An Analysis of Membrane Mirrors For Use in Large Aperture Imaging Systems

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Abstract—The finite element model was used to determine the surface figure of membrane mirrors. The Poisson's ratio of the membrane was varied to determine the most desirable materials. It was found that materials with a Poisson's ratio of 0.45 or greater are desirable. Membrane mirrors alone are poor imaging surfaces, and the effects of radial boundary displacement are investigated. It was found that radial boundary displacement can reduce the rms wavefront error of a 42 inch diameter f/1.9 mirror by 78.7%. A 42 inch diameter f/1.06 membrane mirror would have the rms wavefront error reduced by 65.6%. The primary on-axis and off-axis aberration was found to be 3rd order spherical aberration for all f-numbers and at all radial displacements.

will include an analysis of the different aberrations present for an infinite conjugate as well as examine the off-axis performance of these mirrors.

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1. INTRODUCTION

For space based optical imaging systems, large aperture primary mirrors are desirable due to their capability to provide high resolution imaging. To achieve the required optical quality, traditional mirrors must be extremely heavy and very large which causes these mirrors to be very expensive or impossible to launch into space. Segmenting the primary mirror, using a sparse array, and inflating a membrane mirror are three possible alternatives to conventional mirrors[1]. All of these could achieve the high resolutions desired yet be more feasible for space usage. The advantages of a membrane mirror are its low weight and the potential to achieve extremely large diameters of 20 meters or larger. However, the optical performance of these membranes is largely unknown. In this work, the finite element model was utilized to determine the shape of a membrane surface under uniform pressure. This shape was then used to analyze the optical performance of a membrane mirror to determine its feasibility as an imaging surface. This paper will explore how the material properties of the membrane affect performance, and investigate the effects of radial boundary displacement. This investigation

(a)

(b)

Figure 1 (a) Ray trace of typical membrane mirror and (b) OPD plot of f/1.9 membrane (Scale is +/- 5000 waves).

The behavior of membranes subjected to a uniform pressure has been studied for many years[2-4]. In one such study, Stevens[4] gives the following formula for the displacement of a membrane:

$$w(r) = b(1 - c_1 \frac{r^2}{a^2} - c_2 \frac{r^5}{a^5}) \quad (1)$$

where b is the maximum displacement, c_1 is 0.9, c_2 is 0.1, and a is the aperture radius. The maximum displacement, b , is a function of the f-number and the aperture radius. Figure

1 is a ray trace of typical membrane mirror with its corresponding optical path difference (OPD) plot. Stevens' formula suggests that membranes under uniform pressure will have an aspheric shape which departs from an ideal parabola. Due to this aspheric shape, membrane mirrors will be extremely poor imaging surfaces especially at small f-numbers and large diameters. These surfaces would be excluded from use in imaging systems, even those systems which include adaptive optics. Nevertheless, membrane mirrors could be useful if the surface could be significantly improved. One technique to improve the surface figure is to apply a uniform radial boundary displacement to the membrane mirror. The finite element model allows for the study of a membrane under these conditions.

2. MATERIAL PROPERTIES

Materials used as membrane mirrors must meet certain requirements. First, the materials must have an extremely small thickness on the order of several hundred microns. This allows the membrane to be deformed without requiring unreasonable pressures to be exerted. Second, the material must be highly reflective or be capable of maintaining a highly reflective coating of a material such as aluminum or silver. Third, the surface smoothness must meet optical requirements. Finally, since membranes will require radial boundary displacements, the materials must be capable of withstanding the forces applied.

The effect of the Poisson's ratio (ν) on the membrane mirror was investigated. Poisson's ratio for a material is the ratio of its transverse contraction to its elongation when subjected to a tensile stress. Variations in Young's Modulus were not studied since basic theory and nonlinear finite element analysis both show that a change in this parameter has the effect of only changing the pressures required and does not change the shape of the imaging surface. Also, because the membrane is axisymmetric, the shear modulus is irrelevant since there is no shear strain. However, a change in ν does have an effect on the surface structure.

Using the finite element model to determine the theoretical surface structure, it was found that as ν increases, the optical performance of the membrane improves. Poisson's ratio was varied from 0.25 to 0.49. Since the membrane is a continuum, ν cannot exceed 0.5. The calculated rms (to mean) wavefront error of a 42 inch diameter $f/1.11$ mirror decreases from 2740 waves at 0.55μ for $\nu = 0.25$ to 1814 waves for $\nu = 0.49$. For the remainder of this paper, the rms wavefront error will be referenced to the mean and will be in waves at 0.55μ unless otherwise noted. Figure 2 plots the rms wavefront error at different Poisson's ratios. From this analysis it was determined that the ideal material used for the membrane mirror should have a Poisson's ratio of 0.45 or greater.

At $\nu = 0.45$, the rms wavefront error for the 42 inch diameter $f/1.11$ mirror is 1814 waves at 0.55μ which is in good agreement with Stevens' formula that predicts an rms wavefront error of 1850 waves, a 2% difference. The primary on-axis aberration is 3rd order spherical aberration, and this aberration dominates over primary coma and

astigmatism even at a field of view of four degrees. It should be noted that the on-axis performance is also affected by higher order spherical aberration as well. 1814 waves is an unacceptable amount of aberration for an imaging surface since even a few waves of aberration causes a significant degradation of the image. One possible method for reducing these aberrations is to apply a uniform radial boundary displacement to the membrane which is the subject of the next section.

Figure 2 RMS Wavefront Error at 0.55μ as a function of Poisson's Ratio.

3. RADIAL BOUNDARY DISPLACEMENT

Stevens' formula does not predict the behavior of membrane when the radial boundary is displaced. Therefore, the finite element model was used to predict the surface shape when the membrane is subjected to a uniform radial boundary displacement. The finite element model is a quarter-disk consisting of 304 eight-noded quadrilateral membrane elements and 16 six-noded triangular membrane elements. In all cases, the model was initiated from planar configuration. In order to give a well-conditioned stiffness matrix for small deflections, transverse springs were attached to each node. The model was inflated to a small pressure (5%-10% of final pressure), and the boundary was displaced as called for by each case. The springs were then removed, and the pressure was increased as necessary to obtain the desired center deflection.

All the mirrors in this study had a 42 inch diameter and Poisson's ratio was assumed to be 0.4. The wavelength used for all calculations is 0.55μ . We studied the effects of a radial boundary displacement for mirrors that had an f-number of $f/1.9$, $f/1.44$, and $f/1.06$. To maintain a constant f-number for comparison, the maximum deflection along the z axis was kept constant as the radial boundary was displaced by increasing the pressure on the membrane as is seen in figure 3.

Initially, the case where the radial displacement was applied before the membrane inflation and the case where the membrane was first inflated then the radial boundary was

displaced were both studied. However, it was discovered that both cases give the same result for the surface figure. Therefore, just the former case is presented here.

To analyze the separate aberrations present, the zernike polynomials are used to describe the wavefront. A discussion of these polynomials can be found in any lens design text[5], and a brief description is given here merely for convenience. The zernike polynomials are an orthogonal set of polynomials where each polynomial represents a different aberration. The wavefront variance is defined as [5]

$$\sigma_w^2 = \frac{1}{\pi} \int_0^1 \int_0^{2\pi} (W(\rho, \theta) - W_{avg})^2 \rho d\rho d\theta \quad (2)$$

where ρ, θ are polar coordinates in the entrance pupil, ρ is normalized, and $W(\rho, \theta) = z_{mn} U_{mn}(\rho, \theta)$ where $U_{mn}(\rho, \theta)$ is the mn zernike polynomial and z_{mn} is the mn zernike coefficient. The zernike polynomials which describe 3rd and 5th spherical aberration are given by, respectively[5]

$$U_{42}(\rho, \theta) = 6\rho^4 - 6\rho^2 + 1 \quad (3)$$

$$U_{63} = 20\rho^6 - 30\rho^4 + 12\rho^2 - 1 \quad (4)$$

Since the zernike polynomials are orthogonal, the wavefront variance can be written as

$$\sigma_w^2 = \sum_m \sum_n \sigma_{mn}^2 \quad (5)$$

where σ_{mn}^2 is the contribution from each aberration. The contribution for 3rd and 5th order spherical aberration is $\sigma_{42}^2 = z_{42}^2/5$ and $\sigma_{63}^2 = z_{63}^2/7$, respectively. The remainder of the aberrations are left to the reader.

Figure 3 A plot of the pressure required to maintain a constant f-number as a function of radial boundary displacement.

f/1.9 Membrane Mirror

For the f/1.9 membrane mirror, a small radial displacement of 0.02 inches reduces the rms wavefront error by 22.7%, from 1135 waves to 877 waves. The primary on-axis aberration is 3rd order spherical aberration, and the calculated z_{42} zernike coefficient reduces from 2450.5 to 1965.3, a 22.6% reduction. The 5th order spherical aberration is also reduced by the radial boundary displacement. The z_{63} zernike coefficient is reduced by 46.1% for a radial displacement of 0.02 inches and by 63% for a radial displacement of 0.04 inches while the rms wavefront error is only reduced by 37.6%. This result initially caused us to believe that the higher order aberrations could be eliminated and the primary aberrations could be reduced to an acceptable level by increasing the radial displacement. However, this prediction turned out to be false as the primary and higher order aberrations reached a limit of correction. In other words, the point was reached where an additional radial boundary displacement did not provide any additional optical improvement of the membrane mirror.

Nevertheless, this should not be thought of as a negative result since the aberrations of this surface are significantly reduced. For a uniform radial boundary displacement of 0.24 inches, the rms wavefront error is reduced by 78.1% to 249 waves while z_{42} is reduced by 78.05% and z_{63} is reduced by 90.6%. Figure 4 shows the improvement of the aberrations as the radial boundary displacement is increased. As a function of the radial displacement, the reduction in the rms wavefront error for the f/1.9 mirror is given by

$$w = 242.15 + 876.33e^{-14.86r} \quad (6)$$

where w is the rms wavefront error in waves and r is the radial boundary displacement in inches. The 3rd and 5th order spherical aberration zernike coefficients follow a similar exponential decay given by

$$z_{42} = 542.0 + 1961.6e^{-14.83r} \quad (7)$$

$$z_{63} = 14.25 + 167.25e^{-29.2r} \quad (8)$$

where z_{42} is the zernike coefficient for 3rd order spherical aberration and z_{63} is the zernike coefficient for 5th order spherical aberration. Equations (6), (7), (8) are best fit exponential decay curves to the data points calculated by the finite element model, see figure 4.

From these relationships, it becomes obvious that for small r , a slight increase in r gives rise to a significant decrease in the aberrations. As r increases, a slight change in r results in a less significant improvement in performance. These relationships also show that a limit is reached where the aberrations no longer decrease. The minimum possible rms wavefront error predicted by (6) is 242.15 waves. By comparison, a f/1.9 sphere would have an rms wavefront error of 41 waves. The exponential decay constant in (8) is larger than in (6) and (7) and accounts for the more rapid decrease of the 5th order spherical aberration. The 3rd order

spherical aberration is decreases at a similar rate as the rms wavefront error. This is to be expected since 3rd order spherical aberration is the primary aberration.

Off-axis, the primary aberration is still 3rd order spherical aberration even at a 4° field of view (FOV) yet tilt, astigmatism, and primary coma are all present. The off-axis aberrations are also reduced, but not as significantly as the on-axis aberrations. The z_{10} zernike coefficient corresponding to tilt reduces by a mere 1.04% to 384.5. The z_{22} coefficient corresponding to astigmatism reduces a scant 2.2% while z_{31} , corresponding to primary coma, reduces 1.5%. Therefore, radial boundary displacement does not have a profound effect on off-axis performance.

(a)

(b)

Figure 4 A plot of (a) RMS wavefront error and (b) Z_{42} zernike coefficient versus radial boundary displacement for an f/1.9 membrane mirror.

F/1.44 Membrane Mirror

The exponential decay of the rms wavefront error seen in the case of the f/1.9 mirror also occurs for the f/1.44 membrane mirror. However, some important differences do

take place. As in the previous case, a small radial displacement of 0.02 inches on the f/1.44 mirror reduces the rms wavefront error by 15.0% from 1554 waves to 1320 waves. The z_{42} coefficient is also reduced by 15.0%. The z_{63} zernike coefficient is reduced by 22.8% while a slightly larger radial displacement of 0.04 inch reduces z_{63} by 50.7%. A larger radial displacement of 0.24 inches decreases the rms wavefront error by 67.0%, z_{42} by 66.9%, and z_{63} by 88.0%. Figure 5 shows the decrease in aberrations as a function of the radial boundary displacement. The exponential decay for the rms wavefront error, z_{42} , and z_{63} are given by

$$w_{f/1.44} = 428.6 + 1110.2e^{-10.23r} \quad (9)$$

$$z_{42(f/1.44)} = 959.0 + 2484.9e^{-10.2r} \quad (10)$$

$$z_{63(f/1.44)} = 31.12 + 226.9e^{-19.6r} \quad (11)$$

where r is the radial boundary displacement in inches. As before, (9), (10), and (11) are best fit exponential decay curves. From (9), the minimum rms wavefront error for an f/1.44 membrane mirror is 428.6 waves, a 72.4% decrease. For the f/1.9 mirror, the rms wavefront error could be reduced by 78.7%. For the f/1.44 mirror, the maximum reduction of z_{42} is 72.4% while z_{63} can be reduced by 88.7%. The exponential decay constant in (9), (10), and (11) is smaller than in (6), (7), and (8), respectively. This suggests that a larger radial displacement is needed for an f/1.44 mirror to achieve the desired improvement in performance.

F/1.06 Membrane Mirror

Faster mirrors are desirable for large aperture imaging systems since these mirrors reduce the total optical track. Therefore, the finite element model was used to investigate the performance of an f/1.06 mirror. For this mirror with no radial tension, the rms wavefront error was found to be 2154 waves. However, with a radial boundary displacement of 0.24 inches, the rms wavefront error could be reduced to 1015 waves, a significant improvement. The rms wavefront error exhibits an exponential decay for the f/1.06 membrane mirror similar to the previous two cases. The rms wavefront error as a function of radial boundary displacement is described by

$$w_{f/1.06} = 741.4 + 1407.4e^{-6.74r} \quad (12)$$

Equation (12) suggests that the maximum decrease in the rms wavefront error is 65.6% to 741.4 waves. This result suggests that as the focal length of the membrane mirror increases, radial boundary displacement is capable of higher correction to the rms wavefront error. Therefore, the f/1.9 mirror can achieve a 78.7% reduction in rms wavefront error while the faster mirrors cannot achieve this large of a reduction. In addition to this, the faster mirror requires a larger radial boundary displacement to achieve the desired improvement in optical performance since the exponential decay constant in (12) is smaller than (6) and (9). Figure 6 summarizes the results for an f/1.06 mirror.

(a)

(b)

(c)

Figure 5 (a) RMS wavefront error, (b) Z_{42} coefficient, and (c) Z_{63} coefficient as a function of radial boundary displacement for an f/1.44 membrane mirror.

4. DISCUSSION

The results presented here show the capability of radial boundary displacement to improve the optical performance of a 42 inch membrane mirror. However, one is interested in the performance of membrane mirrors over a broader range of aperture diameters. Since the finite element model has arbitrary units, equations (6), (9), and (12) can be used to generalize the rms wavefront error for the case of no radial displacement and maximum correction. For any aperture diameter, the predicted rms wavefront error for an f/1.9 membrane mirror with no radial tension (i.e. $r=0$) is given by

$$w_{f/1.9} = 1118.5 \cdot \frac{D}{1066.8} = 1.0485 \cdot D \quad (13)$$

where D is the aperture diameter in millimeters and $w_{f/1.9}$ is the rms wavefront error in waves at 0.55μ . The minimum rms wavefront error possible for an f/1.9 membrane mirror with radial boundary displacement is

$$w_{\min f/1.9} = 242.15 \cdot \frac{D}{1066.8} = 0.227 \cdot D \quad (14)$$

where D is in millimeters. These relationships are given below for the f/1.44 and f/1.06 membrane mirrors, respectively.

$$w_{f/1.44} = 1.4424 \cdot D \quad (15)$$

$$w_{\min f/1.44} = 0.4018 \cdot D \quad (16)$$

$$w_{f/1.06} = 2.014 \cdot D \quad (17)$$

$$w_{\min f/1.06} = 0.695 \cdot D \quad (18)$$

where, as before, D is the aperture diameter in millimeters. Therefore, a 42 meter diameter f/1.9 membrane would have an rms wavefront error of over 44,000 waves without radial tension, and achieve a minimum rms wavefront error of 9534 waves with radial boundary displacement. Furthermore, diffraction limited performance can not be achieved with radial boundary displacement even at small diameters of 4 cm where an f/1.9 membrane would have a minimum rms wavefront error of 9.08 waves. This could cause one to immediately conclude that membrane mirrors are not useful. However, this would be a premature conclusion for several reasons which will be discussed in the next paragraph.

First, membranes mirrors could be useful in the microwave and radar regimes where the radial boundary displacement reduction of the rms wavefront error by 66% or greater would improve performance to acceptable levels. For example, a 42 meter diameter f/1.06 membrane would have a minimum rms wavefront error of 0.16 waves at $\lambda = 10\text{cm}$.

(a)

(b)

(c)

Figure 6 (a) The rms wavefront error, (b) Z_{42} Zernike coefficient, (c) Z_{63} Zernike coefficient versus radial boundary displacement for the $f/1.06$ membrane mirror.

Second, the primary aberration is third order spherical aberration which is a well known aberration. This aberration could be corrected, at least partially, by the other optics in the imaging system. The secondary and tertiary mirrors could be constructed with an aspheric shape which removes the aberration. Unfortunately, this technique would have several limitations. The membrane mirror would be required to have a fixed f -number since the secondary optics would be specifically designed for a certain surface shape of the membrane. Also, the secondary optics would be extremely expensive to manufacture, and these optics would be very sensitive to alignment. Finally, other techniques such as a non-uniform membrane thickness and non-uniform pressure distribution could be incorporated in the membrane mirror to further reduce the aberrations. By manufacturing a membrane which has a variable thickness, the thicker portions of the membrane would be deformed differently thus affecting the surface figure of the membrane.

5. CONCLUSIONS

In this paper, it was shown that the Poisson's ratio of a material affects the performance of a membrane mirror, and materials with $\nu = 0.45$ or greater are better suited for use in membrane mirror systems. The imaging capability of membrane mirrors is extremely poor thus requiring techniques to be employed which will improve the optical performance. One technique shown to have the capability to improve the imaging capabilities of the membrane mirror is uniform radial boundary displacement. This technique demonstrated the capability to reduce the rms wavefront error by 65.6% for an $f/1.06$ membrane mirror and 78.7% for an $f/1.9$ mirror. This suggests that longer focal length membrane mirrors can receive more benefit from radial boundary displacement. The primary aberration, even off-axis, for these membrane mirrors was found to be 3rd order spherical aberration.

Even though the rms wavefront error is reduced by 65.6% and greater, the 42 inch membrane mirrors are left with 240 waves or more of aberration depending on the focal length of the mirror. This is an unacceptable amount of aberration for membrane mirrors to be useful. Other techniques such as non-uniform membrane thickness and non-uniform pressure distribution could potentially reduce the aberrations further. Even with these techniques, membrane mirrors will probably never reach diffraction limited performance. However, it is believed that diffraction limited performance is not necessary for membrane mirrors with recent advances in compensated imaging techniques[1,6]. In Gruneisen, *et al.*[1], a six inch diameter, four micron thick, nitrocellulose $f/6$ membrane mirror with >200 waves peak to valley wavefront error was corrected to 1.6 times diffraction limited. Even with advances such as these and specially designed secondary optics, the rms wavefront error needs to be improved to less than a few hundred waves for a large diameter (greater than 5 meters) membrane mirror to be useful in large aperture imaging systems. This performance has not yet been demonstrated, and will require the use of radial boundary displacement combined with the other techniques mentioned earlier.

Future work will include experimental validation of the results presented here. A flexible mirror test cell has been constructed which has multiple pressure chambers and the capability to test the effects of uniform radial boundary displacement. Also, the finite element model will be utilized to study the effects of non-uniform membrane thickness and multiple pressure chambers.

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