Applications of Cluster Analysis in Diagnostics-Related Problems

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Abstract —System diagnostics facilitates reliability enhancement and condition-based maintenance of technical systems. Diagnostic problems can be formulated and solved only on the basis of mathematical models reflecting the complex stochastic nature of the failure development process. This process is affected by numerous continuous factors, but its outcome constitutes a random event. These specifics limit the applications of traditional regression models. The concept of a "continuous process with discrete-event output" is introduced, and cluster analysis is employed as a modeling approach facilitating the solution of various diagnostic problems. Important aspects of cluster analysis are the definition of the informativity criterion, selection of "informative subspaces," definition of separating rules, and finally, formulation of a cluster model are presented. Various enhancements of cluster analysis are proposed. A cluster model is utilized for the definition of so-called "probabilistic space" that, in conjunction with Bayes' technique, facilitates the solution of failure prediction problems. Application of cluster models for failure analysis and prediction is illustrated by numerical examples.

causes of failures of technical systems are traced now to the individual operational and environmental conditions of particular units and to a much lesser extent (due to completely automated manufacturing) individual properties of devices.

Reliable operation of avionics under adverse environmental exposures is crucial for safe and efficient operation of the entire aircraft. While in operation, avionics components are exposed to mechanical vibrations, excessive temperatures, humidity, radiation, etc. These adverse conditions, acting individually and in combination, are known to have cumulative effects leading to avionics performance degradation and failures. Availability of time-stress measurement devices (TSMD) [1] facilitates formulation and solution of important on-line and off-line avionics reliability-related problems. These problems include investigation of the role of various environmental factors in the development of particular failures and combined effects of several factors, reevaluating of probability of failure on the basis of known exposure to particular adverse conditions, as well as development of special types of mathematical models and model-based techniques.

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1. INTRODUCTION

The traditional approach to achieving reliable operation of any technical device (electronics, electrical, electro-mechanical) is aimed at assuring desired reliability characteristics, such as the average time of normal (no-failure) operation. This approach may be considered acceptable as long as the failures are caused by the factors related to manufacturing, not by the factors representing operational and environmental conditions. Nowadays this approach is not always acceptable. Manufacturers of electronics have achieved a very high degree of reliability of its products. Therefore, manufacturing-related factors on failures are gradually become less significant. The average time of normal operation and other "traditional" reliability characteristics are becoming less important. The main

2. CLUSTER ANALYSIS AND DIAGNOSTICS

System diagnostics is understood as detection, identification and prediction of failures by means of statistical analysis of off- and on-line data. These tasks are always based on mathematical models. Mathematical models describing the physical nature of the failure development process are intended for avionics manufacturers. These models are highly sophisticated and are not suitable for diagnostic, especially on-line, applications. Diagnostics is based on stochastic models representing trend-type effects of various operational and environmental factors that may or may not result in a failure. A failure development process can be viewed as a Continuous Process with Discrete Event Output (CPDEO). The word "continuous" reflects the nature of the recorded environmental variables such as intensity of vibration, humidity, radiation, excessive temperature, etc., and exposure time to these factors. "The discrete event output" refers to the occurrence or non-occurrence of various types of failures as the result of the environmental exposures and operational conditions.

Each observation of a CPDEO is interpreted as a point in a multidimensional factor space representing exposure to adverse conditions and the effect of this exposure,

$$\{X(t), Q(t)\} \quad (1)$$

where

$X(t)=[x_1(t), x_2(t), \dots, x_n(t)]$ - is a vector of factors representing cumulative exposure to adverse conditions in hours,

$Q(t)=1$ or $Q(t)=0$ - is the integer variable symbolizing the output discrete event ("the device failed", or "normal operation of the device"), and

$t=1, 2, \dots, N$ - is the observation index.

As a result of this representation, available TSMD data can be divided into two clusters. Analysis of these clusters in the factor space would result in a diagnostic model of the entire class of devices of the same type. This model describes effects of particular factors and combinations of factors on failure development, and allows for the classification of any point of the factor space as belonging to either "normal operation" or "failure" clusters.

Mathematically, such a model could be established as a set of conditions separating the clusters

$$f_i(X)=0, i=1, 2, \dots \quad (2)$$

Then the decision rule implies checking the value (1 or 0, or "true" or "false") of a predicate defined on the basis of functions (2), and the subsequent "recognition" of the point X in the factor space as a point of one or another cluster.

The choice of functions (2) is dependent upon the geometry and mutual positioning of clusters in the factor space. However, analysis of the above properties in n -dimensional space when $n>2$ is impossible, therefore, the problem of choosing a type of classification functions is one of the critical problems in building classification models. The following approach facilitates visualization of the geometry and mutual positioning of clusters by the analysis of projections of clusters on subspaces, strategically selected in the factor space.

The first step of the approach utilizes a special procedure to form a fixed subset of subspaces of the full space of factors. Each subspace is determined by a particular combination of components of the vector of factors. As suggested in [2], the dimension of this combination is assumed to be equal to 2 to make possible the visualization of projections of clusters onto corresponding two-dimensional subspaces (planes). The subspaces included in the fixed subset, and called informative subspaces, are formed based on a special criterion, that provides a measure of informativity of a particular combination of factors for solution of a classification problem. The criterion chosen herein represents the normalized squared distance between a pair of clusters in the corresponding subspace of the factor space. Based on this criterion, a number of two-dimensional subspaces, where the value of the chosen criterion exceeds some margin, are selected.

The next step of the approach includes the definition of separation rules in the selected subspaces and representation

of each of these rules as a predicate,

$$\Phi_i = \Phi_i(f_i(X) \# 0), i=1, 2, \dots, k \quad (3)$$

where

i - is the number assigned to a subspace, and

k - is the number of subspaces included in the fixed subset of subspaces.

It could be seen that unlike most traditional classification schemes resulting in one separation rule defined in the subspace of informative factors, our approach utilizes a number of separation rules. Each informative subspace has its own separation rule. From the mathematical point of view, this implies that a general separation surface is being approximated by number of spline-functions (multi facet surfaces). This is one of the main reasons of high efficiency of the approach.

The above predicates are utilized in a set of 2^k functions of logic, and the entire factor space becomes divided into a number of multi facet regions such that any two regions do not overlap, and the combination of all regions covers the entire factor space. A probabilistic measure is assigned to each of the regions, such that at any point of this region it represents the probability that the point belongs to the cluster "failure." This probability is calculated as a statistical estimate on the basis of experimental data. The approach results in a system of conditional probabilities that facilitate definition of the probability of belonging to the cluster "failure" for any point of the factor space, i.e. defines the probability $P(X/1)$ for any combination of numerical values of exposures X . The utilization of these procedures provides all necessary data for Bayes' formula to compute the probability of failure for a given (current or predicted) value of the exposure vector X^* , $P(1/X^*)$. This constitutes a solution of a prognostic problem.

The general scheme of the approach includes the following steps:

- selection of a set of two dimensional informative subspaces in the factor space, definition of separation rules within each subspace, and predicates of classification,
- definition of a probabilistic space in the factor space,
- definition of a decision making scheme based on the probability of failure of a device for a given value of the vector of adverse exposures.

The first two steps result in the definition of a diagnostic model. The third step constitutes a model-based decision making. The mathematical model, discussed herein, is applicable to solving such practical reliability related problems as:

- ranking particular environmental conditions as factors responsible for general and particular types of failures,
- formation of particular groups of environmental conditions and assessment of their combined effects on failures in general and on particular types of failures,
- tracking the dynamics of development of cluster models and their statistical characteristics in the process of obtaining new experimental data,
- justification of the development of devices protecting avionics from adverse environmental conditions,

- development of the recommendations on the avoidance of the combined effects of adverse conditions.

3. SELECTION OF INFORMATIVE SUBSPACES

Informativity analysis is the most computationally intensive task of any classification problem. In our case this is the procedure resulting in a set of two-dimensional informative subspaces. Consider the informativity criterion

$$F_{lp} = (\sigma_l^2 \sigma_p^2 N_l N_0)^{-1} \sum_{r=1}^{N_l} \sum_{s=1}^{N_0} \{ [x_l^1(r) - x_l^0(s)]^2 + [x_p^1(r) - x_p^0(s)]^2 \} \quad (4)$$

where

N_l, N_0 - are total number of realizations within the clusters "failure" and "normal operation" respectively, marked by superscripts "1" and "0",

$x_l^1(r), x_p^1(r)$ - are exposures to the adverse conditions number l and number p in the realization number r belonging to the cluster "failure," $r=1, 2, \dots, N_l$

$x_l^0(s), x_p^0(s)$ - are exposures to the adverse conditions number l and number p in the realization number s belonging to the cluster "normal operation", $s=1, 2, \dots, N_0$

σ_l, σ_p - are standard deviations of variables x_l and x_p estimated over the entire range of experimental data.

The suggested procedure could be implemented through the exhaustive analysis of all low-dimensional subspaces that could be defined in the factor space X . It is possible to prove that the algorithm implementing the exhaustive analysis of the measure of informativity (4) for two clusters, n factors, with two-dimensional subspaces has a complexity of

$$W_2 = C_1 N_0 N_l n(n-1) \quad (5)$$

and with three-dimensional clusters

$$W_3 = C_1 N_0 N_l n(n-1)(n-2)/2 \quad (6)$$

where

C_1 is a constant, and n is the total number of factors.

Consideration of the informativity criteria for one-, two, and three-dimensional subspaces results in a useful recursive formula that could be easily generalized for higher dimensional subspaces

$$F_{l,p,q} = (\sigma_l^2 \sigma_p^2 \sigma_q^2)^{-1} [\sigma_l^2 \sigma_p^2 F_{lp} + \sigma_q^2 F_q] \quad (7)$$

with the complexity

$$W_3^{RECURS} = C_1 N_0 N_l n + C_2 n(n-1)(n-2) \quad (8)$$

where

$C_2 \ll C_1 N_0 N_l$ - is another constant.

Therefore, one of the ways to enhance the computational efficiency of the considered procedure is to develop more efficient procedures for selecting low-dimensional

informative subspaces. Availability of such procedures would allow one to extend the search of informative subspaces to subspaces of any dimension.

The second approach, still under investigation by the authors, implies that the dimension of the factor space can be significantly reduced by performing cluster analysis not in the original space X but in the reduced dimension space of mutually-orthogonal "artificial" variables Z defined by principle component analysis.

4. DEFINITION OF SEPARATING RULES

When informative subspaces have been selected, separation rules are to be established on the basis of separating functions (2) defined on each informative subspace such that

$$f_l(x_l^0, x_p^0) > 0 \text{ and } f_l(x_l^1, x_p^1) \neq 0$$

where l and p are indexes of factors included in the i -th subspace. Traditionally, in the problem of building separation surfaces it is implied that the clusters are linearly separable. As the experimental results often show, in diagnostic tasks this condition is not met. Therefore, the choice of separation surfaces can be accomplished under less strict conditions and be prompted by the actual topology of cluster projections on informative subspaces. The complexity of this task is significantly reduced by the application of two-dimensional subspaces. Although linear separation surfaces present the simplest solution, much more flexible nonlinear separating surfaces could be suggested as a "general case" solution. The following computer-based procedure for "automatic" definition of separation rules has proven to be quite beneficial.

A separation rule in the space of two variables (factors) x_l and x_p can be defined on the basis of an ellipse that is justified by most common mutual topologies of two clusters

$$f_l(A, x_l, x_p) =$$

$$(x_l - a_1)^2/a_3^2 + (x_p - a_2)^2/a_4^2 + a_5(x_l - a_1)(x_p - a_2)/(a_3 a_4) - 1 \quad (9)$$

where

$A = [a_1, a_2, a_3, a_4, a_5]$ - is the vector of unknown parameters of the ellipse.

Initial values of parameters A can be defined in terms of estimated means and variances of variables $x_l^1(r), x_p^1(r), r=1, 2, \dots, N_l$. The fine-tuning of these parameters, resulting in the optimal position, orientation, size, and shape of the ellipse was achieved by numerical minimization of the following heuristic criterion

$$Q(A) = \sum_{r=1}^{N_l} V[A, x_l^1(r), x_p^1(r)] + \sum_{s=1}^{N_0} V[A, x_l^0(s), x_p^0(s)] \quad (10)$$

where

$$V[A, x_l^1(r), x_p^1(r)] = [x_l^1(r) - a_1]^2 + [x_p^1(r) - a_2]^2$$

Composition of the database is presented in Table 1 of the Appendix. Application of the developed cluster analysis program resulted in a list of two-dimensional subspaces. The first 20 subspaces, listed according to their informativity, can be found in Table 2 of the Appendix. It could be seen that the list of subspaces could be reduced to first five that are most informative. Printout 1 of the Appendix features computer-generated separating functions (ellipses) of the most informative subspaces and presents estimated probabilities of correct and incorrect classification. Printout 2 of the Appendix presents elementary events defined as conjunctions of the 5 original logic functions with and without negation sign, and the estimated conditional probabilities of these events thus providing the definition of the probabilistic space. Printout 3 of the Appendix shows the distribution of representatives of cluster "failure" (F) and cluster "no failure" (O) in the particular informative subspace (note that "X" represents situations when symbols "F" and "O" coincide).

Analysis of the failure process can be performed on the basis of printouts 1 and 2 that feature and rank particular adverse conditions and combinations of adverse conditions resulting in failure.

The defined diagnostic model can be applied for the reevaluation of the probability of failure of the avionics module on the basis of its actual "history of abuse" represented by the vector of adverse exposures,

$$X^* = [1.45, 1.42, .65, 1.2, 1.7, 1.44, 1.1, 0.7, 18., .5, .2, 19., 1.4, .77, .7, 1.5, 15., 1.9, 20.0]$$

Assume that the prior probability of failure is .0001. This task is easily performed by the developed computer code: first it was determined that appearance of vector X^* results in the occurrence of event E_{29} with probabilities

$$P(1/1) = 0.0421, \quad P(1/0) = 0.0095,$$

then posterior probability of failure was defined as

$$(.0001 \times 0.0421) / (.0001 \times 0.0421 + .9999 \times 0.0095) = .00044$$

The posterior probability figure, compared against some marginal values, can facilitate a well-justified maintenance decision.

Testing indicates that the developed software tools are user-friendly and numerically efficient.

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APPENDIX

Table 1

#	Exposure Factors & Status
1	Vibration RMS, 1 - 2 g
2	Vibration RMS, 3 - 4 g
3	Vibration RMS, over 4 g
4	Humidity, 20 - 50%
5	Humidity, 70 - 95%
6	Environmental Temperature -15 - 0EC
7	Environmental Temperature 0 - 15EC
8	Environmental Temperature 50 - 75EC
9	Environmental Temperature 76 - 100EC
10	Power Supply .7 - .9 nominal Vdc
11	Power Supply 1.1 - 1.3 nominal Vdc
12	Power Supply over 1.3 nominal Vdc
13	Functional Overload 5 - 10%
14	Functional Overload 11 - 20%
15	Functional Overload 21 - 30%
16	Functional Overload 31 - 40%
17	Functional Overload 41 - 50%
18	Air Pressure .3 - .7 nominal
19	Air Pressure 1.1 - 1.3 nominal
20	Status of the module (normal / failure)

Table 2

#	X _i	X _j	separation criterion
1	X ₄	X ₉	4.822
2	X ₁	X ₉	4.784
3	X ₉	X ₁₇	4.783
4	X ₉	X ₁₄	4.738
5	X ₃	X ₉	4.726
6	X ₉	X ₁₂	4.725
7	X ₆	X ₉	4.711
8	X ₉	X ₁₈	4.670
9	X ₉	X ₁₆	4.668
10	X ₉	X ₁₉	4.658
11	X ₅	X ₉	4.614
12	X ₉	X ₁₅	4.610
13	X ₉	X ₁₃	4.601
14	X ₉	X ₁₁	4.588
15	X ₈	X ₉	4.580
15	X ₉	X ₁₀	4.575
17	X ₁	X ₄	4.574
18	X ₄	X ₁₇	4.573
19	X ₂	X ₉	4.562
20	X ₇	X ₉	4.553

Printout 1

SUBSPACE OF VARIABLES # 9 & #12

A(1) = 17.573
A(2) = 16.380
A(3) = 4.583
A(4) = 7.278
A(5) = 0.678

NOTE: POINTS "1" ARE INSIDE THE ELLIPSE

$$[X-A(1)]^2/[A(3)^2]+[Y-A(2)]^2/[A(4)^2]+A(5)*[X-A(1)]*[Y-A(2)]/[A(3)*A(4)]=1$$

GENERAL PROBABILITY OF "1" P1 = 0.4750

GENERAL PROBABILITY OF "0" P0 = 0.5250

PROB. OF "1" BEING RECOGNIZED AS "1" P11 = 0.8211

PROB. OF "0" BEING RECOGNIZED AS "1" P01 = 0.2190

SUBSPACE OF VARIABLES # 9 & #17

A(1) = 17.461
A(2) = 16.583
A(3) = 6.333
A(4) = 7.011
A(5) = 0.527

NOTE: POINTS "1" ARE INSIDE THE ELLIPSE

$$[X-A(1)]^2/[A(3)^2]+[Y-A(2)]^2/[A(4)^2]+A(5)*[X-A(1)]*[Y-A(2)]/[A(3)*A(4)]=1$$

GENERAL PROBABILITY OF "1" P1 = 0.4750

GENERAL PROBABILITY OF "0" P0 = 0.5250

PROB. OF "1" BEING RECOGNIZED AS "1" P11 = 0.9053

PROB. OF "0" BEING RECOGNIZED AS "1" P01 = 0.3524

SUBSPACE OF VARIABLES #12 & #17

A(1) = 16.749
A(2) = 17.005
A(3) = 6.652
A(4) = 6.626
A(5) = 0.738

NOTE: POINTS "1" ARE OUTSIDE THE ELLIPSE

$$[X-A(1)]^2/[A(3)^2]+[Y-A(2)]^2/[A(4)^2]+A(5)*[X-A(1)]*[Y-A(2)]/[A(3)*A(4)]=1$$

GENERAL PROBABILITY OF "1" P1 = 0.4750

GENERAL PROBABILITY OF "0" P0 = 0.5250

PROB. OF "1" BEING RECOGNIZED AS "1" P11 = 0.9053

PROB. OF "0" BEING RECOGNIZED AS "1" P01 = 0.4095

Printout 2

CONDITIONAL PROBABILITIES OF INTEREST:

EVENT 1: $P(1/1) = 0.0000$, $P(1/0) = 0.0762$
 EVENT 2: $P(1/1) = 0.0000$, $P(1/0) = 0.0095$
 EVENT 3: $P(1/1) = 0.0000$, $P(1/0) = 0.0286$
 EVENT 4: $P(1/1) = 0.0316$, $P(1/0) = 0.2381$
 EVENT 5: $P(1/1) = 0.0421$, $P(1/0) = 0.0571$
 EVENT 6: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 7: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 8: $P(1/1) = 0.0000$, $P(1/0) = 0.0095$
 EVENT 9: $P(1/1) = 0.0105$, $P(1/0) = 0.0476$
 EVENT 10: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 11: $P(1/1) = 0.0105$, $P(1/0) = 0.0667$
 EVENT 12: $P(1/1) = 0.0211$, $P(1/0) = 0.0095$
 EVENT 13: $P(1/1) = 0.0526$, $P(1/0) = 0.0381$
 EVENT 14: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 15: $P(1/1) = 0.0105$, $P(1/0) = 0.0095$
 EVENT 16: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 17: $P(1/1) = 0.0105$, $P(1/0) = 0.0286$
 EVENT 18: $P(1/1) = 0.0000$, $P(1/0) = 0.0095$
 EVENT 19: $P(1/1) = 0.0105$, $P(1/0) = 0.0000$
 EVENT 20: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 21: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 22: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 23: $P(1/1) = 0.0000$, $P(1/0) = 0.0095$
 EVENT 24: $P(1/1) = 0.0000$, $P(1/0) = 0.0000$
 EVENT 25: $P(1/1) = 0.0000$, $P(1/0) = 0.0095$
 EVENT 26: $P(1/1) = 0.0000$, $P(1/0) = 0.0190$
 EVENT 27: $P(1/1) = 0.0526$, $P(1/0) = 0.0952$
 EVENT 28: $P(1/1) = 0.0105$, $P(1/0) = 0.0000$
 EVENT 29: $P(1/1) = 0.0421$, $P(1/0) = 0.0095$
 EVENT 30: $P(1/1) = 0.0105$, $P(1/0) = 0.0000$
 EVENT 31: $P(1/1) = 0.6842$, $P(1/0) = 0.0381$

Printout 3

INFORMATIVE SUBSPACE # 1: X(9) & X(12) CRITERION = 4.905

```

X( 9) | 0.8787E+01 <-----X(12)-----> 0.2126E+02 |
=====|=====|
8.041>| 0 0 0 | | | | | | | | | | F 0 0 |
8.737>| | 00 | | 0 0 | | 0 0 | | 0 | | | |
9.434>| 0 | 0 | | 0 00 | | 00 00 | | 0F | | X |
10.130>| | | | 0 000 | | 000 | | F | | | |
10.827>| 0 | 00 | | 0 | | F | | 0F0 | | 0 |
11.523>| 00 0 0 | | | | | | | | | | FX 0 00 |
12.220>| 0 | 0 | | 0 | | | | | 0 | | 0 F |
12.916>| 0 0 | | 0 X00 | | FFF | | 0 | | F F |
13.613>| 0 0 0 | | 0 0 | | 0 0 | | 0F | | F F |
14.309>| 0 | | | 0 | | | | | | | | F |
15.005>| 0 | | | F | | | | | | | | F |
15.702>| 0 0 X | | F F | | F F | | FF | | F F |
16.398>| 0 0 | | F 000 | | FFF0 | | F | | F F |
17.095>| 0 | F | | F | | F | | | | | F F |
17.791>| FF F F0 | | | | | | | | | | FF F FF |
18.488>| F | F0F | | 0 | | F | | FF | | F |
19.184>| | 0 | | FFO | | FF | | FFF | | F |
19.880>| X | 0 | | F FF | | F | | FF F | | F |
20.577>| | F 0 | | 0 0 | | | | | F F | | F |
21.273>| F 0 0 | | | | | | | | | | F F |
21.970>| F | | | | | | | | | | | |
=====|=====|
| 0.8787E+01 <-----X(12)-----> 0.2126E+02 |

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