Stokes parameters and spatial correlation lengths for each of the nine incident fields. Note that the $10\lambda \times 10\lambda \times 10\lambda$ cube size was chosen so that coherence's role in the far-zone diffraction patterns was clearly visible. The theory presented above is applicable to objects of any size with the condition that macroscopic electromagnetics holds.

Discussion of results

Dielectric biaxial cube—Figure 2 shows the in-plane ($\theta = 90^\circ$) scattered Stokes parameters for the dielectric biaxial cube versus ϕ . Each row of plots corresponds to an incident-field polarization group defined in Table 1—(a) and (b) (1 1 0 0)^T, (c) and (d) (1 0 0 0)^T, and (e) and (f) (1 0.26 0.24 0.42)^T. The left and right columns of plots show the results on linear and log scale, respectively. The log scale results are included to clearly show the lobing structures of the far-zone patterns. The color of the line denotes the Stokes parameter, i.e., black is S_0 , red is S_1 , blue is S_2 , and green is S_3 . The line symbol denotes the coherence case, i.e., none (solid) is case I, triangle is case II, and square is case III.

Considering coherence, the results in Fig. 2 are in complete agreement with intuition. In the case I results, δ_{xx} , δ_{zz} , and δ_{xz} are significantly less than L —effectively an incoherent incident field. As expected, the scattered sinc(x) patterns are smoothed out. In the case II results, the incident field is more coherent than in case I; yet, δ_{xx} , δ_{zz} , and δ_{xz} are still less than L . The scattered sinc(x) patterns are still smoothed out; however, null locations are generally discernable. In the case III results, δ_{xx} , δ_{zz} , and δ_{xz} are greater than or equal to L . The scattered sinc(x) patterns are recovered.

Considering polarization, the results in Fig. 2 are in good agreement with physical intuition. The incident field for the first polarization group [Fig. 2(a) and (b)] is linearly fully polarized in the x direction. Maxwell's equations (4) and the form of the relative permittivity tensor (20) dictate that only the $\epsilon_{r,xx}$ element contributes to the scattered field. Thus, the object behaves as if it were isotropic and the incident polarization state is maintained upon scattering.

The incident field for the second polarization group [Fig. 2(c) and (d)] is randomly polarized. In this scenario, both $\epsilon_{r,xx}$ and $\epsilon_{r,zz}$ contribute to the scattered field. The horizontal and vertical polarization components of the field are uncorrelated in the case of randomly polarized light; thus, only the scattering-induced amplitude changes to the horizontal and vertical field components are relevant. Since $\epsilon_{r,zz} > \epsilon_{r,xx}$ and only in-plane scattering is considered here, the resulting scattered field is vertically partially polarized in complete agreement with intuition.

The results for the third and final polarization group [Fig. 2(e) and (f)] are included to show the scattered Stokes parameters for a general elliptically partially polarized incident field. As one would expect, the scattered field is elliptically partially polarized. Note that the degree of polarization is different in these results depending on the coherence case. This is evidence of coherence-induced polarization effects [19, 20].

Bianisotropic cube—Figure 3 shows the in-plane ($\theta = 90^\circ$) scattered Stokes parameters for the bianisotropic (SRR metamaterial) cube versus ϕ . The layout of the figure is identical to Fig. 2. The interpretations of the results in Fig. 3 are very similar to those presented above for the dielectric biaxial cube; thus, only a brief discussion of the findings is presented here.

Considering only coherence, the results are in complete agreement with physical intuition. In the least-coherent incident-field case (case I), the scattered sinc(x) patterns are smoothed out, resulting in far-zone diffraction patterns that resemble the Gaussian-shaped correlation functions in (18). As the incident field becomes more coherent (cases II and III),

Figure 2. In-plane ($\theta = 90^\circ$) scattered Stokes parameters for the dielectric biaxial cube. Each row of plots corresponds to an incident-field polarization group defined in Table 1. The left and right columns of plots show the results on linear and log scale, respectively. The Stokes parameters are demarcated by line color; the coherence cases are demarcated by line symbol.

the scattered sinc (x) patterns are recovered.

For the horizontally fully polarized results [Fig. 3(a) and (b)], $\varepsilon_{r,xx}$ and ξ_{xz} contribute to the x -directed scattered electric field, while $\mu_{r,zz}$ and ζ_{zx} contribute to the z -directed scattered magnetic field. This field orientation is identical to the incident field's orientation and therefore, the horizontal linear polarization state is maintained.

For the unpolarized incident field results [Fig. 3(c) and (d)], $\varepsilon_{r,xx}$, $\varepsilon_{r,zz}$, $\mu_{r,xx}$, $\mu_{r,zz}$, ξ_{xz} , and ζ_{zx} contribute to the scattered field. In the case of unpolarized light, the horizontal and vertical components of the field are uncorrelated and therefore, only scattering-induced amplitude changes to the respective field components are germane. Because of the complicated forms of the e and h polarization dyadics (both of which are implicated here, unlike in the biaxial dielectric cube case), it is difficult to intuitively infer whether the scattered field will be horizontally or vertically partially polarized. Nevertheless, intuition dictates that the scattered field be partially polarized, which Fig. 3(c) clearly shows.

For the general elliptically partially polarized results [Fig. 3(e) and (f)], the scattered field is, not surprisingly, elliptically partially polarized. As is the case for the dielectric biaxial cube results above, the degree of polarization is different depending on the coherence case and is evidence of coherence-induced polarization effects.

4. CONCLUSION

In this paper, the scattering of a partially-coherent electromagnetic field from a bianisotropic object was investigated. Assuming a Schell-model form for the incident-field CSD matrix, the far-zone scattered-field CSD matrix was derived using the far-zone vector potentials and the bianisotropic Born approximations. The mathematical form of the scattered-field CSD matrix was very physical and was thoroughly discussed. The theoretical scattered Stokes parameters of two cubes (a dielectric biaxial cube and a bianisotropic cube), illuminated by incident fields with varying coherence and polarization properties, were presented and physically interpreted. The results demonstrated interesting coherence and polarization effects and were in agreement with physical intuition.

Beam-control techniques using designer materials (i.e., meta-materials) or coherence manipulation have been distinct research areas in optics. This work is a first step in merging the two fields and is relevant since it is likely that light control using a combination of advanced materials and coherence manipulation will be utilized in future applications.

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Figure 3. In-plane ($\theta = 90^\circ$) scattered Stokes parameters for the bianisotropic cube. Each row of plots corresponds to an incident-field polarization group defined in Table 1. The left and right columns of plots show the results on linear and log scale, respectively. The Stokes parameters are demarcated by line color; the coherence cases are demarcated by line symbol.

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