Monitoring Functional Integrity in Critical Control Computers Subjected to Electromagnetic Disturbances

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ABSTRACT

Verifying the integrity of control computers operating in harsh electromagnetic environments is a key issue in the development, certification, and operation of control systems performing flight critical functions for future transport aircraft. Harsh electromagnetic environments (EME) are caused by sources such as lightning, high-intensity radiated fields (HIRF) from radars and radio frequency (RF) transmitters, personal electronic devices carried onto the aircraft by passengers, electromagnetic interference and incompatibilities with onboard equipment, onboard or ground-based terrorists using RF weapons, and erroneous ground-based or airborne military activities.

This paper considers the problem of applying analytical redundancy, distributed detection techniques, and decision fusion to monitoring the integrity of a fault tolerant flight control computer with redundant processors for real-time laboratory and in-flight applications. This work is in the initial phase and focuses on the formulation of the design problem. The analytical redundancy for each control command is based on the nominal control law calculation using linear parameter varying (LPV) modeling techniques. Malfunctions in the control law calculation of each processor are detected with statistical decision rules. The threshold of the decision rules is based on the residual between the measured control law calculation (that may result from computer malfunctions) from the control computer and the estimate of the correct command obtained from an LPV Kalman filter. Therefore, the decision rules are scheduled over the varying parameters. Malfunctions in each processor are detected by fusing the decisions from M control law calculation monitors. Malfunctions in the fault tolerant controller are detected by fusing the decisions from the monitors for N processors. Analysis of the performance of the distributed monitoring system is discussed.

1. INTRODUCTION

Future advanced commercial aircraft will require systems for stability augmentation and flutter suppression, as well as guidance and control [1]. Such systems will be flight-critical since the flight of the aircraft will depend on reliable operation of these systems. The problem of verifying the integrity of the control computer in adverse, as well as nominal, operating conditions is a key issue in the development, certification, and operation of critical control systems. One adverse operating condition is the electromagnetic environment (EME) caused by sources such as lightning, high-intensity radiated fields (HIRF) associated with radar and radio frequency (RF) transmitters, personal electronic devices carried onto the aircraft by passengers, onboard or ground-based terrorists using RF weapons, and erroneous ground-based or airborne military

activities. Electromagnetic fields may cause analog electrical transients to be induced on the aircraft's wiring, and these signals can propagate to the onboard electronic equipment and result in a failure of multiple processors in the control computer that affect performance and reliability. The failure phenomena associated with electromagnetically induced transient signals that affect performance and reliability of a digital system without component damage are collectively known as digital system upset. The occurrence of digital system upset in critical flight systems can cause catastrophic deviations from the flight path of the aircraft that compromise safe flight [2].

Upset phenomena can include ([3] - [7]) changes in data values of the input/output circuitry, logic changes on the data bus, address bus, and control lines of the processors, logic changes in registers of the central processing unit (CPU) of the processors, and logic changes in the arithmetic logic unit (ALU) within the CPU of the processors. Upset phenomena such as these can interfere with normal operation of the processors within a control computer and result in degraded performance and reliability at the closed-loop system level. Since there are numerous error modes that can occur in a digital controller, it is impractical to determine and model each of the failure modes and then design a corresponding detector. The most practical strategy is to model the nominal function of the controller and detect when it is not performing nominally.

Previous work on detecting computer upset includes a processor-level scheme [8], as well as a systems-level scheme [9] - [10]. The monitor presented in [9] - [10] is based on linear state space models. The design was demonstrated on the longitudinal control laws of a simulated quad-redundant B737 Autoland control computer, from glideslope engaged until flare, using a detailed simulation of the B737 aircraft, engines, Autoland control laws, sensors, actuators, and atmosphere for the case of landing in clear air turbulence. The monitor design formulation presented in this paper is an improvement to the monitor design presented in [10]. The analytical redundancy of the previous monitor was based on a series of linear models for the nominal control law calculations. The analytical redundancy for each control command of the monitor design formulation presented in this paper is based on the nominal control law calculation using linear parameter varying (LPV) model techniques. Malfunctions in the control law calculation of each processor are detected with statistical decision rules. The threshold for each decision rule is compared to the residual between the measured control law calculation (that could result from computer malfunctions) from the control computer and the estimate of the correct command obtained from an LPV Kalman filter, assuming that the values of the varying parameters are available for

computation. Therefore, the decision rules are scheduled according to the values of the varying parameters for each Autoland flight simulation.

The objective of this paper is the design formulation of a Control Law Calculation Malfunction detector using an improved analytical redundancy scheme that enables the decision rules to be scheduled according to varying parameters. The modeling problem is discussed in Section 2. The formulation of the controller malfunction problem is presented in Section 3. The design formulation of the monitor is presented in Section 4. A summary and some conclusions are presented in Section 5.

2. MODELING PROBLEM

A model-based monitoring scheme has been chosen for the formulation of the design problem because it is desired to have the capability of analytically determining the performance of the detector. The model developed must depict the dynamic variation in the control commands that occur over the flight condition of interest, the stochastic variation that occurs when the aircraft is subjected to atmospheric disturbances and when the measurements are noisy, and the uncertainty associated with the tracking of the aircraft for each flight. The monitor that will be designed with this formulation will be demonstrated using a detailed simulation of the B737 aircraft, engines, Autoland control laws, sensors, actuators, and atmosphere for the case of landing in clear air turbulence. Once the monitoring scheme is demonstrated via simulation, it will be implemented in the laboratory for demonstration with real-time flight systems subjected to electromagnetic disturbances. In-flight applications are ultimately planned.

The monitor design presented in [10] was demonstrated on the longitudinal control laws of a simulated quad-redundant B737 Autoland control computer, from glideslope engaged until the initiation of the flare sequence. The commands were modeled using a series of linear state space models. The flight sequence from glideslope engaged to the initiation of flare was divided into segments. A linear state space model of the control command was developed for each segment along with the associated monitor. The model and decision rule were constant throughout each segment, but varied from segment to segment and were scheduled by data frame. This scheduling approach is inadequate for implementation in the laboratory for demonstration on real-time flight systems. Therefore, a model-based detection scheme will be formulated that will allow the decision rule to be scheduled according to parameter variations that occur during each flight simulation.

3. CONTROLLER MALFUNCTION PROBLEM

The fault tolerant controller has N processors that each calculate M control laws. The control commands calculated by processor i are:

$$\vec{x}_i(k) = [x_i^1(k) \ x_i^2(k) \ \dots \ x_i^M(k)]^T \quad (1)$$

Time step k corresponds to one data frame of the controller in which all control laws are calculated and output. The input vector for the M control law calculations of the ith processor is:

$$\vec{u}_i(k) = [\vec{u}_i^1(k) \ \vec{u}_i^2(k) \ \dots \ \vec{u}_i^M(k)]^T \quad (2)$$

Each vector element in equation (2) is the input vector for the jth command calculation of processor i:

$$\vec{u}_i^j(k) = [u_{i1}(k) \ u_{i2}(k) \ \dots \ u_{iL}(k)]^T \quad (3)$$

Each processor in the controller has its own input circuitry. Therefore, the inputs for the command calculations of processor i may have different values than those of processor j even when they are from the same sensors.

The vector of measurements $\vec{z}_i(k)$ from the ith processor that is input to the detector is:

$$\vec{z}_i(k) = [z_i^1(k) \ z_i^2(k) \ \dots \ z_i^M(k)]^T \quad (4)$$

$$z_i^j(k) = H_i^j x_i^j(k) + v_i^j(k); \ i = 1, \dots, N; \ j = 1, \dots, M \quad (5)$$

where:

$z_i^j(k)$ = meas. of jth command from processor i; $z_i^j(k) \in \mathcal{R}$

$v_i^j(k)$ = meas. noise for jth command of processor i; $v_i^j(k) \in \mathcal{R}$

H_i^j = measurement weighting coefficient

Three types of malfunction are defined.

DEFINITION 1: The jth control law calculation of the ith processor in a fault tolerant control computer is the result of a malfunction if:

$$|\Delta x_i^j(k)| > \epsilon_x^j(k) \text{ for } K \text{ time steps} \quad (6)$$

where:

$\Delta x_i^j(k)$ = change in the jth control law calculation of the ith processor due to malfunction

$\epsilon_x^j(k)$ = maximum allowable variation of $x_i^j(k)$

$|\cdot|$ = absolute value

The change $\Delta x_i^j(k)$ in the calculation of the jth control law of the ith processor due to malfunction is defined as:

$$\Delta x_i^j(k) = x_i^j(k) - E[\hat{x}_i^j(k) | \mathcal{X}_i^{j, k-1}] \quad (7)$$

where:

$x_i^j(k)$ = actual jth control law calculation at time k of the ith processor (may reflect a malfunction)

$\hat{x}_i^j(k)$ = correct (no malfunction) control law calculation j of the ith processor at time k

$\mathcal{X}_i^{j, k-1}$ = set of all past $\hat{x}_i^j(k)$ up to time k-1

$E[x_i^j(k)|x_i^j(k-1)]$ = conditional expectation of correct (no malfunction) control calculation j at time step k given all past $x_i^j(k)$ up to time $k-1$

DEFINITION 2: The i th processor in a fault tolerant control computer is malfunctioning if equation (7) holds for any j in $\{1, \dots, M\}$.

DEFINITION 3: The fault tolerant control computer is malfunctioning if equation (7) holds for any i in $\{1, \dots, N\}$.

Definition 3 considers the most conservative case in detecting malfunctions. By this definition, the fault tolerant controller is malfunctioning if any of the processors is malfunctioning. This is because the reliability of the fault tolerant computer is degraded if even one processor is malfunctioning. Other definitions of malfunction in the fault tolerant controller could also be considered.

The correct (no malfunction) command calculation, $x_i^j(k)$, in the reference term of equation (7) is defined:

$$x_i^j(k+1) = \bar{F}_i^j(\vec{\rho}) x_i^j(k) + \bar{G}_i^j(\vec{\rho}) \bar{u}_i^j(k) + \zeta_i^j w_i^j(k) \quad (8)$$

where:

$x_i^j(k) \in \mathcal{R}$; $i = 1, 2, \dots, N$; $j = 1, \dots, M$

$\bar{u}_i^j(k)$ = inputs to j th command calculation of *processor* i from sim. ;

$$\bar{u}_i^j(k) \in \mathcal{U}$$

$w_i^j(k)$ = process noise for j th command calculation of *processor* i ;

$$w_i^j(k) \in \mathcal{W}$$

$\bar{F}_i^j(\vec{\rho})$ = system matrix for correct command calculation j of *processor* i that is dependent on parameter vector $\vec{\rho}(k)$

$\bar{G}_i^j(\vec{\rho})$ = input matrix for correct command calculation j of *processor* i that is dependent on parameter vector $\vec{\rho}(k)$

The system matrix $\bar{F}_i^j(\vec{\rho})$ and input matrix $\bar{G}_i^j(\vec{\rho})$ are dependent on the parameter vector $\vec{\rho}(k)$ ([12] - [14]). The parameter vector $\vec{\rho}(k)$ may contain the values of the varying parameters, as well as the rate of change of the variations. The process noise $w_i^j(k)$ in equation (8) accounts for modeling error, noise in the input vector $\bar{u}_i^j(k)$ from the plant caused by signal conversions, and stochastic variations in the command that result from exogenous disturbances to the simulated plant.

Command calculation j of the i th processor (which may reflect malfunction) is defined as:

$$x_i^j(k+1) = F_i^j(\vec{\rho}, k) x_i^j(k) + \bar{G}_i^j(\vec{\rho}, k) \bar{u}_i^j(k) + \zeta_i^j w_i^j(k) \quad (9)$$

where:

$x_i^j(k) \in \mathcal{R}$; $i = 1, 2, \dots, N$; $j = 1, \dots, M$

$\vec{\rho}$ = the vector of varying parameters

The initial state of the j th command calculation of the i th processor is denoted as $x_i^j(k_0)$.

ASSUMPTION 1: The initial state $x_i^j(k_0)$ is a Gaussian random variable with mean $\bar{x}_i^j(k_0)$ and variance $P_i^j(k_0)$. The initial state of the calculations of the i th processor are independent so that

$$\text{cov}[x_i^a(k_0), x_i^b(k_0)] = 0 \quad \text{for } a \neq b \quad \text{where } \text{cov}[\bullet] \text{ is the covariance.}$$

The initial state of the j th calculation of *processor* i is independent of that of *processor* j so that

$$\text{cov}[x_i^c(k_0), x_i^d(k_0)] = 0 \quad \text{for } c \neq d. \quad \#$$

ASSUMPTION 2: The process noise $w_i^j(k)$ is zero-mean, Gaussian, and white with variance Q_i^j . The process noise of the calculations of the i th processor are independent so that

$$\text{cov}[w_i^a(k), w_i^b(k)] = 0 \quad \text{for } a \neq b.$$

The process noise of the j th calculation of *processor* i is independent of that of *processor* j so that

$$\text{cov}[w_i^c(k), w_i^d(k)] = 0 \quad \text{for } c \neq d.$$

Process noise $w_i^j(k)$ is independent of the initial state. #

ASSUMPTION 3: Malfunction phenomena in a digital control computer that result in errors in the j th control law calculation of the i th processor, modeled by equation (9), can be represented by parameter changes $\Delta F_i^j(k)$ and $\Delta \bar{G}_i^j(k)$ in the nominal values of matrices $F_i^j(\vec{\rho}, k)$ and $\bar{G}_i^j(\vec{\rho}, k)$, respectively, so that:

$$F_i^j(\vec{\rho}, k) = \bar{F}_i^j(\vec{\rho}) + \Delta F_i^j(k) \quad (10)$$

$$\bar{G}_i^j(\vec{\rho}, k) = \bar{G}_i^j(\vec{\rho}) + \Delta \bar{G}_i^j(k) \quad (11)$$

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The terms $\bar{F}_i^j(\vec{\rho})$ and $\bar{G}_i^j(\vec{\rho})$ are the nominal values of the matrices $F_i^j(\vec{\rho}, k)$ and $\bar{G}_i^j(\vec{\rho}, k)$, respectively, are dependent on the varying parameter vector $\vec{\rho}(k)$, and are used in determining $x_i^j(k)$ for the reference signal. The time-varying terms $\Delta F_i^j(k)$ and $\Delta \bar{G}_i^j(k)$ reflect the perturbation in matrices $F_i^j(\vec{\rho}, k)$ and $\bar{G}_i^j(\vec{\rho}, k)$, respectively, that occur due to malfunction. Substituting equations (10) and (11) into equation (9) yields:

$$x_i^j(k+1) = [\bar{F}_i^j(\vec{\rho}) + \Delta F_i^j(k)] x_i^j(k) + [\bar{G}_i^j(\vec{\rho}) + \Delta \bar{G}_i^j(k)] \bar{u}_i^j(k) + \zeta_i^j w_i^j(k) \quad (12)$$

Since the malfunction is stochastic, the perturbations $\Delta F_i^j(k)$ and $\Delta \bar{G}_i^j(k)$ are modeled as stochastic processes [10].

4. MONITOR DESIGN FORMULATION

The malfunctions to be detected are defined by Definitions 1 - 3 relative to (1) the control law calculation j of *processor i*, (2) *processor i*, and (3) the fault tolerant controller. Detection of the phenomenon in each of these definitions is binary and can, therefore, be defined in terms of general binary hypotheses:

$$\begin{aligned} H_1 &: \text{Malfunction Condition} \\ H_0 &: \text{Nominal (No Malfunction) Condition} \end{aligned} \quad (13)$$

The calculations of each processor are observed via noisy measurements $\vec{z}_i(k)$ with probability density $p[\vec{z}_i(k)|H_i]$ from each of the processors.

ASSUMPTION 4: Measurement noise $v_i^j(k)$ is zero mean,

Gaussian, and white with covariance matrix R_i^j . The measurement noise of the calculations of the i th processor are independent so that $\text{cov}[v_i^a(k), v_i^b(k)] = 0$ for $a \neq b$. The measurement noise of the j th calculation of *processor i* is independent of that of *processor j* so that

$$\text{cov}[v_i^j(k), v_d^j(k)] = 0 \text{ for } c \neq d. \quad \#$$

ASSUMPTION 5: Measurement noise $v_i^j(k)$ is assumed to be statistically independent of the initial state $\vec{x}_i^j(k_0)$ and the process noise $w_i^j(k)$. #

3.1 Malfunction Detector for the j th Control Law Calculation of Processor i

The approach for detecting malfunctions in the control law calculations is shown in Figure 1:

Figure 1: Design Approach for Detecting Malfunctions in Control Law Calculation j of *Processor i*.

In the design approach of Figure 1, control law calculation j of the i th processor is monitored using the residual $r_i^j(\vec{p}, k)$:

$$r_i^j(\vec{p}, k) = z_i^j(k) - H_i^j \hat{x}_i^j(\vec{p}, k|k-1) \quad (14)$$

where $z_i^j(k)$ and H_i^j are defined by equation (5), and $\hat{x}_i^j(\vec{p}, k|k-1)$ is an estimate of $E[\vec{x}_i^j(k)|\mathcal{X}_i^{j, k-1}]$ defined in equation (7) for the linear parameter varying model of the control law calculation defined in (8). The estimate $\hat{x}_i^j(\vec{p}, k|k-1)$ is produced using an LPV Kalman filter whose inputs are the input vector $\vec{u}_i^j(k)$ from the plant, the varying parameter vector $\vec{p}(k)$, and the previous

measurement $z_i^j(k-1)$ of the calculation estimate under the nominal hypothesis, H_0 . Formulation of the Kalman Filter for LPV systems is given in [14]. A method for determining the parameterized model is given in [15], and a method for obtaining bounds on the uncertain parameters is given in [16].

A separate single binary upset decision $d_i^j(\vec{p}, k)$ is made for each command calculation of *processor i*:

$$d_i^j(\vec{p}, k) = \begin{cases} 1 \Rightarrow \text{calculation } \vec{x}_i^j(k) \text{ of eqn. (9)} \\ \quad \text{reflects malfunction } (H_1) \\ -1 \Rightarrow \text{calculation } \vec{x}_i^j(k) \text{ of eqn. (9)} \\ \quad \text{reflects no malfunction } (H_0) \end{cases} \quad (15)$$

The mapping for each error decision of the local detector is:

$$d_i^j(\vec{p}, k) = \begin{cases} 1, & f[r_i^j(\vec{p}, k)] \geq TH_i^j(k) \\ -1, & f[r_i^j(\vec{p}, k)] < TH_i^j(k) \end{cases} \quad (16)$$

where:

$f[r_i^j(\vec{p}, k)]$ = function of the residual of the j th control law calculation of the i th processor

$TH_i^j(k)$ = threshold for the decision rule that optimizes some performance criterion

Since the residual $r_i^j(\vec{p}, k)$ is based on the estimate $\hat{x}_i^j(\vec{p}, k|k-1)$ for the LPV model, the decision rule is scheduled over the vector of varying parameters $\vec{p}(k)$. The performance of the detector is determined in terms of the probability of a false alarm and the probability of a missed detection. The probability of false alarm can be written as:

$$P_{F_i^j}(k) = \int_{\lambda_i^j(k)}^{\infty} p_{H_0}[z_i^j(k)|Z_i^{j, k-1}] dz_i^j(k) \quad (17)$$

where $p_{H_0}[z_i^j(k)|Z_i^{j, k-1}]$ is the conditional probability under the nominal hypothesis of the measurement from the i th processor at time k given all past measurements, and $\lambda_i^j(k)$ is the threshold of the decision rule. The probability of a missed error detection in the j th calculation of the i th processor is:

$$P_{M_i^j}(k) = 1 - \int_{\lambda_i^j(k)}^{\infty} p_{H_1}[z_i^j(k)|Z_i^{j, k-1}] dz_i^j(k) \quad (18)$$

where $p_{H_1}[z_i^j(k)|Z_i^{j, k-1}]$ is the conditional probability under the malfunction hypothesis of the measurement from the i th

processor at time k given all past measurements, and $\lambda_i^j(k)$ is the threshold of the decision rule.

3.2 Malfunction Detector for Processor i

The decision $d_i(k)$ for the i th processor is produced by a weighted sum of the decisions $d_i^j(\vec{p}, k)$ [$j = 1, \dots, M$] from the detectors for the control law calculations of the i th processor. The weight for decision $d_i^j(\vec{p}, k)$ is based on the performance of the corresponding detector. The performance of the detector for processor i is given in [10].

3.3 Malfunction Detector for the Controller

The decision $d(k)$ for the controller is produced by a weighted sum of the decisions $d_i(k)$ [$i = 1, \dots, N$] from the detector for the i th processor. The weight for each decision $d_i(k)$ is based on the performance of the detector for the i th processor as characterized by the probability of false alarm and the probability of a missed detection. The performance of the detector for the controller is in [10].

5.0 SUMMARY AND CONCLUSIONS

Verifying the integrity of control computers operating in harsh electromagnetic environments is a key issue in the development, certification, and operation of control systems performing flight critical functions for future transport aircraft. The integrity of critical fault-tolerant control computers can be viewed as the reliable system-level operation of controller functions. This paper considers the problem of applying analytical redundancy, distributed detection techniques, and decision fusion to monitoring the integrity of a redundant flight control computer. A model-based monitoring scheme was chosen for the formulation of the design problem because it was desired to have the capability to analytically determine the performance of the monitor. This work is in the initial phase and focuses on the formulation of the design problem. The contribution of this paper is the formulation of the monitoring problem using LPV modeling techniques to provide analytical redundancy for detecting malfunctions in a fault tolerant control computer, and in any one of the individual processors. The analytical redundancy for each control command is based on an estimate of the nominal control law calculation using linear parameter varying (LPV) modeling techniques. Malfunctions in the control law calculation of each processor are detected with statistical decision rules. The decision rules are based on the residual between the measured control law calculation (that may result from computer malfunctions) from the control computer and the estimate of the correct command obtained from an LPV Kalman Filter. Therefore, the decision rule is scheduled over the varying parameters. Malfunctions in each processor are detected by fusing the decisions from M control law calculation monitors. Malfunctions in the fault tolerant controller are detected by fusing the decisions from the monitors for N processors. Analysis of the performance of the distributed monitoring system is discussed.

Future plans include the design of the monitor for a quad redundant flight control computer executing B737 Autoland control laws. The design will be demonstrated using a detailed simulation of the B737 aircraft, engines, Autoland control laws, sensors, actuators, and atmosphere for the case of landing in clear air turbulence. Once the design is demonstrated using the simulation, the monitor will be implemented in the laboratory for demonstration with real-time flight control system hardware subjected to electromagnetic disturbances.

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