Deadlock Avoidance in Manufacturing Systems with a Multiple Resource Request Model

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ABSTRACT

A deadlock avoidance method for flexible manufacturing systems with both free choices and multiple resource requests allowed in part routing is presented. Based on a digraph model of the system, the method classifies whether moving a part to its next step is safe, unsafe, or undetermined. An undetermined part movement is further analyzed using a very efficient system simulation, which attempts to empty the system virtually to determine whether the move is safe. This classification algorithm is shown to be polynomial in complexity. To avoid deadlocks, only safe part movements should be allowed to proceed. An example is provided to illustrate how the method can be applied.

1. INTRODUCTION

In the past decade, considerable research has been performed on the topic of developing deadlock control policies for flexible manufacturing systems. Some of the significant works have adopted Petri net models as a formalism to describe a manufacturing system. Banaszak and Krogh [1] proposed a *deadlock avoidance algorithm* (DAA) that developed a restriction policy to guarantee that no circular wait situations would occur. Viswanadham *et al.* [9] developed a deadlock avoidance algorithm that used a look-ahead policy. Structural properties of PNs such as siphons are used in [2,4] to determine potential deadlock situations.

Another formalism is to describe the manufacturing system using graphs. In this approach, the vertices represent resources and the edges represent part flows between resources. Cho *et al.* [3] developed the concept of bounded circuits with empty and non-empty shared resources to detect deadlock. Fanti *et al.* [5] developed a simple graph-theoretic method for deadlock detection and recovery in systems with multiple capacity resources. Judd and Faiz [6] derived a set of static linear inequalities that, when satisfied, avoid deadlock. Lipset *et al.* [8] expanded upon [6], and quantified both necessary and sufficient conditions for deadlock to occur in a manufacturing system. Lipset *et al.* [7] developed a polynomial complexity deadlock avoidance algorithm by classifying part movements.

Very few papers are found to have studied the deadlock avoidance problems with a multiple resource request model in the manufacturing context. Reveliotis *et al.* [10] developed a deadlock avoidance policy for a conjunctive resource allocation model by extending their previous result based on the resource upstream neighborhood concept, where a conjunctive resource allocation model is a multiple resource request model and a resource vector is used to represent multiple resource requests in a process plan.

In this paper, we will extend [7] to allow flexible manufacturing systems with both free choices and multiple resource requests in process plans (FCMRR). Section 2 discusses the FCMRR model and properties. Section 3 reviews and modifies the classifications. Section 4 presents the new algorithm, and Section 5 gives an example using the proposed algorithm.

2. FCMRR SYSTEM MODEL

A FCMRR system consists of a set R of resources such as machines, robots, AGVs, etc., and a set P of product types. Each resource has a capacity, denoted as $Cr \geq 1$, which can be considered as Cr identical units. At any time, the system contains a set Q of parts. Each part q belongs to a product type $p \in P$ and each product type p possesses a process plan that describes the resource sequence required to produce it. We assume that a process plan is of finite length. A process plan consists of process steps. In a FCMRR system, a process step can be: i) a simple step that requests exactly one unit of a specific resource; ii) a free choice step that requests one resource unit out of a subset of resources (the OR [11] resource allocation model), or iii) a multiple resource request (MRR) step that requests one unit from each resource in a subset of resources (the AND [11] model). By *free choice* in a process plan, we mean that the part's next resource can be arbitrarily chosen from the subset of resources and the system controller will dispatch the part to the chosen resource. While at a MRR step, a part needs the exclusive use of all the resources in the subset. The implication is that we need to redefine an "enabled" part and the deadlock state of the system.

2.1 The Wait Relation Graph

A FCMRR system can be described by a digraph, called a *wait relation graph* (WRG), which consists of a set of nodes to represent resources of the system and a set of arcs to represent individual process sequences for all product types.

For a choice step, we draw an arc (labeled with a '+') from the current resource to every resource in the subset. When the system is simulated, a part can be simply moved to the resource chosen by the system controller.

For a MRR step, we also draw an arc (labeled with a '•') from the current resource to every resource in the subset. When the system is simulated, a part will be split to all the resources in the subset. Naturally, we will also have a converse operation; that is, to *merge* a set of part splits. If every part split's next resource is the same, then these splits can be merged at the next resource.

A *path* in a wait relation graph G is a set of nodes that are connected to one another within the graph. A *closed path* starts and ends on the same node. A *simple circuit* is a closed path with no repeated nodes, except for the common starting and ending node. A *semi-closed* path is a path that is not closed but contains a closed path. The portion of a semi-closed path that is not on the contained closed path is a *tail*. The resource that connects the tail and the closed portion is the *key resource*.

2.2 Process Plan Representation

Our method to represent a process plan with choices is to "straighten" the plan. Simple steps are listed resource by resource sequentially, and steps that have choices are listed branch by branch sequentially. Each branch further consists of sequences of resources and other branches in an iterative manner. In this way, each process plan can still be represented as a sequence of resources. To unify the various types of process steps, we

decompose a MRR step. That is, we consider each resource of a MRR step as a separate simple step/branch, as in the choice case, and we split the part into every resource of the MRR step. In this way, our method allows a process step to acquire/release resources while keeping some current resources. The method associates each process step with a step type: *s*—for simple, *c*—for choice and *m*—for MRR. In addition to the sequence of resources, our method also associates each process plan with three data structures: i) *T*, a vector that specifies the step type; ii) *N*, a vector that counts the number of branches (choice/MRR) at each step; and iii) *B*, a matrix that identifies the next step(s), where each step has a corresponding column of the matrix. With choices, a process plan can have more than one terminal step. All terminal steps are given the same next step number—0. Several functions are defined. Function *len(q)* returns the plan length of part *q*. Function *branches(q,n)* returns the number of branches at step *n* of the plan of part *q*. Function *step_type(q,n)* returns the type of the step as one of simple, choice or MRR. Function *next(q,n,b)* returns the next step of part *q* after step *n* on branch *b*. See the example in Section 5.

2.3 Deadlock In A FCMRR System

Normally, a flexible manufacturing system would start from an empty state with no parts in the system; that state is usually referred to as the idle state.

Each part *q* has a current step *n* ($0 \leq n \leq \text{len}(q)$), where $n=0$ means the part is waiting to be loaded into the system or has exited the system. A system state is a vector of dimension ΣCr , $\forall r \in R$, denoted as $s = [n: q\text{'s current step}, \forall q \in Q \mid 0 \text{ for empty resource units}]$. The idle state is denoted as s_0 .

A part is *enabled* if its next resource has an empty unit for a simple step, one of its next resources has an empty unit for a choice step or all of its next resources have an empty unit for a MRR step. An enabled part can be moved into its next resource(s). A state s_i is *reachable* from state s_j if there exists a sequence of part movements and the system state changes from s_j to s_i after the sequence of part movements; otherwise, it is *unreachable*.

A system state s_i is called a *deadlock* if state s_0 is unreachable from s_i . Deadlock states can be further categorized into two major types: i) a *primary* deadlock state in which there exist parts in a circular wait situation which can involve parts at a choice or MRR step; and ii) an *impending* deadlock state which is not a primary deadlock state, but must evolve into a primary deadlock state in a finite number of part movements. The algorithm to be presented herein will be able to efficiently avoid both types of deadlocks.

3. CLASSIFICATIONS OF PART MOVES

In the following, we will divide part movements into two major categories, i.e. *live* moves and *dead* moves. See Figure 3.1. A *dead* move will cause a system in a non-deadlock state to enter a deadlock state, and a *live* move will not cause a non-deadlock system to enter a deadlock state. Our algorithm categorizes a large class of moves as live in linear time. These moves are termed *safe* moves. The algorithm also identifies a class of dead moves, which are termed *unsafe*, through the introduction of the canonical semi-closed path concept. Moves that cannot be determined to be either safe or unsafe are classified as *undetermined*. Those will be analyzed further through a virtual system simulation.

When the manufacturing system is in any particular state s , there usually exists an enabled part. Let s' denote the state of the system after an enabled part is moved. An enabled part must be analyzed to determine whether the move will result in deadlock. The following subsections summarize the eight classes of part

Figure 3.1. Classification of moves

movements developed in [7] along with appropriate modifications to accommodate the multiple resource request model.

3.1 Basic Classes

There are three classes of safe moves that can be determined immediately by examining the current and next resources in the process plan.

Class 1. Advancing a part/split that is at its last step is safe.

Class 2. Advancing a part/split onto a non-converging resource (one that has only one input arc) is safe.

Class 3. Advancing a part/split that does not move onto any new closed paths is safe.

Proof. The same as in [7]. ■

3.2 Dynamic Subgraph (Path)

To categorize other classes of part moves, it is necessary to build a dynamic subgraph. A dynamic subgraph is formed by concatenating the resources in the plan of the candidate part until a different part is encountered. The subgraph is continued by appending the resources of that new part's process plan. When a part at a choice/MRR step or a multiple capacity resource with a part is seen, the dynamic subgraph will be split into several branch paths. If the resource is free (not filled to capacity), then a special path corresponding to the part entering the resource is also constructed. That part is called the *ghost part* with respect to this resource because the resource does not actually contain the part. Branch paths are also constructed for all the parts contained in the resource.

For each part, there are three possible types of paths that can exist in such a dynamic subgraph:

- i) an open path,
- ii) a closed path that includes the original part, and
- iii) a semi-closed path in which the original part is on the tail.

Paths on the dynamic subgraph are terminated when

- i) the terminal resource of the plan for the current part is reached,
- ii) a part is encountered for the second time (PST), or
- iii) a resource is encountered for the second time (RST).

3.3 Canonical Semi-Closed Path

A *canonical semi-closed path* (CSC) is a semi-closed path with a part on the tail that enters the closed portion of the CSC, and a part on the closed portion that leaves the CSC through its key resource. This concept is developed to detect/avoid impending deadlocks. Details can be found in [7].

3.4 More Classes For Part Moves

Based on the dynamic subgraph, several other classes can be determined. In the following, let *q* be the candidate part, r_0 be the resource that contains *q* and r_1 be a next resource.

Class 4. Advancing a part/split that has a free exit is safe. A free exit means that all resources needed by the rest of the part's process plan have at least one free unit.

Proof. Same as in [7]. ■

Class 5. Let S_{CSC} be the set of all moves for a CSC such that the pivot part has not yet entered the key resource. Then moving the initiating part on the CSC into its key resource is a class 5 move if it deadlocks the closed portion of the CSC for all moves in S_{CSC} . Since, by definition, the pivot part must leave the system through the key resource, a class 5 move is a dead move and will

be classified as unsafe.

For the purpose of classifying part moves, a CSC can be reduced by replacing the closed portion of the CSC with only the key resource [7]. More complicated class 5 moves can be analyzed through the process of CSC reduction. This reduction will be performed logically by the algorithm introduced later. The system graph will not be physically reduced.

The remaining three classes are all *undetermined*. They can not be classified as either safe or unsafe from only the information developed by building the dynamic subgraph. See [7] for details.

Class 6 (undetermined_0), is a part that moves onto a closed path that contains the part, but is neither a class 2 nor a class 3.

Class 7 (undetermined_1), is a part on an open path but is not a class 4 move.

Class 8 (undetermined_1), is a part that is on the tail of a semi-closed path, but does not fall into any other class.

3.5 A New Class Related to MRR

A new class will be identified based on a MRR step. This class is safe. The class is numbered 2b because the move does not involve any closed path examination.

Class 2b. Advancing a part split whose next step is n_1 to a resource that contains a split n_2 of the same part will cause the two splits to merge, and is, therefore, safe.

Proof. Once this type of move is made, the part will occupy no further resource space but will release the resource space it is occupying. ■

4. DEADLOCK AVOIDANCE ALGORITHM

The modified method to determine whether advancing a candidate part results in a live state or not is presented in the following three algorithms: *dynamo_MRR*, *empty_system* and *live*. *Dynamo_MRR* performs the classification, *empty_system* performs the virtual system simulation and *live* is the main function. For a part at a choice step, the best class (if we order from best to worst as safe, undetermined, and unsafe) among all branches should be chosen since the system controller can determine which choice to select. On the other hand, for a part at a MRR step, the worst classification should be chosen, which means the part is safe only if the part is safe on every branch, and the part is unsafe if the part is unsafe on any branch.

4.1 Dynamic Path Status

Each of the stopping conditions is associated with a status value. *Dynamo_MRR* examines all possible paths in a dynamic subgraph and keeps track of the one with the best-suited status. It then makes conclusions on whether the proposed move is live or not based on this optimal path within the dynamic subgraph.

1) *PST*. A PST status is defined when a part is seen for the second time. PST means that a part was found on either a closed dynamic path or on a semi-closed path.

2) *RST*. An RST status is defined when a free resource is encountered for the second time, indicating a (component) CSC was found. The resource is the key resource. The part leading the path to the RST resource for the first time is the initiating part of the CSC. The part leading the path to the RST resource for the second time is the pivot part. Associated with an RST status is the space flag used to determine whether the space in the key resource is needed to avoid deadlock. A false space flag means that the key resource was seen for the second time without seeing any other free resource after the first time it is seen. This means moving the initiating part into the key resource is a class 5 move for this component CSC. After all component CSCs sharing the same key resource have been found, the space flags of all component CSCs

are Ored/ANded together. If they are all class 5 moves, then algorithm *dynamo* declares that the entire CSC is a class 5 move.

3) *EXIT*. This is the best possible status. EXIT status means either the candidate part or another part can exit. Even though EXIT as a status is better than RST, the algorithm gives higher priority to detect RST over EXIT unless the candidate part is at its terminal step. The reason is to make sure that the dynamic path has enough space to accommodate the candidate part, so that the part that is found to exit actually can exit. A part at a choice step is given an EXIT status as long as EXIT is found on one of the branches. On the other hand, a part at a MRR step receives an EXIT status only if EXIT is found on every branch.

To help build the dynamic subgraph, an AND-OR tree will be created and expanded while the dynamic subgraph is being constructed. The AND-OR tree is composed of nodes. A node is recursively defined with a set of child nodes or (sub) trees,

$$C = (\text{type}, \text{status}, \text{part}, \text{space}, \{C_1, C_2, \dots, C_k\}),$$

where: i) *type* is the type of the node; ii) *status* is the classification status at the node; iii) *part* is the part corresponding to the status; iv) *space* is the space flag to indicate whether free space is available or not on the dynamic path; v) k is the number of child nodes and $C_i, i = 1 \sim k$, is a child node. The set of child nodes is empty only for a terminal node. An internal node is created only for a part at a choice/MRR step and for a non-empty multiple capacity resource. The tree can be graphically represented.

There are three node types:

- i) END for a terminal node, represented as a blank oval;
- ii) OR for a choice step or a non-empty multiple capacity resource, represented as an oval with a "+" in it;
- iii) AND for a MRR step, represented as an oval with a "•" in it.

There are four values for node status:

- i) EXIT for a part that sees a free exit;
- ii) PST for a part that is seen for the second time;
- iii) RST for a free resource that is seen for the second time;
- iv) UNSAFE for the root node that is concluded unsafe.

The node status is set according to the stopping condition for terminal nodes and can be calculated for internal nodes as follows (using the above order),

$$\text{status} = \begin{cases} \text{the lowest child status, for an OR node} \\ \text{the highest child status, for an AND node} \end{cases}$$

The *space* is set to FALSE for an RST terminal node when it is created, TRUE for an EXIT or PST because no CSC is involved and is calculated for an internal node as in the following,

$$\text{space} = \begin{cases} \text{logical OR of child spaces, for OR nodes} \\ \text{logical AND of child spaces, for AND nodes} \end{cases}$$

The space can become TRUE when the algorithm returns to a free resource that has multiple capacity or is not a key resource.

4.2 The Stop-Check-and-Go

The stop-check-and-go mechanism is performed when status RST is detected. This mechanism implements the CSC reduction.

The stop. If the path encounters a resource for the second time that has at least one free unit (RST), then: i) the path stops; ii) the last part (the pivot part) that would have entered the key resource and its process step are recorded in tree C , and; iii) returns to the point when the key resource was visited for the first time.

The check. As the algorithm returns to the key resource, any resources with free capacity will be flagged.

The go. When the algorithm returns to the key resource, the flag indicates whether space on the CSC can be used. If the flag is true, all the pivot parts in C will be chased further to determine which one will produce the optimal path. Otherwise, the algorithm continues to return.

4.3 Algorithm *dynamo_MRR*

Algorithm *dynamo_MRR* consists of a group of procedures: *dynamo_MRR*, *check_resource* and *check_part*. The algorithm defines for class 2b a separate status, called MERGE, for use in the main procedure *dynamo_MRR* and for virtual system simulation only. MERGE is a class of safe moves.

The algorithm first checks whether the move is one of the first four classes of safe moves. If none of those classes applies, then the algorithm starts the construction of the dynamic subgraph which is stored in AND-OR tree *C* by calling *check_resource*. *Check_resource* and *check_part* form an indirect recursion. *Check_resource* calls *check_part* for every part in the resource, including the ghost part, and *check_part* calls *check_resource* for the next resource on each branch. Because of this recursive nature, the construction of the dynamic subgraph consists of two stages: the *forward process* and the *backward process* (processing after returning from a recursive call).

As the construction of the dynamic subgraph progresses forward, the algorithm tracks the free resources it encounters in *R* to check for a RST condition and the parts it sees in *Q* to check for a PST condition. During the backward process, in addition to filling internal nodes with child nodes, the algorithm checks whether any free resources can be used by a component CSC.

The go of the stop-check-and-go is also implemented in the backward process. It actually starts a new forward process, called a *sub-forward process*, which takes the form of a sub-tree and eventually replaces the corresponding RST terminal node. This sub-forward process can be considered the continuation of the main forward process. Since the main forward process never sees the sub-forward process, to get a correct combined AND-OR tree, the parent nodes' data needs to be adjusted based on the sub tree. The adjustment of status is made to every ancestor node until the adjusted status of a node remains unchanged, or the root node is reached. The adjustment of space is also made to every ancestor node until the adjusted space remains unchanged or the root node is reached. Figure 4.1 illustrates the adjustment process.

Figure 4.1 Sub tree and status/space back propagation

4.3.1 Listing of *dynamo_MRR*

```

Let q0 be the candidate part
if q0 is at a simple step then
  if q0 belongs to one of the first four classes then
    return the class
  else call check_resource on next resource r1 to construct
    the AND-OR tree C and return the class found
else (q0 is at a choice/MRR step)
  for next resource r1 on each branch do
    if q0 belongs to one of the first four classes then

```

```

      record the class
    else call check_resource on r1 to construct the AND-OR
      tree C and record the class
  end for
return the best/worst class for choice/MRR
end

```

4.3.2 Listing of *check_part*

```

if the part being checked is at a simple step then
  if the part is at the last step then
    create an END node C with status EXIT
  else call check_resource to construct a sub tree C
else (the part is at a choice/MRR step)
  create an OR or AND node C
  for next resource r on each branch do
    if the part is at the last step then
      create a child END node Ch with status EXIT
    if a choice step then break out the for loop
    else call check_resource on r to construct a sub tree Ch
      and add child node Ch to C
  end for
  set C's status to the best/worst of child node status
  set C's space to the logical OR/AND of child node space
end

```

4.3.3 Listing of *check_resource*

```

1. The forward process
let q be the part being checked
let r be the resource being checked
if r is in R then
  create an END node C with status RST and return
if r is free then (ghost part)
  add r to R, call check_part on q to construct a sub tree Ch
if r has multiple capacity and is not empty then
  create an OR node C and
  if Ch is created then add child node Ch to C
if r is not empty then
  for each part p in r do
    if p is in Q then
      create an END node C with status PST and return
    else add p to Q, call check_part on p to construct a sub-
      tree Ch and if C is created then add child node Ch to C
  end for
if C is created then
  set C's status to the best of child node status
  set C's space to the logical OR of child node space
else set C to Ch (single capacity or empty resource)

2. The stop and check
if r is free but has a FALSE space flag then
  if r is not an key resource or r has more than 1 free space
    or (r has multiple capacity and |R| > 1) then
    set C's space to TRUE
if C's status is RST and space is FALSE and |R| is 1 then
  update C's status to UNSAFE and return

3. The go
while C has an RST node and the key resource is r do
  let p be the pivot part
  call check_part on p to construct a sub tree C1
  replace the RST node with C1 and update all parent
  nodes' status and space
end while
end

```

4.3.4 Analysis of *dynamo_MRR*

The following theorem establishes that the algorithm *dynamo_MRR* terminates and has linear complexity.

Theorem 4.1. The complexity of *dynamo_MRR* is linear in the system size, $L = \sum(\text{len}(p)), \forall p \in P$.

Proof. The same as of *dynamo* in [7]. ■

The following lemma will help to establish the correctness of algorithm *live*.

Lemma 4.2. If algorithm *dynamo_MRR* classifies a part move as safe, then the proposed move is a live move.

Proof. We will show this result in the following three cases:

If the candidate part is at a simple step, then the proposed move is classified as safe only if it belongs to either the first four classes (class 1, 2, 2b and 3) or class 4.

If the candidate part is at a choice step, then the proposed move is classified as safe only if on one of its branches it belongs to either the first four classes (class 1, 2, 2b and 3) or class 4.

If the candidate part is at a MRR step, then the proposed move is classified as safe only if on every branch it belongs to either the first four classes (class 1, 2, 2b and 3) or class 4.

According to choice/MRR definitions and proofs for classes 1, 2, 2b, 3 and 4, the proposed move is indeed a live move. ■

4.4 Algorithm *empty_system*

Given a set of parts Q in state s , *empty_system* attempts to find a set of moves that will remove all parts from the virtual system using only safe moves. It invokes algorithm *dynamo_MRR* on every enabled part. *Empty_system* will execute all the safe moves first. Parts returned with status EXIT from *dynamo* will be taken out of the virtual system immediately. Parts returned with status MERGE or SAFE will be merged or advanced and the set of enabled parts will be re-evaluated. If only undetermined_0 moves exist, then it will arbitrarily pick one and proceed. The algorithm continues this cycle until it empties all parts from the system; it then returns true. It returns false if the system cannot be emptied. It will be shown that the algorithm has polynomial complexity.

4.4.1 Listing of *empty_system*

```

let Q be the set of parts in the system
while Q is not empty do
  let E be the set of enabled parts,  $E \subseteq Q$ 
  call dynamo_MRR to classify each part  $p \in E$ 
  if any part classified as exit, safe, merge or
  undetermined_0 then
    remove all parts classified as exit from Q
    advance/split all parts classified as merge
    if there is a part classified as safe then
      advance/split the part
    else if there is a undetermined_0 part then
      advance/split the part
    else return false (system can not be emptied)
  end while
return true
end

```

4.4.2 Analysis of *empty_system*

The following result shows that the algorithm has polynomial complexity.

Theorem 4.3. The complexity of algorithm *empty_system* is $O(L^3)$, where $L = \sum(\text{len}(p)), \forall p \in P$.

Proof. The same as in [7]. ■

4.5 Algorithm *live*

Among the three algorithms, *live* is the algorithm to be called first to determine whether a proposed move is live. It further

invokes *dynamo_MRR* to perform the classification. Its complexity is linear in L unless *dynamo* returns undetermined, in which case it will invoke *empty_system* and the complexity is therefore $O(L^3)$. *Live* begins by classifying the proposed part move. A virtual move is made if the proposed move is found to be undetermined. *Live* then calls *empty_system* to try to find a way to empty the virtual system to determine whether the original proposed move is indeed live. The algorithm is listed below.

Listing of *live*

```

Let q be the proposed part
call dynamo_MRR to classify q
if q is classified as exit or safe then return true
else if q is classified as unsafe then return false
else (undetermined)
  advance q and return empty_system
end

```

Theorem 4.4. If algorithm *live* returns TRUE (classifies the part move as safe), then the proposed move is a live move.

Proof. *Live* returns TRUE only in the following two cases:

- dynamo* classifies the part as exit or safe;
- dynamo* classifies the part as undetermined, the virtual move is made and *empty_system* returns TRUE to indicate it can find a sequence of moves to empty the system.

In case i), the proposed move is a live move according to Lemma 4.2. In case ii), since *empty_system* can empty the system, the proposed move must be a live move. ■

The above theorem establishes the correctness of algorithm *live*. Unfortunately, the algorithm does not provide optimality. In a few cases, it will be possible for *live* to return false for a live move. This will happen if, while emptying the virtual system, a state is reached where no enabled parts are safe moves, multiple undetermined_0 moves are detected, and the undetermined_0 move that is chosen results in deadlock of the virtual system, whereas selecting a different undetermined_0 move would not have.

5. EXAMPLE

An example illustrating choices in process plans was given in [7]. In this section, we will present an example system with MRR steps in the process plans.

Example 5.1. Let the manufacturing system be composed of three resources r_1, r_2 , and r_3 : All resources are of unit capacity. Suppose the system manufactures two types of products, p_A and p_B , specified by the following process plans: $p_A = r_1 r_2 r_3$, $p_B = r_3 r_2 r_1$. After applying phase I of [2], that is to add the 3 control places to control the 3 minimal uncontrolled siphons identified ($\{p_{A2}, p_{B3}, r_1, r_2\}$, $\{p_{A3}, p_{B2}, r_2, r_3\}$ and $\{p_{A3}, p_{B3}, r_1, r_2, r_3\}$), an augmented Petri net is obtained as in Figure 5.1. However, it is known that this controlled net can still run into deadlock. Now let's apply our method to the augmented system. Part of type p_A will need first r_1, c_1 and c_3 , then r_2, c_2 and c_3 , finally r_3 . The process plan is given in Table 5.1.

Table 5.1. Process plan for p_A

step No.	0	1	2	3	4	5	6
resource list	r_0	r_1	r_2	r_3	c_1	c_2	c_3
step type T	m	m	s	s	m	s	s
N	3	2	1	1	2	1	1
B	1	2	3	0	2	3	3
	4	5			5		
	6						

Parts of type p_B will need first r_3 , c_2 and c_3 , then r_2 , c_1 and c_3 , finally r_1 . The process plan can be given similarly.

Suppose part a of type p_A is already loaded, as shown in Figure 5.2, that is we have three part splits a_1 , a_2 and a_3 . We want to know whether introducing part b of type p_B is live. We first call *live*. *Live* calls *dynamo_MRR* to classify part b and *dynamo_MRR* will check for part b along each of its 3 branches beginning with r_3 , c_2 and c_3 , respectively. The branch with r_3 will return a class 5 through building the AND-OR tree, see the tree built in Figure 5.3.

Figure 5.1 Petri net for example 5.1

Figure 5.2 The WRG for example 5.1

The branch with c_2 will return a class 5 similarly and the branch with c_3 will return a class 7 (a_1 can exit). So, *dynamo_MRR* will return the worst class, which is a class 5. Therefore, loading part b should be avoided.

Note if applying *dynamo_MRR* to the original system without the added three control places, then the algorithm can run much more efficiently because of no MRR steps, avoid all the deadlock states and keep all live states.

6. DISCUSSION

To improve our algorithm, the change would involve trying all undetermined_0 moves if no safe moves existed, instead of selecting one arbitrary move of this type. If that change is made, however, *live* may no longer terminate in polynomial time. A counter could be used in *empty_system* to limit the number of searches of undetermined_0 moves. This counter would then become a tuning parameter to trade off the algorithm's complexity versus the degree of completeness to which the algorithm can determine all live moves. One of future research subjects is to extend our method to include the AND/OR resource request model [11].

Figure 5.3 The AND-OR tree built for example 5.1

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