REFERENCES

- R. K. Brown et al., "A lightweight and self-contained airborne navigational system," Proc. IRE, vol. 47, no. 5, pp. 778-807, May 1959.
- [2] G. B. Bush et al., "An analysis of a satellite multibeam altimeter," Marine Geodesy, vol. 8, nos. 1–4, pp. 345–384, 1984.
 [3] R. L. Sternberg, "Successive approximation and expansion methods in
- [3] R. L. Sternberg, "Successive approximation and expansion methods in the numerical design of microwave dielectric lenses," J. Math. Phys., vol. XXXIV, no. 4, Jan. 1955.
- [4] —, "Surface-series expansion coefficients for numerical design of scannable aspherical microwave and acoustic lenticular antennas," *Proc. Inst. Elec. Eng.*, vol. 121, no. 11, Nov. 1974.
- [5] W. V. T. Rusch and A. C. Ludwig, "Determination of the maximum scan-gain contours of a beam scanning paraboloid and their relationship to the Petzval surface," *IEEE Trans. Antennas and Propagat.*, vol. AP-21, pp. 141-147, Mar. 1973.
- [6] W. A. Imbriale et al., "Large lateral feed displacements in a parabolic reflector," *IEEE Trans. Antennas Propagat.*, vol. AP-22, pp. 742– 745, Nov. 1974.
- [7] A. V. Mrstik and P. G. Smith, "Scanning capabilities of large parabolic cylinder reflector antennas with phased-array feeds," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 455-462, May 1981.

A Novel Principle for Optimization of the Instantaneous Fourier Plane Coverage of Correlation Arrays

T. J. CORNWELL

Abstract—A novel principle for the design of correlation arrays is introduced, based upon the maximization of the distance between samples. A relatively new optimization technique, simulated annealing, is applied to the problem of finding solutions for moderate numbers of elements: up to 12. The resulting arrays have beautiful, symmetric crystalline structures.

INTRODUCTION

One very important part of the design of radio-interferometric (i.e., correlation) arrays is the choice of the optimum layout of the receiving elements. The set of element separations as seen by a celestial object, and tracked over some period of time, determines the sampling of the Fourier transform of the sky brightness performed by such an array, and, in turn, the sampling determines the fidelity of image reconstruction possible [7]. Over the years, many different design principles have been proposed: for example, minimum redundancy [1], [3], [6], pseudorandomness [5], power laws [2], and minimization of holes in the sampling [8]. This communication describes a new principle motivated by the desire to spread samples across the Fourier plane as evenly as possible. This principle, which has an interesting relationship to minimum energy configurations in electrostatics, leads to beautiful, symmetric crystal-like sampling of the Fourier plane, arising from simple circular configurations of elements. The methodology used to find solutions is based upon simulated annealing [4]. Note that this methodology applies only to correlation arrays, which must be used for the imaging of incoherent objects, rather than arrays in which the signals are sampled directly. In the next section, the problem is summarized. In the subsequent

Manuscript received April 7, 1987; revised December 3, 1987.

The author is with the National Radio Astronomy Observatory, P.O. Box O, Socorro, NM 87801.

IEEE Log Number 8821193.

two sections, the optimization strategy used and its application to this problem are discussed. Finally, some results for moderate numbers of array elements are given.

SUMMARY OF THE PROBLEM

Radio interferometers are correlation devices: they sample the time-averaged product of the electric field at two widely separated points. Each sample is an estimate of the Fourier transform of a region of the sky brightness [7]. A given pair of elements in an array will sample the Fourier plane at a point given by the projected vector separation of the elements measured in wavelengths, where the projection is onto a plane perpendicular to the line of sight. For a general source location, a given pair of elements will trace out an ellipse in the Fourier plane as the earth rotates. However, for fast imaging, the instantaneous coverage is more important, and so we will concentrate on it, rather than the full coverage. To simplify the analysis further we will concentrate on arrays seen face-on. This is not really a significant restriction since, for instantaneous coverage, any obliquity can be corrected easily in the layout of the elements on the ground.

We wish to find positions for N elements within some area such that a measure of the Fourier plane coverage is optimized. The form of the measure could be very complicated, e.g., rms sidelobes, mean-square separation of Fourier plane points, geometric mean separation of Fourier plane points, number of empty cells in a grid of appropriate size, etc. Before describing the choice for this measure, we will outline the method used to find the solution.

Let \mathbf{r}_i be the vector position of the *i*th element, and let K be the set of allowed positions for elements: it may consist of a grid, a compact region, or a number of disconnected regions. Let the measure of Fourier plane coverage be $m(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$. The problem is then to optimize globally the value of $m(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$ such that $(\mathbf{r}_i \in K: i = 1, N)$.

Although this seems a simple optimization problem, derivatives may thus be difficult to obtain, or simply not defined, and there may also be multiple extrema which could trap a gradient-based optimization algorithm. A recently developed alternative to conventional methods, simulated annealing [4], can be used to attack this problem.

OPTIMIZATION STRATEGY

Simulated annealing is essentially a statistical approach: configurations are tried at random and accepted according to the following rules. Let E be the function to be minimized, and let T be a usercontrolled "temperature" (the meaning will become clear).

- If $E_r < E_{r-1}$, then the new, rth configuration is always accepted.
- Otherwise, accept the configuration with some probability: compute $p(E_r) = e^{-E_r/T}$, and E_r , a random number drawn from a uniform distribution ranging from 0 to 1. If $p(E_r) < X_r$ then accept the new *r*th configuration; otherwise, reject it.

By the analogy with annealing, $p(E_r)$ is the probability of this change being consistent with random fluctuations. This latter aspect is crucial; it allows the algorithm to go uphill occasionally. Some fraction of the time, depending upon the temperature, the algorithm can therefore escape from local minima if the temperature is varied sufficiently slowly. The art of this algorithm consists in choosing the appropriate "annealing schedule"; many of the usual statistical mechanics tricks can be used to aid in this choice [4]. For example, the specific heat can be monitored for signs of the onset of freezing. We have not found such sophistication to be necessary for this problem and so we resorted to a simple cooling law: multiply T by

U.S. Government work not protected by U.S. copyright

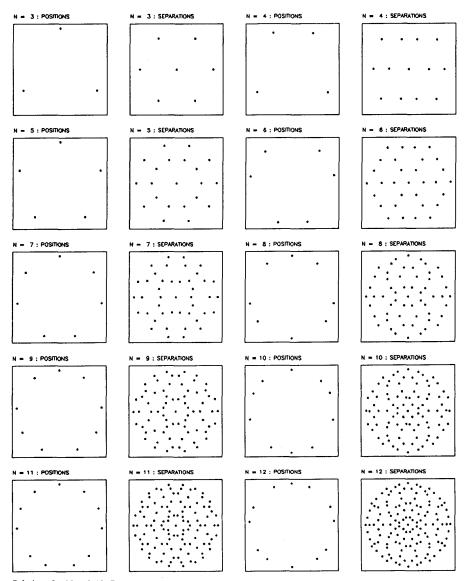


Fig. 1. Solutions for N = 3-12. Element positions and separations are plotted in adjacent diagrams. All separations for given set of element positions are shown. Scale is arbitrary.

some factor g, e.g., g = 0.9, after a given number of new configurations have been accepted at any given temperature.

APPLICATION TO THE OPTIMIZATION OF INSTANTANEOUS COVERAGE

Simulated annealing is a powerful technique for performing optimization of functions which are poorly suited to conventional gradient methods. In this application, the energy function E is to be identified with the measure function $m(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$, and the element positions \mathbf{r}_i are to be varied.

One can argue for any number of different choices for the measure. For radio-astronomical applications, e.g., [7], it is desirable to optimize the imaging ability of the array. However, if one simply requires that the sidelobe level be minimized, then it can be shown by Parseval's theorem that the only requirement on the Fourier plane samples is that no two samples be redundant. Many different configurations will obey this rule, and so to get a unique solution, a stronger principle is required. To carry the nonredundancy one step further, it was decided to maximize the distance between the Fourier plane points for the instantaneous coverage. To concentrate more on closer points, we use the logarithm of the distance between Fourier plane points rather than the square.

For observations of a source at the north pole, the instantaneous sampling is given by the set of difference vectors:

$$(\mathbf{u}_{i,j} = \mathbf{r}_i - \mathbf{r}_j : i, j = 1, N).$$

The measure is thus

$$m(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_N) = \sum_{i,j,k,l} \log \left(|\mathbf{u}_{i,j} - \mathbf{u}_{k,l}| \right)$$

where self-terms are ignored in the sum. One can regard this measure as allowing a generalization of nonredundancy: the resulting arrays will certainly be nonredundant if possible, but, in addition, whatever samples are present will be spread evenly over the Fourier plane. In this form of the measure, equal weight is given to all elements, corresponding to the case where all elements are equally sensitive to IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 36, NO. 8, AUGUST 1988

	TABLE I N												
N													
3	0.0	120.2	- 120.0										
4	28.0	- 28.0	123.1	- 123.8									
5	0.0	72.0	- 71.9	144.0	- 144.0								
6	39.7	- 39.7	80.2	- 80.2	159.7	- 159.8							
7	0.0	52.9	-53.1	98.2	- 98.4	158.8	- 158.9						
8	0.0	36.4	- 36.2	- 99.0	99.2	126.6	- 126.4	- 179.8					
9	0.0	35.0	- 35.6	84.4	- 85.2	120.0	~ 120.0	154.9	- 155.6				
10	0.0	43.8	- 43.6	64.0	- 64.7	102.1	- 101.2	151.0	- 150.7	- 179.6			
11	0.0	35.6	- 35.4	66.1	- 66.0	93.8	- 93.6	136.3	- 136.3	160.5	- 160.5		
12	0.0	24.5	- 24.5	64.9	-65.3	84.9	-85.4	126.2	- 126.4	143.7	- 144.2	179.9	

the incoming radiation. Differences in sensitivity can be easily accomodated by using a weighted sum in the measure. For the N elements, there will be N(N - 1) separation vectors, all of which are included in the summation.

The elements were constrained to lie within a circle. This seems appropriate for the application at hand: the design of a small array to operate at millimeter wavelengths. For other applications, different boundary conditions would be appropriate and can be easily incorporated. Trial configurations may be constructed randomly, but we have found that it is best to change one element location at a time and by an amount $\delta \mathbf{r}_i$ such that the corresponding δE is comparable to T. This does not affect the result obtained but merely the rate of convergence. To verify that consistent true minima had been obtained, all of the annealings were performed a few times with different random initial conditions.

DISCUSSION

Solutions were obtained for N ranging from three to 12. For larger numbers of elements, the computing effort required becomes prohibitive because the work per iteration goes roughly as the fourth power of N, and also because the minima become harder to locate. In all the annealings performed the elements migrated to the edge of the boundary, leading to configurations based upon the circle. It should be emphasized that this was not required a priori but rather arose naturally. The resulting arrays are listed in the Appendix and shown in Fig. 1. All have beautiful crystalline structure, with bilateral symmetry. The arrays are very redundant in rotation when seen faceon and may therefore be unsuitable for some uses such as spaceborne optical arrays. By design, these arrays will be of greatest use in correlation arrays for which the instantaneous coverage must be very good. The principles involved could be extended to noninstantaneous coverage provided that analytic forms for the measure can be calculated. The major disadvantage of these arrays, which is common to all minimum redundancy arrays, is the sensitivity to temporarily missing or nonfunctional elements.

The chief virtue of the simulated annealing method in this application is its versatility. One could easily apply the procedures described here to more complicated measures, in the presence of complex boundary conditions. For example, geographical boundaries are easily incorporated, or one could produce "sedimentation" of the more important elements to the center of an array by changing the weight of each element.

Finally, it should be admitted that the choice of the measure function made here is somewhat arbitrary. However, it is interesting to note that it corresponds to an electrostatics problem in two dimensions, wherein each element is represented by an equal charge, and the minimum energy configuration is sought subject to a boundary condition that no charges may move outside a given region.

APPENDIX

LISTINGS OF THE ELEMENT POSITIONS

All elements lie on a circle. The position angles (in degrees) of the elements are as shown in Table I.

REFERENCES

- J. Arsac, "Nonveau reseau pour l'observation radioastronomique de la brilliance de la soleil a 9350 Mc/s," Acad. Sci., vol. 240, pp. 942-945, 1955.
- [2] Y. L. Chow, "On designing a supersynthesis antenna array," IEEE Trans. Antennas Propagat., vol. AP-20, pp. 30-35, 1972.
- [3] M. Ishguro, "Minimum redundancy linear arrays for a large number of antennas," *Radio Sci.*, vol. 15, pp. 1163–1170, 1980.
- [4] S. Kirkpatrick, C. D. Gelatt, Jr., and M. P. Vecchi, "Optimization of simulated annealing," Science, vol. 220, pp. 671-679, 1983.
- [5] N. C. Mathur, "A pseudodynamic programming technique for the design of correlator super-synthesis arrays," *Radio Sci.*, vol. 4, pp. 235-244, 1969.
- [6] A. T. Moffet, "Minimum-redundancy arrays," IEEE Trans. Antennas Propagat., vol. AP-16, pp. 172-175, 1968.
- [7] A. R. Thompson, J. M. Moran, and G. W. Swenson, *Interferometry and Synthesis in Radio Astronomy*. New York: Wiley-Interscience.
- [8] R. C. Walker, "VLBI array design," in *Indirect Imaging*, J. A. Roberts, Ed. Cambridge, England: Cambridge Univ. Press, 1984, pp. 53-65.

A Compensation Technique for Positioning Errors in Planar Near-Field Measurements

OVIDIO M. BUCCI, SENIOR MEMBER, IEEE, G. SCHIRINZI, AND G. LEONE

Abstract—Probe positioning errors are among the major sources of inaccuracy in the planar near-field-far-field transformation technique, as their presence destroys the Fourier transform relationship between the tangential components of the near field and those of the plane-wave

Manuscript received June 6, 1986; revised April 14, 1987. This work was supported in part by Selenia S.p.A., Rome, Italy.

O. M. Bucci and G. Schirinzi are with the Dipartimento di Ingegneria Elettronica, Universitá di Napoli, Via Claudio 21, 80125 Naples, Italy.

G. Leone is with the Istituto di Ingegneria Elettronica, Universitá di Salerno, 84100 Salerno, Italy.

IEEE Log Number 8821194.

0018-926X/88/0800-1167\$01.00 © 1988 IEEE