An Improved Hybrid Interferometer

Robert P. Penno, Ph.D.
University of Dayton
Dayton, Ohio, 45469-0226
(937) 229 3984
Robert.Penno@notes.udayton.edu

Krishna M. Pasala, Ph.D.
University of Dayton
Dayton, Ohio 45469-0226
(937) 229 2683
Krishna.Pasala@notes.udayton.edu

Stephen Schneider, Ph.D.
AFRL/SNRR
Wright Patterson Air Force Base
WPAFB, OH 45433-7333
(937) 255 6427, X4329
Stephen.Schneider@afrl.af.mil

Abstract—In this paper[1,2,3], three alternatives to AOA determination are considered: the Hybrid Interferometer using MUSIC resolution, the Hybrid Interferometer using the comparison approach to resolution and the Hybrid-MUSIC approach. It is seen that the Hybrid Interferometer (MUSIC) is more accurate than Hybrid Interferometer (comparison). On the other hand, Hybrid Interferometer (MUSIC) is not as accurate as the Hybrid-MUSIC, but does not require excessive computational resources to achieve AOA estimates whose bias and standard deviations are much improved over other hybrid approaches as well as the linear phase interferometer.

TABLE OF CONTENTS

1 INTRODUCTION

There are many applications that require the estimation of the Angle of Arrival (AOA) of an unknown electromagnetic wave, or signal. Furthermore, such AOA estimates must be made over a wide band of signal frequencies. Typically, such estimates are made in the presence of a significant amount of corrupting noise that is introduced into the system both internally and externally. The classic approach to this problem has been the five-element Linear Phase Interferometer (LPI), which offers AOA estimates whose accuracy is derived from the length of the long baselines.[1-4] More recently, it has been shown that the ability to achieve the accuracy available from an LPI is also determined by the ability of the unambiguous, short pair to discern between the multiple ambiguous solutions obtained from the long baseline [5]. Additionally, the LPI is inherently effective over only a narrow band of frequencies. Hence, a number of LPI's, each operational over a narrow band of frequencies, is required for wide band operation. This results in a large number of antennas that form an antenna "farm", which requires a large amount of space on an airborne platform and a large amount of electronics to support each LPI. Thus, the sheer number of apertures required for wide band operation makes maintainability, reliability and application versatility, along with issues of cost, size and weight, major considerations.

With advances in receiver technology, coupled with the implementation of high speed computational power, alternatives to the LPI have emerged. The single, multi-mode antenna is a class of antennas that can produce AOA estimates at least as accurate as the short baseline pair of an LPI [6,7]. When such a multi-mode antenna is also of a class of antennas referred to as "frequency independent", the accuracy of these AOA estimates become invariant over a wide band of frequencies. The N-arm, spiral antenna is such an antenna, combining multi-mode characteristics over a wide band of frequencies. It was seen that such multi-mode antennas can provide accurate estimates of AOA using classical phase (and magnitude) comparison techniques. Such techniques require the use of mode-forming hardware such as a Butler matrix. To offset the expense of such mode-forming hardware, parameter estimation techniques were employed to produce AOA estimates at least as accurate as those using the mode-forming hardware. These parameter estimation techniques, such as Maximum Likelihood Method (MLM) and MUSIC [8], while eliminating the need for beam-forming hardware, require increased computational power and multi-channel receivers. These single aperture, N-arm spirals, however, could not match the accuracy of the LPI at its design frequency.

The Hybrid Interferometer [5] combines the advantages of multi-mode, frequency independent antennas such as the N-arm spiral, with the accuracy of the long baseline used in the LPI. Comprised of three, N-arm spiral antennas, the Hybrid Interferometer uses the separation between spiral antennas to produce ambiguous AOA estimates. These ambiguous estimates are then resolved by using AOA estimates obtained from the individual spirals using the Phase Comparison approach. Unlike the LPI, a single Hybrid Interferometer requires only three spirals, but operates over a wide band of frequencies. Like the LPI,

[1] U.S. Government work not protected by U.S. Copyright

[2] IEEEAC paper #1275, Version 6, Updated December 22, 2003

[3] This work was supported by AFRL/SNRR, WPAFB, OH

however, it was observed that for a fixed Signal-to-Noise Ratio (SNR), there was a long baseline separation beyond which the AOA estimates obtained from the individual spiral antennas cannot discern between adjacent ambiguous solutions. Likewise, for a fixed long baseline separation, there was an SNR below which accurate resolution of ambiguous solutions could not consistently be performed. This threshold exists for both the LPI and the Hybrid Interferometer, but the Hybrid Interferometer produced AOA estimates much more robust in the face of this thresholding.

To deal with the threshold effect, the signals from all three spirals were concatenated to form a single data matrix. Parameter estimation techniques were then applied to this composite data matrix. It was seen that this process, previously described as Hybrid-MUSIC, eliminated the threshold phenomenon, producing AOA estimates completely free of the effects of the ambiguities that were problematic with the LPI and the Hybrid Interferometer. Moreover, the AOA estimates of Hybrid-MUSIC [9] were much improved over a wide band of frequencies beyond those of the LPI and the Hybrid-Interferometer. Unfortunately, application of MUSIC to the three spiral interferometer requires a significant increase in computational power and more complex, expensive receiver technology. Three, 4-arm spirals, configured in Hybrid-MUSIC, would require a twelve-channel receiver operating on a twelve-row data matrix!

This work, then, seeks to find a middle ground that improves upon the Hybrid-Interferometer, which is itself an improvement on the LPI in terms of AOA accuracy and bandwidth consideration. At the same time, it seeks to avoid the computational and hardware requirements of Hybrid-MUSIC.

2 THEORY

2.1—AOA Determination Using a Single Spiral

The basic problem is described by Figure 1, where electromagnetic radiation is illuminating a single, N-arm spiral at unknown AOA, (θ_o, ϕ_o). It is well-documented [7] that the outputs of the modeformer associated with this spiral have the form

$$\mathbf{M}_\theta^n = \mathbf{C}_{n,0}\, \mathbf{j}^n \mathbf{e}^{-jn\phi_0}\left[\cos\theta_0\left(\mathbf{J}_{n-1}(n\sin\theta_0)+\mathbf{J}_{n+1}(n\sin\theta_0)\right)\right],$$
$$\mathbf{M}_\phi^n = \mathbf{C}_{n,0}\, \mathbf{j}^{n+1} \mathbf{e}^{-jn\phi_0}\left[\left(\mathbf{J}_{n+1}(n\sin\theta_0)-\mathbf{J}_{n-1}(n\sin\theta_0)\right)\right]. \quad (1)$$
$$\text{where } \mathbf{C}_{n,0} = \mathbf{E}_0 \mathbf{i}_{n,0}\left(\frac{n\lambda}{2\pi}\right)\square\pi$$

for θ-polarized and φ-polarized signals, respectively. Here, $J_n(x)$ is a Bessel function of the first kind of order n, and E_o is the strength of the incident signal. For both polarizations, the elevation information is contained in the magnitude, while azimuth information is contained in the phase of these modal outputs. Comparison of the magnitude and phase of these modal outputs provide an estimate of AOA. The frequency of operation of the spiral is determined by the radii over which the spiral elements comprising the spiral antenna retain their relative angular displacement. Hence, the spiral is wideband. Alternatively, it was seen that that using the terminal outputs directly, in conjunction with parameter estimation techniques, produced AOA estimates at least as accurate as those of the comparison method, but with improved coverage in the elevation. These parameter estimation approaches employed steering vectors derived from the field equation (1). The estimates of AOA $(\hat{\theta}_0, \hat{\phi}_o)$, obtained by using a single, N-arm spiral were seen to vary according to SNR. Figure (2) shows the variation of the standard deviation of AOA estimates with respect to SNR [7] for the comparison method, MUSIC and MLM. While the standard deviation of the azimuth estimates obtained from the 4-arm spiral are essentially the same for all three methods, the standard deviation of the MUSIC and MLM estimates of elevation angle obtained from the single, 4-arm spiral were much improved over those of the comparison method, especially in a high noise environment.

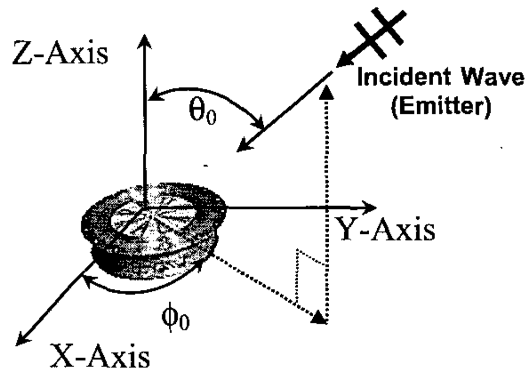
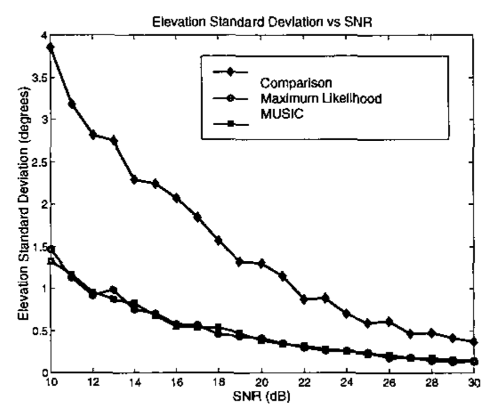


Figure 1: Plane Wave Illumination of an N-Arm



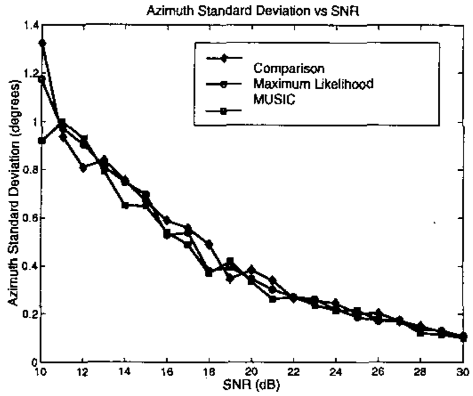


Figure 2: Standard Deviation of the Three Angle Estimators Applied to a 4-Arm Spiral

2.2—AOA Determination Using the Hybrid

Hybrid-MUSIC—The techniques described above use the phase difference between two widely spaced antennas to identify multiple ambiguous AOA estimates. A coarse estimate produced by one of two methods is then used to isolate the correct AOA solution. An alternative approach described in [9] involves concatenating the data matrices obtained from all three spirals in to a single data matrix. Let

$$\bar{\mathbf{x}}_i = \bar{\mathbf{g}}^n_{\theta,\phi} e^{j\psi_i} + \bar{\eta}, \quad i = 1,2,3 \tag{2}$$

where

$$\psi_i = \begin{cases} 0, & i = 1, \\ \beta d_{12}\sin(\theta)\cos(\phi), & i = 2, \\ \beta d_{13}\sin(\theta)\sin(\phi), & i = 3; \end{cases} \tag{3}$$

and

$$\begin{aligned} g^n_\theta &= [\cos(\theta_o)(J_{n+1}(n\sin(\theta_o)) + J_{n-1}(n\sin(\theta_o)))], \\ g^n_\phi &= [J_{n+1}(n\sin(\theta_o)) - J_{n-1}(n\sin(\theta_o)))] \end{aligned} \tag{4}$$

It is assumed that the noise present is white, Gaussian and zero-mean. The concatenated data vector, then, is given by

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{\mathbf{x}}_1 \\ \bar{\mathbf{x}}_2 \\ \bar{\mathbf{x}}_3 \end{bmatrix} \tag{5}$$

The autocorrelation matrix is

$$\mathbf{R}_{\bar{x}} = E\{\bar{\mathbf{x}}\bar{\mathbf{x}}^H\}, \tag{6}$$

The noise vectors, collected together in the noise matrix, Q_N, can be found accordingly. Applying the MUSIC technique to this problem produces the pseudospectrum

$$P_{MUSIC} = \frac{(\bar{g}(\theta_s,\phi_s)^H \bar{g}(\theta_s,\phi_s))}{\bar{g}(\theta_s,\phi_s)^H (Q_N(\theta,\phi)Q_N^H(\theta,\phi))\bar{g}(\theta_s,\phi_s)}. \tag{7}$$

The steering vector, $\bar{g}(\theta_s,\phi_s)$, corresponds to any possible AOA, (θ_s, ϕ_s); when this angle corresponds to the true AOA, (θ_o, ϕ_o), the scalar value of the pseudospectrum is a maximum.

Resolving the Ambiguity Using MUSIC—The accuracy of AOA estimates obtained from a single spiral are not as accurate as those obtained from a LPI operating at its design frequency. The LPI provides increased accuracy due to its use of a large separation between antennas. Thus, to incorporate the increased accuracy of the LPI, but over a wide band of frequencies, three spirals were arranged as shown in Figure (3). By establishing two long, perpendicular baselines, estimates of the AOA with respect to each baseline are determined. From these estimates, estimates of the unknown AOA (θ_o, ϕ_o) are determined.

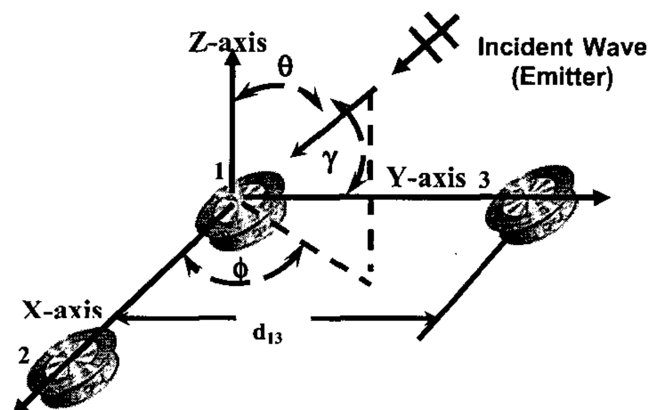


Figure 3: Three-Element Hybrid Interferometer

Since the N-arm spiral can be considered as an array of spiral wire elements, the phase difference between the k[th] pair of these elements is the k[th] measured phase difference. The average of these measured phase differences is denoted by χ, and is necessarily a number of modulo-2π. The large separation between antennas, however, would dictate that the actual value of phase difference, $\tilde{\chi}_o$, which also includes some noise, would be related to the measured value as

$$\tilde{\chi}_o = \chi + 2\hat{n}_o\pi = 2\pi\frac{d_{13}}{\lambda}v_o + \varepsilon, \quad \text{for } n = 0,\pm1,\pm2,\cdots; \; k = 1,2,\cdots,(N-1) \tag{8}$$

where ε is the measurement noise, $v_o = \sin(\theta_o)\sin(\phi_o)$, is the true value of the direction cosine and $\hat{n}_o$ is an estimate of the unknown integer, n_o, to be determined by resolving the phase ambiguity.

Another estimate of χ is available from the individual spirals, $\hat{\hat{\chi}}$. This estimate is much more coarse, but is unambiguous, and is given by

$$\hat{\hat{\chi}} = 2\pi \frac{d_{13}}{\lambda} \hat{v}_o . \tag{9}$$

where

$$\hat{v}_o = \sin(\hat{\theta}_0)\sin(\hat{\phi}_0) \tag{10}$$

An estimate of n_o is obtained from

$$\hat{n}_o = \frac{\hat{\hat{\chi}} - \chi}{2\pi} . \tag{11}$$

It should be noted that $\hat{n}_o, \hat{v}_o$ and $\hat{\hat{\chi}}$ are all random variables, displaying characteristics of a Gaussian distribution. When the distribution of $\hat{\hat{\chi}}$, obtained from the individual spiral antennas, is wider than the distribution of multiple, adjacent, ambiguous estimates of $\tilde{\chi}_o$, obtained from the phase differences between spirals, the value of n_o selected is prone to error. This results in erroneous estimates of v_o, and, therefore,γ. As was shown in Figure (2), the standard deviation of the estimates of elevation angle obtained from a single spiral using MUSIC or MLM are much improved over those obtained using the comparison approach. Lower standard deviation implies a narrower distribution. Moreover, the estimate of n_o obtained using MUSIC (or MLM) is more accurate, as is the resulting estimate of γ. Figure (4) compares the distributions of $\hat{n}_o$ obtained using the comparison and MUSIC approaches, for the case where SNR is 7 dB. The resulting estimates of γ are shown in Figure (5), where the comparison approach produces a significant number of estimates of γ, which are incorrect. Thus, it is expected that the AOA estimates obtained from the spirals using MUSIC will have improved bias, but, more importantly, will have much improved standard deviations over SNR, frequency and AOA than AOA estimates provide by the classic comparison technique.

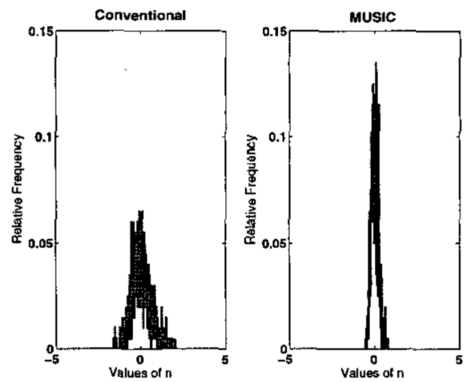


Figure 4: Distribution of N

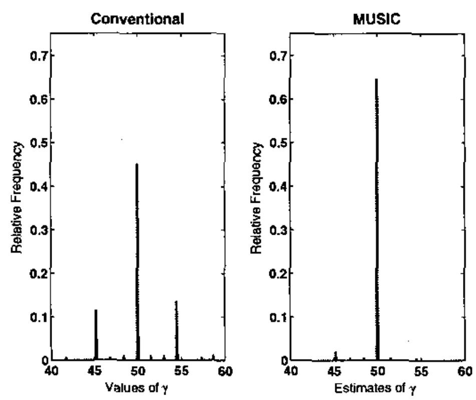


Figure 5: Distribution of Estimates of γ

3 RESULTS

Several simulations were run and evaluated to provide a basis of comparison for the three techniques described herein. These results were obtained using a computer simulation and Monte Carlo analysis techniques. Each test point was evaluated over 100 trials to establish convergent statistical data.

The first results shown in Figures (6)-(9), deal with the performance of these AOA determinations systems over a range of baselines, but for two different SNR values of 10 dB and 4 dB. Here, the AOA was fixed at θ=40° and ϕ=90°, equivalent to γ=50°. An alternative viewpoint is to describe the baseline as fixed at $d_{13} = 16.2\lambda_{10}$, where λ_{10} is one wavelength at 10 GHz. In this case, a variation of baselines from 5.2λ to 25.2λ is equivalent to a variation of frequency from 3.2 GHz to 15.5 GHz for a fixed d_{13}.

In Figures (6)-(7), the bias and standard deviations of AOA estimates are shown respectively for the case where SNR = 10 dB. These results reveal that the ambiguities associated with the AOA estimates obtained from the long baseline are resolved with the coarse estimates obtained from the spirals using either the comparison method or MUSIC.

Furthermore, there is a threshold beyond which unambiguous resolution occurs, and the threshold observed by resolution using MUSIC is higher than that achieved by resolution using the comparison method. In the case of the conventional comparison technique, this threshold occurs at approximately $d_{13} = 7.5\lambda$, while for the case of MUSIC resolution this threshold occurs at a much larger $d_{13} = 17.5\lambda$. Even beyond threshold it should be noted that accuracy of the AOA estimates using MUSIC is superior to that using the comparison approach. It is also noted that the application of MUSIC to the concatenated data matrix provides the best accuracy, and does not exhibit the threshold effect.

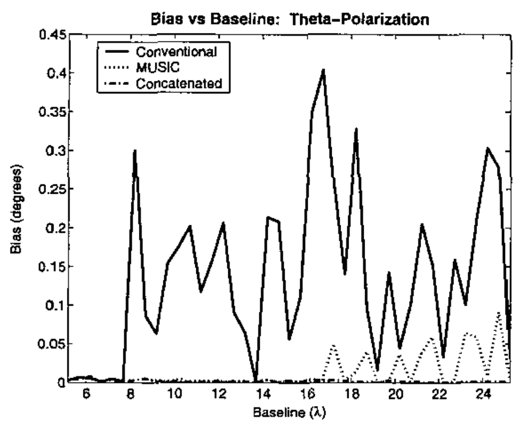


Figure 6: γ Bias vs. Baseline, SNR = 10dB

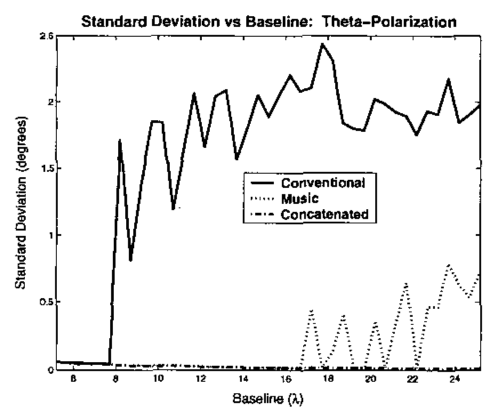


Figure 7: γ Standard Deviation vs. Baseline, SNR = 10dB

Figures (8) and (9) display the bias and standard deviations respectively for the identical case as Figures (6) and (7), but for SNR = 4 dB. For this case, the SNR and the range of baselines were chosen to explicitly avoid the threshold effect. As such, it is evident that the accuracy of AOA estimates (and the standard deviation of those estimates) obtained using MUSIC resolution is much improved over that obtained using the comparison approach.

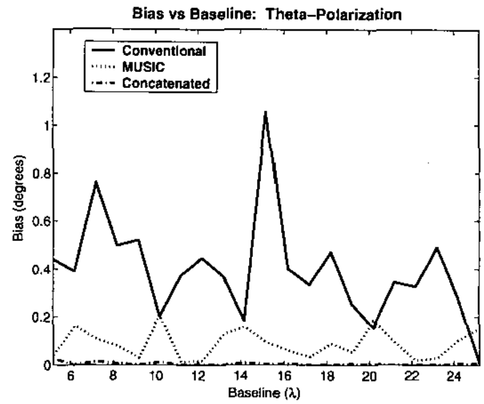


Figure 8: γ Bias vs. Baseline, SNR = 4dB

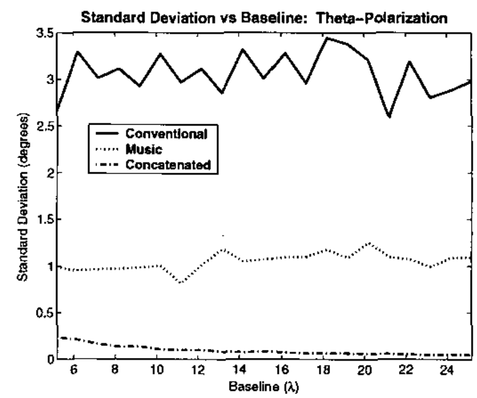


Figure 9: γ Standard Deviation vs. Baseline, SNR = 4dB

Figures (10) and (11) show the impact of varying SNR for a fixed frequency and baseline separation ($d_{13} = 16.2\lambda_{10}$). As SNR increases, less system noise is present, providing narrower distribution of AOA estimates from the spirals used to resolve the ambiguous Interferometric AOA estimates. However, an SNR threshold still exists, below which such resolution is not always possible. In the case of the comparison approach, this threshold occurs at approximately 18 dB, while for MUSIC resolution, it occurs at 9 dB.

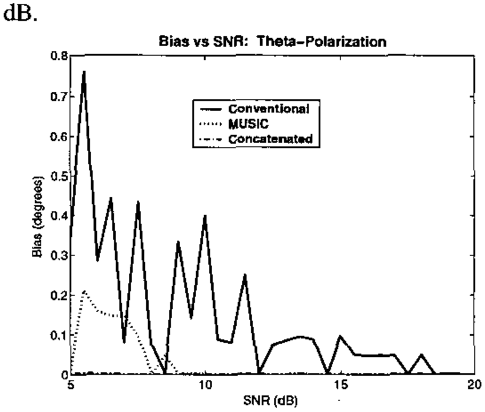


Figure 10: γ Bias vs. SNR

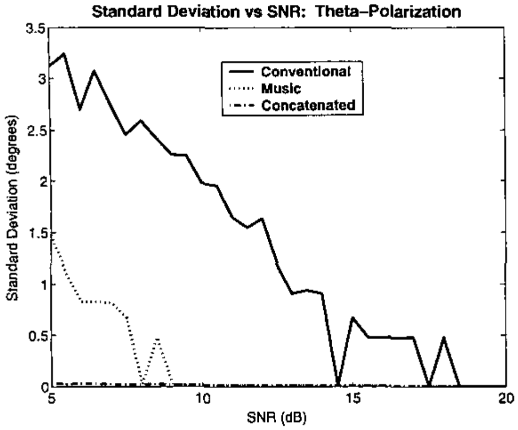


Figure 11: γ Standard Deviation vs. SNR

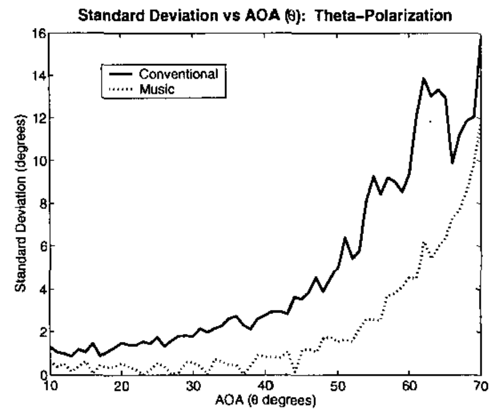


Figure 13: γ Standard Deviation vs. Elevation

Recognizing that omni directional elements are not being used, it is useful to evaluate the effectiveness of the comparison and MUSIC approaches to resolution for a variety of AOA's. In fact, the azimuthal symmetry of the fields pattern of the spiral, in conjunction with the arrangement of the spirals as a hybrid interferometer, make variation of the azimuth angle,ϕ, very uninteresting. The elevation angle, θ, poses different considerations, since the modal field patterns obtained from the individual spirals very considerably. In fact, it is this variation that provides the multi-mode spiral with so much of its special ability to estimate AOA. To examine this aspect, azimuth was fixed at ϕ=90°, rather the AOA was set in the y-z plane of Figure (3). For a fixed frequency (10GHz) and baseline (d_{13}=16. $2\lambda_{10}$), the elevation angle,θ, was varied from 10° to 70°. The case of θ=0°, coincides with boresight or an AOA incident from the +z axis, while the case of θ=90° is the case of grazing incidence. Figures (12) and (13) display the results of this comparison for the case of SNR = 10 dB. Clearly, MUSIC resolution outperforms comparison resolution, providing much lower bias and standard deviation statistics.

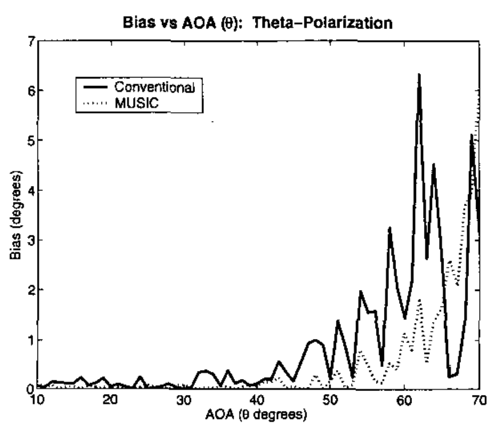


Figure 12: γ Bias vs. Elevation

4 CONCLUSIONS

In this work, three relatively innovative techniques to estimate the AOA of an unknown source have been considered. The first technique, the Hybrid Interferometer using the comparison method, had the worst accuracy, but also the lowest computational requirements. The second technique, Hybrid-MUSIC, has the best accuracy, but requires computational power that is excessive. The final alternative is the Hybrid Interferometer where ambiguity resolution has been obtained by applying MUSIC to extract a coarse AOA estimate from the spirals comprising the interferometer. This method was seen to excellent results with better thresholds, while not requiring excessive computational power.

5 REFERENCES

1. E. Jacobs and E. W. Ralston, "Ambiguity Resolution in Interferometry," IEEE Trans. AES Vol. AES-17, No.6, Nov. 1981.
2. Stansfield, R.G., "Statistical Theory of D.F. fixing," Journal of IEE, (London), 94 (k) 3A, 762-770.
3. Kendall, W.B.,"Unambiguous accuracy of an interferometer angle-measuring system," IEEE Transactions on Space Electronics and Telemetry, SET-11, pp.62-70, June 1965.
4. Hansen, R.C (Ed). Microwave Scanning Antennas, Vol. III: Array Systems, New York: Academic Press, 1966.
5. "A Novel Multi-Mode Interferometer System", R. Penno, K. Pasala, S. Schneider, IEEE Aerospace Conference Proceedings, March 10-17, 2001, Track 3.0203
6. R.G. Corzine, J.A. Mosko, "Four-Arm Spiral Antennas", Artech House, 1990, Norwood, Ma.

7. "Theory of Angle Estimation Using a Multiarm Spiral Antenna", R. Penno, K. Pasala, IEEE Transactions, AESS, Vol. 37, No 1, January, 2001
8. Schmidt, R.O.(1981), "A Signal Subspace Approach to Multiple Emitter Location and Spectral Estimation", Ph.D. dissertation, Stanford University, Stanford, CA, 1981
9. Penno, R. P, Pasala, K.M. and Stephen Schneider, "The Application of Hybrid Interferometer-MUSIC to Eliminate Grating Lobe Effects", Proceedings of IEEE Aerospace Conference held at Big Sky, Montana, March, 2003

6 BIOGRAPHIES:

Dr. Krishna M. Pasala is a Professor in the Department of Electrical and Computer Engineering at the University of Dayton, Dayton, Ohio. He received the B.E. degree from Andhra University in 1970 and the Ph.D. in Electronics and Communications from the Indian Institute of Science in 1975. Dr. Pasala is a Senior Member of IEEE.

Dr. Robert Penno is an Associate Professor in the Department of Electrical and Computer Engineering at the University of Dayton, Dayton, Ohio. He received a B.S.M.E. degree from Rose Polytechnic Institute in 1971, the M.S.E.E. from Rose-Hulman Institute of Technology in 1984, and the Ph.D. from the University of Dayton in 1987. He is a member of Sigma Xi and a Senior Member of IEEE.

Stephen W. Schneider received his B.S.E.E. from Arizona State University (1985), and his M.S. (1988) and Ph.D. (1992) in Electrical Engineering from the Ohio State University. He has over 15 years of experience in the area of applied electromagnetics. From 1985 to 1992, Dr. Schneider was employed at the Ohio State University ElectroScience Laboratory where he was involved in the analysis, design and measurement of periodic surfaces for frequency selective surfaces and phased arrays. Since 1992, Dr. Schneider has been employed at Wright-Patterson AFB, OH, where he is a Principal Engineer performing research and development in Applied Radio Frequency Aperture Technology for the Sensors Directorate of the Air Force Research Laboratory. Dr. Schneider is a member of Tau Beta Pi, Eta Kappa Nu, the Association of Old Crows, the Technical Coordinator for the Antenna Measurement Techniques Association (AMTA) and is a Senior Member of IEEE.