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GLOBAL NONLINEAR PARAMETRIC MODELING WITH APPLICATION TO F-16 AERODYNAMICS

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ABSTRACT

A global nonlinear parametric modeling technique is described and demonstrated. The technique uses multivariate orthogonal modeling functions generated from the data to determine nonlinear model structure, then expands each retained modeling function into an ordinary multivariate polynomial. The final model form is a finite multivariate power series expansion for the dependent variable in terms of the independent variables. Partial derivatives of the identified models can be used to assemble globally valid linear parameter varying models. The technique is demonstrated by identifying global nonlinear parametric models for nondimensional aerodynamic force and moment coefficients from a subsonic wind tunnel database for the F-16 fighter aircraft. Results show less than 10% difference between wind tunnel aerodynamic data and the nonlinear parameterized model for a simulated doublet maneuver at moderate angle of attack. Analysis indicated that the global nonlinear parametric models adequately captured the multivariate nonlinear aerodynamic functional dependence.

NOMENCLATURE

$\bar{c}$	wing reference chord, ft
b	wing span, ft
C_x, C_y, C_z	aerodynamic force coefficients
C_ℓ, C_m, C_n	aerodynamic moment coefficients
p, q, r	body axis angular rates, rad/sec
PSE	predicted squared error
V	airspeed, ft/sec
α	angle of attack, rad
β	sideslip angle, rad
$\delta_e, \delta_a, \delta_r$	elevator, aileron, rudder deflections, rad
x_{cg}	longitudinal c.g. position
$x_{cg_{ref}}$	reference longitudinal c.g. position = 0.35

1. INTRODUCTION

An important aspect of accurately modeling nonlinear functional dependence is determining the mathematical form relating independent variables to a dependent variable. In general, the goal is to find a compact model structure which still has adequate complexity to capture the nonlinearities. Keeping the number of terms in the model low improves model parameter identifiability, resulting in a more accurate model with good prediction capability.

Models can be loosely classified as local or global. Local models are identified using data from a relatively small region of the independent variable space. It follows that local models are valid for small ranges of the independent variables. A global model results when the range of validity for the identified model covers a large portion of the independent variable space. Given this latter type of (generally nonlinear) model, operations can often be streamlined by replacing many local models with a single global model.

When the global nonlinear model can be identified in a parametric form using simple analytic terms, it is possible to formulate Linear Parameter Varying (LPV) models that are globally valid. Partial derivatives of the global analytical models with respect to the independent variables provide global linear model parameter variations for LPV models. Such models are useful in robust and nonlinear control design. This type of model also provides insight not available from a family of linear constant coefficient models obtained from local linearization of the nonlinear functional dependence at various operating points. Calculation of finite difference linear constant coefficient models requires that perturbation size be chosen carefully to ensure proper linear characterization of a nonlinear functional dependency and to avoid inaccuracies due to local measurement noise. These considerations are avoided with a global nonlinear model. Globally valid analytical models and their associated smooth gradients are also useful for optimization and global nonlinear stability and control analysis.

Recently, a method has been developed which identifies an adequate multivariate polynomial model structure with accurate parameter estimates based on orthogonal modeling functions generated from the data[1]. Selected orthogonal modeling functions are included in the model based on minimizing the predicted squared error, and then are decomposed into ordinary multivariate polynomials. The final identified model consists of selected terms from a multivariable power series expansion for the dependent variable in terms of the independent variables.

The purpose of this work was to investigate the suitability of the orthogonal function modeling technique for global nonlinear modeling on a realistic problem, namely a nonlinear aerodynamic database[2] obtained from wind tunnel tests of a modern fighter[3]. The investigation focuses on the degree to which a realistic nonlinear functional dependence embodied in large tables of values can be modeled using compact multivariate polynomial expressions while retaining good predictive capability. The next section outlines the theoretical development. Following this, the nonlinear modeling technique is applied to a wind tunnel aerodynamic database for the F-16 aircraft.

2. THEORETICAL DEVELOPMENT

Assume an N-dimensional vector of dependent variable values, $y = [y_1, y_2, \dots, y_N]^T$, modeled in terms of a linear combination of n modeling functions p_j, $j = 1, 2, \dots, n$. Each p_j is an N-dimensional vector which in general depends on the independent variables. Then,

$$y = c_1 p_1 + c_2 p_2 + \dots + c_n p_n + \varepsilon \qquad (1)$$

The c_j, $j = 1, 2, \dots, n$, are constant model parameters and ε denotes the modeling error vector. We put aside for the moment the question of how to determine the modeling functions p_j, as well as how to select which functions should be included in the model of Eq. (1) (which implicitly determines n). Now define an Nxn matrix **P**,

$$P = [p_1, p_2, \dots, p_n] \qquad (2)$$

and let $c = [c_1, c_2, \dots, c_N]^T$. Then using Eq. (1),

$$y = P\,c + \varepsilon \qquad (3)$$

The goal is to determine c which minimizes the least squares cost function

$$J = (y - P\,c)^T (y - P\,c) \qquad (4)$$

The least squares estimate of c is computed from[4]

$$\hat{c} = [P^T P]^{-1} P^T y \qquad (5)$$

and the estimated parameter covariance matrix is[4]

$$\mathrm{var}(\hat{c}) = E\{(\hat{c} - c)(\hat{c} - c)^T\} = \frac{\hat{J}}{(N - n)} [P^T P]^{-1} \qquad (6)$$

where E is the expectation operator, and $\hat{J}$ is the cost calculated from Eq. (4) with $c = \hat{c}$.

Reference [1] describes a procedure for using the independent variable data to generate orthogonal modeling functions, which have the following important property:

$$p_i^T p_j = 0 \quad , \quad i \neq j \quad , \quad i, j = 1, 2, \dots, n \qquad (7)$$

Using Eqs. (2) and (7) in Eq. (5), the jth element of the estimated parameter vector $\hat{c}$ is given by

$$\hat{c}_j = \left(p_j^T y\right) / \left(p_j^T p_j\right) \qquad (8)$$

Combining Eqs. (2), (4), (7), and (8),

$$\hat{J} = y^T y - \sum_{j=1}^{n} \left(p_j^T y\right)^2 / \left(p_j^T p_j\right) \qquad (9)$$

Eq. (9) shows that when the modeling functions are orthogonal, the reduction in the estimated cost resulting from including the term $c_j p_j$ in the model depends only the dependent variable data y and the added orthogonal modeling function p_j. This decouples the least squares estimation, and makes it possible to evaluate each orthogonal modeling function in terms of its ability to reduce the least squares model fit to the data, regardless of which other orthogonal modeling functions are present in the model. The orthogonal modeling functions are chosen to minimize predicted squared error PSE, defined by[5]

$$PSE = \frac{\hat{J}}{N} + \sigma_o^2 \frac{n}{N} \qquad (10)$$

where

$$\sigma_o^2 = \frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{y})^2 \quad , \quad \bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i \qquad (11)$$

In Eq. (10), the PSE depends on the mean square fit error $\hat{J}/N$, and a term proportional to the number of terms in the model, n. The latter term prevents overfitting with too many model terms, which is detrimental to model prediction accuracy[5]. Note that while the mean square fit error $\hat{J}/N$ must decrease with the addition of each orthogonal modeling function by Eq. (9), the overfit penalty term $\sigma_o^2\, n/N$ increases with each added model term (n increases), so that PSE always has a single global minimum value. Ref. [5] contains details on the statistical properties of the PSE metric, including justification for its use in modeling problems.

The orthogonal functions are generated in a manner that allows them to be decomposed without ambiguity into an expansion of ordinary multivariate polynomials[1]. The process can be repeated to generate orthogonal functions of arbitrary order in the independent variables, subject only to limitations related to the information contained in the data.

Using orthogonal functions to model the dependent variable made it possible to evaluate the merit of including each modeling function <u>individually</u> as part of the model, using the predicted squared error, PSE. This approach made model structure determination a well-defined and straightforward process. After the orthogonal modeling functions that minimized PSE were selected, each retained orthogonal function was expanded into an ordinary polynomial expression, and common terms in the ordinary polynomials were combined using double precision arithmetic to arrive finally at a multivariate model using only ordinary polynomials in the independent variables. Ordinary polynomial coefficients with absolute value less than 10^{-8} were dropped from the final model.

Orthogonal modeling functions are useful in determining the model structure for the dependent variable using the PSE metric, by virtue of the benefits of orthogonal functions and the resultant decoupling of the associated least squares problem. The subsequent decomposition of the retained orthogonal functions is done to express the results in physically meaningful terms and to allow analytic differentiation for partial derivatives of the dependent variable with respect to the independent variables.

3. RESULTS

Wind tunnel aerodynamic data for a 16% scale model of the F-16 aircraft flying at relatively low Mach numbers (< 0.6), out of ground effect, with landing gear retracted and no external stores, is given in Reference [3]. The data used in this work was a slightly simplified version[2] of the original wind tunnel database. Nondimensional coefficients that vary nonlinearly with flow angles (α, β), aircraft angular velocities (p, q, r), and control surface deflections $(\delta_e, \delta_a, \delta_r)$ characterize the aerodynamic forces and moments acting on the aircraft. Dependence of the nondimensional coefficients on $\dot{\alpha}$ is included in the q dependencies, due to the manner in which the data is collected in the wind tunnel. Each nondimensional aerodynamic force and moment coefficient was built up from a set of component functions, where each component function was determined by a table look-up in the wind tunnel database. The expressions for the nondimensional aerodynamic force and moment coefficients were

$$C_x = C_x(\alpha, \delta_e) + C_{x_q}(\alpha)\tilde{q} \tag{12}$$

$$C_y = C_y(\beta, \delta_a, \delta_r) + C_{y_p}(\alpha)\tilde{p} + C_{y_r}(\alpha)\tilde{r} \tag{13}$$

$$C_z = C_z(\alpha, \beta, \delta_e) + C_{z_q}(\alpha)\tilde{q} \tag{14}$$

$$\begin{aligned} C_\ell = {} & C_\ell(\alpha, \beta) + C_{\ell_p}(\alpha)\tilde{p} + C_{\ell_r}(\alpha)\tilde{r} \\ & + C_{\ell\delta_a}(\alpha, \beta)\delta_a + C_{\ell\delta_r}(\alpha, \beta)\delta_r \end{aligned} \tag{15}$$

$$C_m = C_m(\alpha, \delta_e) + C_{m_q}(\alpha)\tilde{q} + C_z\left(x_{cg_{ref}} - x_{cg}\right) \tag{16}$$

$$\begin{aligned} C_n = {} & C_n(\alpha, \beta) + C_{n_p}(\alpha)\tilde{p} + C_{n_r}(\alpha)\tilde{r} \\ & + C_{n\delta_a}(\alpha, \beta)\delta_a + C_{n\delta_r}(\alpha, \beta)\delta_r \\ & - C_y\left(x_{cg_{ref}} - x_{cg}\right)\left(\frac{\bar{c}}{b}\right) \end{aligned} \tag{17}$$

where

$$\tilde{p} = pb/2V \qquad \tilde{q} = q\bar{c}/2V \qquad \tilde{r} = rb/2V \tag{18}$$

Each function in Eqs. (12)-(17) was modeled individually using the orthogonal function modeling technique described above for the independent variable ranges shown in Table 1. These independent variable ranges represent all the data available in the wind tunnel database.

For example, the $C_x(\alpha, \delta_e)$ function was modeled using tabulated values of that function as the dependent variable and corresponding tabulated values of α and δ_e as the independent variables. The first row of Table 2 specifies the resulting identified model structure, with estimated parameter values from Table 3.

Model structures for all the component functions in Eqs. (12)-(17) appear in Table 2. Estimated parameter values are given in Table 3. Global LPV models can be assembled via analytic partial differentiation of these polynomial aerodynamic models in combination with the equations of motion.

Figure 1 shows results from a simulated lateral/directional doublet maneuver at 10° angle of attack, Mach 0.26 at sea level, with $x_{cg} = 0.25$. The control surface inputs shown in the first two plots of Figure 1 were taken from actual flight test data for the F-16 aircraft. The lower plots in Figure 1 show time histories for sideslip angle, roll rate, and nondimensional yawing moment coefficient, obtained by applying the control inputs shown to a full nonlinear F-16 simulation[2] using the wind tunnel aerodynamic database (solid lines) and the identified polynomial aerodynamic models (dashed lines). Less than 10% difference was seen in these time histories, indicating that the polynomial modeling was successful in capturing the nonlinear aerodynamic functional dependence. Time histories for the other aircraft responses and nondimensional coefficients exhibited a similar level of agreement.

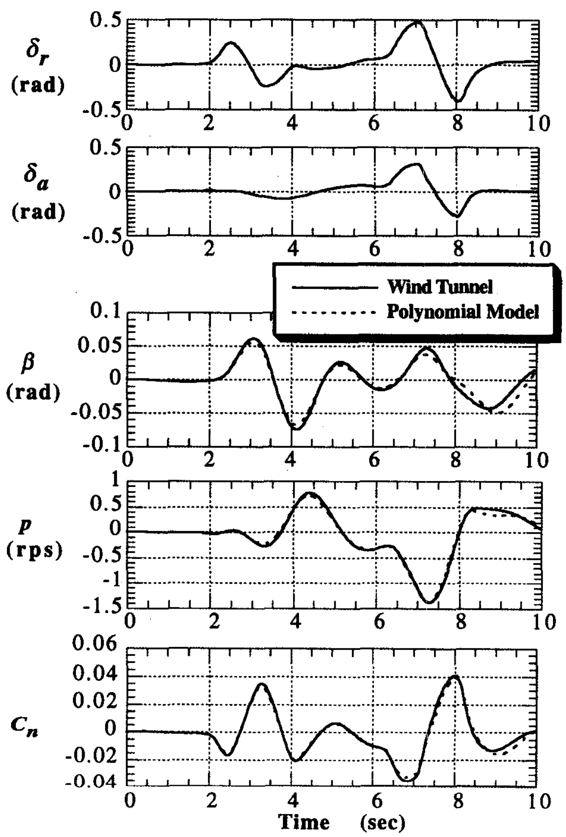

Figure 1 Lateral Directional Doublet Maneuver

4. CONCLUDING REMARKS

Multivariate orthogonal functions generated from the data were used to construct global analytical models for nondimensional aerodynamic force and moment coefficients of the F-16 aircraft based on a subsonic wind tunnel database. Each model was a single ordinary multivariate polynomial in the independent variables, valid for the entire flight envelope encompassed by the wind tunnel data. For a realistic simulated lateral/directional doublet maneuver, aircraft response variables and aerodynamic coefficients computed using the identified polynomial models matched those obtained using the wind tunnel database within 10%. Global nonlinear aerodynamic models like those identified here are useful in many applications, including flight simulation, control system design, and dynamic analysis.

The modeling technique described and demonstrated here is general and can be applied to data from other physical systems. The final result is a compact, global analytical model of the nonlinear functional dependence embodied in the data, with good predictive capabilities. Smooth global analytic derivatives of any order can be easily calculated.

5. REFERENCES

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Table 1 - Independent Variable Ranges for a Compact Global Nonlinear Aerodynamic Model of the F-16

Lower Bound	Variable	Lower Bound
–0.1745 rad (–10 deg)	α	0.7854 rad (45 deg)
–0.5236 rad (–30 deg)	β	0.5236 rad (30 deg)
–0.4363 rad (–25 deg)	δ_e	0.4363 rad (25 deg)
–0.3752 rad (–21.5 deg)	δ_a	0.3752 rad (21.5 deg)
–0.5236 rad (–30 deg)	δ_r	0.5236 rad (30 deg)

Table 2 - Model Structure for a Compact Global Nonlinear Aerodynamic Model of the F-16

Function	Model Structure
$C_x(\alpha,\delta_e)$	$a_0+a_1\alpha+a_2\delta_e^2+a_3\delta_e$ $+a_4\alpha\delta_e+a_5\alpha^2+a_6\alpha^3$
$C_{x_q}(\alpha)$	$b_0+b_1\alpha+b_2\alpha^2+b_3\alpha^3+b_4\alpha^4$
$C_y(\beta,\delta_a,\delta_r)$	$c_0\beta+c_1\delta_a+c_2\delta_r$
$C_{y_p}(\alpha)$	$d_0+d_1\alpha+d_2\alpha^2+d_3\alpha^3$
$C_{y_r}(\alpha)$	$e_0+e_1\alpha+e_2\alpha^2+e_3\alpha^3$
$C_z(\alpha,\beta,\delta_e)$	$\begin{pmatrix} f_0+f_1\alpha+f_2\alpha^2 \\ +f_3\alpha^3+f_4\alpha^4 \end{pmatrix}(1-\beta^2)+f_5\delta_e$
$C_{z_q}(\alpha)$	$g_0+g_1\alpha+g_2\alpha^2+g_3\alpha^3+g_4\alpha^4$
$C_l(\alpha,\beta)$	$h_0\beta+h_1\alpha\beta+h_2\alpha^2\beta+h_3\beta^2$ $+h_4\alpha\beta^2+h_5\alpha^3\beta+h_6\alpha^4\beta+h_7\alpha^2\beta^2$
$C_{l_p}(\alpha)$	$i_0+i_1\alpha+i_2\alpha^2+i_3\alpha^3$
$C_{l_r}(\alpha)$	$j_0+j_1\alpha+j_2\alpha^2+j_3\alpha^3+j_4\alpha^4$
$C_{l_{\delta_a}}(\alpha,\beta)$	$k_0+k_1\alpha+k_2\beta+k_3\alpha^2$ $+k_4\alpha\beta+k_5\alpha^2\beta+k_6\alpha^3$
$C_{l_{\delta_r}}(\alpha,\beta)$	$l_0+l_1\alpha+l_2\beta+l_3\alpha\beta$ $+l_5\alpha^2\beta+l_6\alpha^3\beta+l_7\beta^2$
$C_m(\alpha,\delta_e)$	$m_0+m_1\alpha+m_2\delta_e+m_3\alpha\delta_e+m_4\delta_e^2$ $+m_5\alpha^2\delta_e+m_6\delta_e^3+m_7\alpha\delta_e^2$
$C_{m_q}(\alpha)$	$n_0+n_1\alpha+n_2\alpha^2+n_3\alpha^3$ $+n_4\alpha^4+n_5\alpha^5$
$C_n(\alpha,\beta)$	$o_0\beta+o_1\alpha\beta+o_2\beta^2+o_3\alpha\beta^2$ $+o_4\alpha^2\beta+o_5\alpha^2\beta^2+o_6\alpha^3\beta$
$C_{n_p}(\alpha)$	$p_0+p_1\alpha+p_2\alpha^2+p_3\alpha^3+p_4\alpha^4$
$C_{n_r}(\alpha)$	$q_0+q_1\alpha+q_2\alpha^2$
$C_{n_{\delta_a}}(\alpha,\beta)$	$r_0+r_1\alpha+r_2\beta+r_3\alpha\beta$ $+r_4\alpha^2\beta+r_5\alpha^3\beta+r_6\alpha^2$ $+r_7\alpha^3+r_8\beta^3+r_9\alpha\beta^3$
$C_{n_{\delta_r}}(\alpha,\beta)$	$s_0+s_1\alpha+s_2\beta+s_3\alpha\beta$ $+s_4\alpha^2\beta+s_5\alpha^2$

Table 3 - Parameter Values for Global Nonlinear F-16 Aerodynamic Model

a_0	–1.943367e–02
a_1	2.136104e–01
a_2	–2.903457e–01
a_3	–3.348641e–03
a_4	–2.060504e–01
a_5	6.988016e–01
a_6	–9.035381e–01
b_0	4.833383e–01
b_1	8.644627e+00
b_2	1.131098e+01
b_3	–7.422961e+01
b_4	6.075776e+01
c_0	–1.145916e+00
c_1	6.016057e-02
c_2	1.642479e-01
d_0	–1.006733e-01
d_1	8.679799e-01
d_2	4.260586e+00
d_3	–6.923267e+00
e_0	8.071648e-01
e_1	1.189633e-01
e_2	4.177702e+00
e_3	–9.162236e+00
f_0	–1.378278e–01
f_1	–4.211369e+00
f_2	4.775187e+00
f_3	–1.026225e+01
f_4	8.399763e+00
f_5	–4.354000e–01
g_0	–3.054956e+01
g_1	–4.132305e+01
g_2	3.292788e+02
g_3	–6.848038e+02
g_4	4.080244e+02

h_0	–1.058583e–01
h_1	–5.776677e–01
h_2	–1.672435e–02
h_3	1.357256e–01
h_4	2.172952e–01
h_5	3.464156e+00
h_6	–2.835451e+00
h_7	–1.098104e+00
i_0	–4.126806e–01
i_1	–1.189974e–01
i_2	1.247721e+00
i_3	–7.391132e–01
j_0	6.250437e–02
j_1	6.067723e–01
j_2	–1.101964e+00
j_3	9.100087e+00
j_4	–1.192672e+01
k_0	–1.463144e–01
k_1	–4.073901e–02
k_2	3.253159e–02
k_3	4.851209e01
k_4	2.978850e–01
k_5	–3.746393e–01
k_6	–3.213068e–01
l_0	2.635729e–02
l_1	–2.192910e–02
l_2	–3.152901e–03
l_3	–5.817803e–02
l_4	4.516159e–01
l_5	–4.928702e–01
l_6	–1.579864e–02
m_0	–2.029370e–02
m_1	4.660702e–02
m_2	–6.012308e–01
m_3	–8.062977e–02
m_4	8.320429e–02
m_5	5.018538e–01
m_6	6.378864e–01
m_7	4.226356e–01

n_0	–5.159153e+00
n_1	–3.554716e+00
n_2	–3.598636e+01
n_3	2.247355e+02
n_4	–4.120991e+02
n_5	2.411750e+02
o_0	2.993363e–01
o_1	6.594004e–02
o_2	–2.003125e–01
o_3	–6.233977e–02
o_4	–2.107885e+00
o_5	2.141420e+00
o_6	8.476901e–01
p_0	2.677652e–02
p_1	–3.298246e–01
p_2	1.926178e–01
p_3	4.013325e+00
p_4	–4.404302e+00
q_0	–3.698756e–01
q_1	–1.167551e–01
q_2	–7.641297e01
r_0	–3.348717e–02
r_1	4.276655e–02
r_2	6.573646e–03
r_3	3.535831e–01
r_4	–1.373308e+00
r_5	1.237582e+00
r_6	2.302543e–01
r_7	–2.512876e–01
r_8	1.588105–01
r_9	–5.199526e–01
s_0	–8.115894e–02
s_1	–1.156580e–02
s_2	2.514167e–02
s_3	2.038748e–01
s_4	–3.337476e–01
s_5	1.004297e–01