A Local Description of the Directional Characteristics of the Propagation of Electromagnetic Fields

FRANCIS X. CANNING

Abstract—A method is presented for determining a local measure of the amount of the electromagnetic field propagation as a function of direction. This determination is made for the region near a given point by using the known field in the vicinity of that point within a given plane. The method is developed by assuming the parabolic equation approximation. Then the resulting equations are cast in the path integral form. A reformulation of the path integral formulation is then developed. This reformulation contains a quantity which is interpreted as a reasonable measure of the amount of field (amplitude) propagating in each direction. The relationship of this method to the plane-wave expansion technique is noted. The use of the method in two dimensions is illustrated by several numerical calculations. These numerical calculations are shown to be consistent with the predictions of the geometrical theory of diffraction.

I. INTRODUCTION

THE GEOMETRICAL theory of diffraction (GTD) and its variants allow a ray picture of electromagnetic fields and their propagation to be given in a large range of situations. At present, though, applications are somewhat restricted. First, in some geometries the necessary diffraction coefficients are not accurately known. Second, in certain complicated geometries, such as cavities, it may be very difficult to follow all the rays. Lastly, in some geometries, such as those containing small discontinuous shapes and small cavities, a ray description such as GTD is not valid. This paper develops a method of generating a "raylike" picture which is valid when GTD is difficult to apply or even for many situations in which one cannot use a ray description.

The method developed here requires that numerical values of the field (phase and amplitude) are known. They might be known from a measurement or, more likely, from a numerical calculation. Thus what is discussed here is not in itself a new method of calculation, but rather a new way of presenting the results of such a calculation.

The fields often are determined by complicated interactions. In these situations, when one has a specific purpose in mind, it is difficult to isolate the essential effects that need to be understood. For instance, in a cavity, changing the orientation of one of the walls or of some interior scatterer may cause a large change in the resulting fields. Thus a method is needed

that presents the known fields in a way that allows such an effect to be evident. This work presents such a method by giving a raylike description of the net fields whenever they can be computed or measured. However, the approximations of the ray description are avoided.

In this paper we will avoid unessential complications in the presentation by considering only two-dimensional problems in homogeneous space. However, the generalization to three-dimensional problems in homogeneous and smoothly varying media is immediate for regions removed from boundaries.

The organization of the rest of this paper is as follows. Section II develops a method of calculating the raylike description mentioned above. Bounds on the precision of any such method are also given. Section III presents numerical results illustrating the use of this method. Finally, Section IV summarizes our results.

In Section II one formula from quantum mechanics is used (see (6)). Since this formula may not be familiar to some readers, an outline of a simple derivation (using Gaussian integrals) is given. Based on this formula, all of the results about electromagnetic scattering are found without further use of quantum mechanics.

Some of the results we find (our equations) may be somewhat difficult to understand conceptually. Many of these equations have analogies to well-known results of quantum mechanics. We hope that by mentioning these analogies, the reader will have the opportunity to gain a deeper understanding of the equations in this paper, if he so desires. Occasionally, these analogies also provide a motivation for our approach, though they are not used directly.

II. THE THEORY

The theoretical development begins with the Helmholtz equation

$$\nabla^2 F + k^2 F = 0. \quad (1)$$

We allow k to be a function of position, i.e., a function of (x, z) . By making the substitution

$$F(x, z) = \exp(ik_1 z) f(x, z), \quad (2)$$

(1) becomes (exactly)

$$\frac{\partial^2}{\partial x^2} f + \frac{\partial^2}{\partial z^2} f + 2ik_1 \frac{\partial f}{\partial z} + (k^2 - k_1^2) f = 0. \quad (3)$$

Manuscript received January 5, 1987; revised July 6, 1987.

The author was with the Naval Weapons Center, China Lake, CA. He is now with the Rockwell Science Center, P. O. Box 1085, Thousand Oaks, CA 93160.

IEEE Log Number 8821174.

Next, we make the parabolic equation approximation, which is the assumption that the second derivative of f with respect to z is negligible. The resulting equation is

$$i \frac{1}{k_1} \frac{\partial f}{\partial z} = -\frac{(1/k_1)^2}{2} \frac{\partial^2 f}{\partial x^2} + 1/2 \left[1 - \left(\frac{k^2}{k_1^2} \right) \right] f. \quad (4)$$

At this point, several comments are in order about the approximation and the resulting equation. The parabolic equation approximation has been used many times before, and its range of validity is understood [1]. Thus only the salient facts need to be mentioned here. The approximation is used for fields propagating roughly in the direction of the z axis. Throughout this paper the small angle assumption will be made. It is known how to relax the assumption to include larger angles (but always less than 90° [2]). The constant k_1 is chosen so that the exponential in (2) captures the majority of the variation in the z direction. Therefore, the second derivative of f with respect to z should be small.

A limitation of (4) is that it cannot predict any backscatter. That is, we have assumed that the fields are propagating roughly in the z direction. Although it is possible that some radiation would actually scatter in the negative z direction, this approximation says that cannot happen. Fortunately, this limitation is not a serious one for our application. Equation (4) will be used only to iterate a very small distance Δz in the z direction. Thus even if a significant amount of backscatter occurs globally (i.e., over a long distance in the z direction), there will not be a significant amount over a short distance Δz .

Because (4) does not allow scattering in the negative z direction, it is possible to write a simple iteration formula in the z direction. This formula has already been derived in the context of quantum mechanics and is the basis of the Feynman path integral [3]. To see how those results apply, we notice that (4) has the same form as Schrodinger's equation of quantum mechanics, and these equations are identical with the identifications

$$(1/k) \leftrightarrow \hbar/2\pi \quad z \leftrightarrow t \quad 1 \leftrightarrow m. \quad (5)$$

This connection is described in some detail by Keller and McLaughlin [4]. The iteration formula that one finds is

$$f(x, z + \Delta z) = \left[\frac{-ik_1}{2\pi\Delta z} \right]^{1/2} \int_{-\infty}^{\infty} \exp \left\{ ik_1 \left[\frac{(x - x')^2}{2\Delta z} - \frac{\Delta z}{2} \left(1 - \frac{k^2}{k_1^2} \right) \right] \right\} f(x', z) dx'. \quad (6)$$

Actually, (6) is quite simple to prove, quite independent of any results of quantum mechanics. It is to be understood that this equation is true only for small Δz , by which we mean that terms of order $(\Delta z)^2$ have been ignored. To show the equivalence of (4) and (6), expand $f(x', z)$ of (6) in a Taylor series in x' about $x' = x$. If k varies with position, then also expand $(1 - k^2/k_1^2)$ as a Taylor series in x' about $x' = x$. This reduces the right side of (6) to well-known Gaussian integrals. Evaluating these integrals reduces (6) to the finite difference version of (4).

To summarize our findings so far, we have found the path integral description of an approximation to the Helmholtz equation. We believe that this description provides information about the paths that the radiation takes. We have not yet defined what is meant by the paths that the radiation takes, nor considered how to extract that information from the iteration formula (6).

Just for the moment, to simplify the discussion, assume that k is real. All equations, however, will continue to be true for general k . Clearly, then, the argument of the exponential is a pure imaginary quantity. This is to say, the magnitude of the exponential is always equal to one. The magnitude of the kernel must then be independent of x . Simply by looking at the magnitude of the kernel, therefore, one cannot determine the relative strengths of the propagation of the field to different locations.

As an aside at this point, we mention that the analogous problem has been studied in quantum mechanics. In the language of quantum mechanics, one would say that the states used in the path integral formulation are those of definite position. Therefore by the uncertainty principle, their momentum must be completely unknown. This is, the uncertainty in x' is zero, or

$$\Delta x' = 0. \quad (7)$$

By the uncertainty principle,

$$\Delta x' \Delta p \geq \hbar/2\pi > 0. \quad (8)$$

Therefore,

$$\Delta p = \infty \quad (9)$$

or one can have no knowledge of the momentum. The obvious solution is to introduce states which are not (completely) localized in space.

Returning to electromagnetics, we notice the analogous statement. That statement is that an observation of the field at one point tells nothing about the direction in which it is traveling. Thus we want to develop an equation which contains a term giving information about the field in a region near a point, not just at that point. To this end, consider the identity

$$\int_{-\infty}^{\infty} p(x, y) dy = A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x, s) g(s - y) ds dy \quad (10)$$

where

$$A^{-1} = \int_{-\infty}^{\infty} g(t) dt \quad (11)$$

and $g(t)$ may be any function that dies off quickly enough at infinity.

The inside integral, which is the convolution of $p(x, \cdot)$ with $g(\cdot)$, is interpreted as an average of $p(x, \cdot)$ over values of its second argument near y . The function g is arbitrary but is assumed to be peaked near zero. Therefore, the inside integral in (10) represents an average of $p(x, \cdot)$ in the vicinity of y . This equation permits integrals over a function to be replaced by integrals over an average of that function.

Using identity (10) in (6) gives

$$f(x, z + \Delta z) = A \left[\frac{-ik_1}{2\pi\Delta z} \right]^{1/2} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \exp \left\{ ik_1 \left[\frac{(x-s)^2}{2\Delta z} - \frac{\Delta z}{2} \left(1 - \frac{k^2}{k_1^2} \right) \right] \right\} \cdot f(s, z) g(s - x') ds \right] dx'. \quad (12)$$

Notice that the form of (12) is

$$f(x, z + \Delta z) = \int F(x|x') dx' \quad (13)$$

where the $F(x|x')$ represents a constant times the inside integral in (12). We make the interpretation that $F(x|x')$ represents the field near x' that is traveling to near x . The magnitude of $F(x|x')$ will not be constant, as long as g is not a delta function.

If we can pick g well enough, we might hope that the magnitude of $F(x|x')$ in (13) is much more significant in the evaluation of the integral than is its phase. That is, the integral of $F(x|x')$ might be well approximated by the integral of its absolute value. In that case, we call $|F(x|x')|$ the "amount" of field (amplitude) propagating from near x' to near x . Depending on whether electric or magnetic field was used in the Helmholtz equation (1), the amplitude considered here is that of the electric or magnetic field, respectively.

A previous study [5] considered the specific case which corresponds, in our notation, to picking g to be the function

$$\begin{aligned} g(x) &= 1/2\epsilon, & \text{for } |x| \leq \epsilon \\ g(x) &= 0, & \text{for } |x| > \epsilon. \end{aligned} \quad (14)$$

It was found that the resulting variation in the magnitude of F allowed an importance sampling Monte Carlo method to be used to evaluate (numerically) the path integral. In the context of that study, it was not evident that there was a freedom to pick the weighting function g at will.

The goal of this paper is to make explicit which paths in a given region carry the most field, once that field is known. Because that is our final goal, we do not apply the quantitative information on how much energy flows along each path to develop an importance sampling Monte Carlo method to evaluate the path integrals, thereby calculating the field. Clearly, such a method would work, though it may not be computationally efficient, even with an optimal choice of g , as in (15).

We realize that the function g can be chosen at will. However, it is desirable to choose g so that the regions for x and x' where $F(x|x')$ is large are as small (localized) as possible. The optimal g may be defined as the one which minimizes the product $(\Delta x)(\Delta x')$. By picking specific definitions for Δx and $\Delta x'$ that make use of the concept of standard deviation, we can show that the optimal g is given by

$$g(s) = e^{-(\alpha + ik_1/2\Delta z)s^2}. \quad (15)$$

The width $\Delta x'$ is proportional to $1/\sqrt{\alpha}$. That is, the width of the window can be set as desired. We also can show that, for an averaging distance of width $\Delta x'$, the width of the scattered

beam $\Delta\theta$ must obey

$$\Delta x' \Delta\theta \geq 1/2k_1. \quad (16)$$

Later on, the results of many numerical calculations will be given. In every case, we find that this inequality is fairly close to being an equality.

These results are analogous to basic results in quantum mechanics. Our inequality is just another form of the Heisenberg uncertainty principle for position and momentum. Our equation for g is related to the fact that the minimum uncertainty state in quantum mechanics is a Gaussian. We have not included the proofs of these results here, since it would be necessary either to depend heavily on quantum mechanics or to take up several pages. In addition, the rest of this paper does not depend on (16), though that equation may be helpful in understanding the meaning of our results.

Now, $g(s)$ has been chosen. Using this choice in the transformed iteration formula (12) gives

$$\begin{aligned} f(x, z + \Delta z) &= \left[\frac{-ik_1}{2\pi\Delta z} \right]^{1/2} A \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \exp \left\{ ik_1 \left[\frac{(x-s)^2}{2\Delta z} - \frac{\Delta z}{2} \left(1 - \frac{k^2}{k_1^2} \right) \right] \right\} \right. \\ &\quad \cdot f(s, z) \exp \left\{ -\left(\frac{ik_1}{2\Delta z} + \alpha \right) \right. \\ &\quad \left. \left. \cdot (s - x')^2 \right\} ds \right] dx'. \end{aligned} \quad (17)$$

An equivalent form of this equation is

$$\begin{aligned} f(x, z + \Delta z) &= \left[\frac{-ik_1}{2\pi\Delta z} \right]^{1/2} A \exp \left[-\frac{ik_1\Delta z}{2} \left(1 - \frac{k^2}{k_1^2} \right) \right] \\ &\quad \cdot \int_{-\infty}^{\infty} \left[\exp \left[ik_1 \frac{x - x'}{\Delta z} \frac{x + x'}{2} \right] \int_{-\infty}^{\infty} \right. \\ &\quad \cdot \exp \left[ik_1 \frac{x' - x}{\Delta z} s \right] \\ &\quad \left. \cdot \exp [-\alpha(s - x')^2] f(s, z) ds \right] dx'. \end{aligned} \quad (18)$$

The inner integral is a Fourier transform of $f(\cdot, z)$ with a Gaussian window. The exponential in the outer integral has an argument of approximately $i(k_1 \tan \theta)x'$ where θ is an angle from the z axis.

At this point, it is perhaps appropriate to describe the mathematical connection between our method and ray approximations. The ray approximation results from assuming that $\lambda \rightarrow 0$. Those familiar with path integrals may know that in this limit, the path integral collapses to a sum over terms representing the contributions of the specific paths which "minimize the action," which are the ray paths. Our method uses the actual λ and thus generates a spread of directions.

In the limit $\lambda \rightarrow 0$, one can show that our raylike description reduces to the usual ray description, at least within the small angle approximation. To do this calculation, we first note that for $\Delta\theta$ in (16) to approach zero, we must require

that $\lambda/\Delta x' \rightarrow 0$. Thus we are motivated to write our small λ condition as $k_1^2/\alpha \rightarrow \infty$. Now, assume that in some region the field is given by

$$\exp [i(k_1 z + k_1 x' \tan \phi)]. \quad (19)$$

Then the corresponding $f(x', z)$ may be put into (18), and a tedious calculation shows

$$|F(x|x')| \propto \exp \left[-\frac{k_1^2}{\alpha} \frac{1}{4(\Delta z)^2} (x - x' - \Delta z \tan \phi)^2 \right]. \quad (20)$$

Therefore, as $k_1^2/\alpha \rightarrow \infty$ our description produces a delta function peak in $|F(x|x')|$ at an angle of ϕ from the z axis, capturing the ray description.

If we had chosen to approach this problem by using the plane-wave expansion method [6], it would have been found that $F(x|x')$ is related to a Fourier transform of the field, similar to what we found above (where our $(x' - x)/\Delta z = \tan \theta$ would be replaced by $\sin \theta$). However, the plane-wave expansion method does not readily admit using a window to derive local information. Furthermore, the plane wave expansion method does not apply in inhomogeneous media.

It is important to notice that by the method presented here, we can show that the Gaussian window is optimum and is not just arbitrarily chosen. This result is analogous to the fact that [7] "Gaussian beams have the smallest transverse extent-angular divergence product of all optical beams." The plane-wave expansion method does not, in itself, allow any window function. A more important point, however, is that the present technique immediately generalizes to inhomogeneous materials. It does so primarily because k need not be a constant, though we have implicitly treated it as such. Indeed, k does not have to be purely real. The allowance of k depending on position raises the possibility of this method being applied when boundaries (conductors) are present. However, there are difficulties in treating cases where the material properties change quickly or discontinuously.

In this paper, we have assumed that one already has obtained values of the total field. These values must correctly take into account scattering from any surfaces. By restricting our applications to cases where our observation point is somewhat more than both $\Delta x'$ and Δx away from any surfaces, we can see that the direct contribution of those surfaces to the answer is negligible (exponentially small). Of course, the surfaces contribute indirectly by affecting the total field everywhere. Special care is required to expand the range of validity of the method so it may be applied right on the surfaces of conductors. Thus that topic will be saved for a later paper.

III. NUMERICAL RESULTS

This section contains the results of several numerical calculations illustrating the use of the method developed above. A quantitative comparison of this method with the results of GTD is made, with favorable results. It is found that the numerical results can be made to be quite precise.

The TE and TM modes of scattering of a plane wave by

a conducting circular cylinder will be considered. This case has been chosen because it has a well-known accurate GTD description. In generating numerical values of the appropriate field, the eigenfunction solution was used for convenience. However, it must be stressed that only numerical values of the field, generated by any technique, are all that is needed to apply our method.

In our notation, propagation occurs in the x and z directions. Therefore, the TM mode is the one in which the magnetic field is transverse to the y axis, which is the axis of the cylinder. This is also called the E -polarized (E -pol) mode, and it has its E -field along the y axis. The eigenfunction solution for the total field due to an incident plane wave is given by

$$E_y = \sum_{n=-\infty}^{\infty} i^n \left[J_n(k\rho) - \frac{J_n(ka)}{H_n^{(1)}(ka)} H_n^{(1)}(k\rho) \right] e^{-in\phi}. \quad (21)$$

Similarly, for the TE or H -polarized (H -pole) mode in the H -field is given by

$$H_y = \sum_{n=-\infty}^{\infty} i^n \left[J_n(k\rho) - \frac{J'_n(ka)}{H_n^{(1)'}(ka)} H_n^{(1)'}(k\rho) \right] e^{-in\phi}. \quad (22)$$

The geometrical theory of diffraction is so named because it describes diffraction effects in terms of rays. These rays are similar to the rays of geometrical optics. We will compare the results of our method in the shadow region behind a circular cylinder to the predictions of GTD. In this case, the GTD says that the total field is the sum of the fields due to many modes. Each mode represents a creeping wave excited on the surface at the shadow boundary, which creeps along the surface and sheds tangential rays as it goes. Therefore, at an observation point in the shadow region one would expect to observe two rays, coming from the tangents on each side of the cylinder. Near the edge of the geometrical shadow boundary it is necessary to include another term, which describes the finite width of the shadow boundary. When this is done, the theory is called the uniform theory of diffraction (UTD). The numerical results that follow are comparisons of GTD or UTD with the application of our method to the field calculated using the eigenfunction solution.

Fig. 1 defines the coordinate system used to define the test cases studied. Notice that it shows an incident plane wave, whose field is equal to $\exp(ikz)$. The field refers to E_y or H_y in the E -pol or H -pol cases, respectively. There is a perfectly conducting circular cylinder, centered at $(0, 0)$ and with a radius of two units. The wavelength λ is defined in terms of these same units.

Fig. 2 is a (linear) polar plot of $|F(x|x')|$ for the E -pol case. The wavelength λ is fixed at one-fourth unit. The ratio $k_1/\Delta z$ is 40, which is much greater than all of the values of α considered. The plots on this figure, each for a different value of α are separately normalized to fill the figure. By considering $\alpha = 1.5, 4, 16$, and 32, it is possible to see clearly the effect of α (or more accurately, of the ratio of distances $(1/\sqrt{\alpha})/\lambda$), on the widths of the patterns produced. The smallest α , 1.5, implies the largest averaging distance ($= 1/\sqrt{\alpha} = 0.82$). The largest α , 32, implies an averaging

Fig. 1. Geometry used on all succeeding figures.

Fig. 2. Polar plot of $|F(x|x')|$ as a function of x for E -pol, $x' = 0$, $z = 5$, $\lambda = 0.25$, and $k_1/\Delta z = 40$, where α takes on values indicated.

distance of only 0.18, which is less than a wavelength. Thus through (16) we would expect the large angular spread seen on the figure for $\alpha = 32$. The reader should note that the definitions of $\Delta x'$ and $\Delta \theta$ in (16) are in terms of standard deviations. For quantitative results, these definitions must be used in (16). However, for qualitative results, one could approximate $\Delta \theta$ by a half-width at half-height.

It can also be seen that the directions of the peaks change slightly with α . As $1/\sqrt{\alpha}$ decreases from 0.82 to 0.5 to 0.25, the peaks seem to be moving outward. The reason for this quite simple. Because we are averaging over a distance in the x -direction, we are looking at places where the field is traveling in different directions. However, the field for $|x|$ larger will both have a larger magnitude and have a shallower angle (from the z axis). Thus when we average over a larger distance larger $1/\sqrt{\alpha}$, we should expect the smaller angles as is seen, because the regions of larger field magnitude tend to dominate.

Fig. 3. Polar plot of $|F(x|x')|$ for E -pol, $x' = 0$, $z = 5$, $\lambda = 0.5$, $k_1/\Delta z = 20$, and $\alpha = 1.5$.

In GTD one does not find several of the features found here. In our terms, GTD must be the case $\alpha \rightarrow \infty$. However, we have already seen that as α becomes large, the peak's width increases for fixed λ . Thus the perfectly narrow peaks of GTD correspond, in our notation, to the simultaneous limit $\alpha \rightarrow \infty$ and $(1/\sqrt{\alpha})/\lambda \rightarrow \infty$. This makes sense since GTD is a small λ approximation.

The polar plot of Fig. 2 and all succeeding plots was made by attributing an angular structure to $|F(x|x')|$ through the relation $\tan \theta = (x - x')/\Delta z$. This relation follows from our theory based on a small θ approximation. The two radial lines at the top of Fig. 2 indicate the directions of the two rays tangent to the cylinder and passing through point (x', z) . It is gratifying to note that, as $1/\sqrt{\alpha}$ is decreased from 0.82 to 0.5 to 0.25, the peaks move outward until they are very close to the directions indicated by the radial lines. When $1/\sqrt{\alpha}$ is equal to 0.18, we see that the pattern is so broad that the directions of the peaks are not readily discernible.

By using the plane-wave expansion method and not making the small angle approximations we made, one finds the relation

$$\sin \theta = (x - x')/\Delta z. \quad (23)$$

Using this fact, we have indicated on Fig. 2 using the arrows on top of the figure, a corrected direction to compare our peak directions against. One can see that for $1/\sqrt{\alpha}$ equal to 0.25, the peak points at the arrows within the accuracy that one can determine the average direction of the field near the peak.

Figs. 3-6 are for $\lambda = 0.5$ and the E -pol case, with $\alpha = 1.5$. The ratio $(k_1/\Delta z)$ was chosen to be 20, which is larger than α which provides better resolution. On these figures we show a (linear) polar plot of $|F(x|x')|$. As on all these plots the distance Δz implies the angular structure shown, through the relation $\tan \theta = (x - x')/\Delta z$. On Fig. 3 there are two equal peaks, due to the rays coming equally around each side of the cylinder, since the observation point is in the center, at $x' = 0$. The hint of a small peak at $\theta = 0$ is a feature which goes away when the resolution is improved (i.e., when $1/\sqrt{\alpha}$ is increased). As the observation point is moved to the side

Fig. 4. Polar plot of $|F(x|x')|$ for E -pol, $x' = 0.5$, $z = 5$, $\lambda = 0.5$, $k_1/\Delta z = 20$, and $\alpha = 1.5$.

Fig. 5. Polar plot of $|F(x|x')|$ for E -pol, $x' = 1.0$, $z = 5$, $\lambda = 0.5$, $k_1/\Delta z = 20$, and $\alpha = 1.5$.

Fig. 6. Polar plot of $|F(x|x')|$ for E -pol, $x' = 2.0$, $z = 5$, $\lambda = 0.5$, $k_1/\Delta z = 20$, and $\alpha = 1.5$.

(to $x' = 0.5$, $x' = 1.0$, and finally to $x' = 2.0$), we see the expected features in the directions and magnitudes of the two peaks.

The graphs shown are all normalized to fill the figure. However, in Table I we give a numerical comparison of the

magnitudes involved in Figs. 3–6 against uniform theory of diffraction calculations. The numerical (NUM) results in Table I were found by integrating the field strengths over the region of each peak. The dividing line between the peaks was taken to be the place where there is a relative minimum in $|F(x|x')|$.

The UTD calculation was done by adding the transition function given by a Fresnel integral to the GTD creeping wave contribution. For simplicity, only the first two modes were used in calculating the creeping wave contribution. In Table I the paths S and L refer to the short and the long ray paths to the observation position (x', z) indicated.

The comparison between our numerical method and UTD shows that in all but one case the fields strengths agree within about 0.5 dB. The case where the difference is 2.1 dB corresponds to the long path in Fig. 6. This peak is not apparent on this linear plot. However, it would be apparent on a logarithmic (dB) plot of the data. We have done well numerically to find this peak which is over 40 dB down in power.

The rest of the cases considered are for λ equal to one-quarter. First, consider the position (1, 2.5). The E -pol case is shown in Figs. 7 and 8 using linear and logarithmic scales, respectively. Table II shows that the smaller peak region on Fig. 8 contains an energy down about 70 dB in power from the incident field.

Figs. 9 and 10 show linear and logarithmic plots for the H -pol case. Notice that while the large peak for this case is closer to the tangent ray direction than for the E -pol case, it is still slightly more shallow than the tangent direction. This may be explained as due to the fact that in the H -pol case the magnitude of the field varies less rapidly in the x direction. Thus the discrepancy (between numerical and UTD results) due to averaging should be less in the H -pol case.

One can extend this argument. It would seem reasonable that whenever the magnitude of the field is varying more slowly in the x direction, the comparison with UTD should show closer agreement. An inspection of Table II shows this to be the case. The first four entries in Table II correspond to the figures just considered. It can be seen that the differences between the numerical and UTD calculations are less in the H -pol cases than in the E -pol cases, as mentioned before. The entries going further down the table are for larger values of z . This means that, since the observation position is moving further from the cylinder, the creeping wave distance is depending less sensitively on the x position. By the reasoning given, the numerical method should overestimate the field and should overestimate the field less as z increases. Table II shows that this is indeed the case; the errors monotonically decrease as z increases. The second to last and last entries in Table II are for rays that creep the same distance along the cylinder. This shows that by moving away from the cylinder along a ray, more accurate results are found.

The comparisons of this section have shown an excellent agreement between the numerical method derived in this paper and UTD. The numerical method developed here gives a more discriminating test of GTD and UTD than the usual comparison in terms of the total field. It allows the parts of

TABLE I
FIELD STRENGTHS FOR $\lambda = 0.5$, $\alpha = 1.5$ AND $k/\Delta z = 20$, E-POLARIZED

Path	x	Position z	Field Strength		Difference (dB)
			NUM	UTD	
S, L	0.0	5	0.044	0.048	-0.4
S	0.5	5	0.081	0.085	-0.2
L	0.5	5	0.025	0.028	-0.5
S	1.0	5	0.147	0.168	-0.6
L	1.0	5	0.016	0.017	-0.3
S	2.0	5	0.504	—	—
L	2.0	5	0.0047	0.0076	-2.1

Fig. 7. Polar plot of $|F(x|x')|$ for E-pol, $x' = 1$, $z = 2.5$, $\lambda = 0.25$, $k_1/\Delta z = 40$, and $\alpha = 4$.

Fig. 8. Logarithmic scale plot of data in Fig. 7.

the field to be compared individually, which we believe has not been done before.

However, the widths of the pictures presented here gives real information that is not available from GTD, since GTD uses the limit $\lambda \rightarrow 0$. We saw that the effect of finite wavelength is manifestly evident in these pictures, in terms of the spread around the peak directions. Differences were found between the magnitudes and directions predicted by the numerical method and by UTD. Nearly all of these differences

were qualitatively explained in terms of the variation of the magnitude of the field over a distance of approximately a wavelength. This means that the differences are due to the approximation $\lambda \approx 0$ in GTD not being made in the numerical method. One cannot say that either method is more accurate. The two methods supply similar but different information.

IV. CONCLUSION

A method for redescribing electric and magnetic fields has been derived. This alternative description allows the direction of flow of the fields to be manifestly evident to the extent possible for the wavelength considered. That is, for a given wavelength, we did not find a perfectly sharp peak, all of our figures and our theory give a finite width.

The method of derivation used helped to explain why a ray picture is only possible as $\lambda \rightarrow 0$, and to explain what happens for a finite λ . It was mentioned how some of the equations found could be suggested by the plane-wave expansion technique. The way to generalize our equations to (smoothly) inhomogeneous media was stated. An important advantage of the technique used to derive our equations, other than the insight it provided, is that it allows for inhomogeneities and that it should be possible to extend the derivation to treat the case of an observation point very close (much closer than a wavelength) to a perfect conductor.

It was found that to obtain sharp directional information about the propagation of E - and H -fields, one must consider the fields not just at a point but also in a region of some width about that point. This width is measured in terms of wavelengths, so it can become negligible in the high-frequency limit.

Calculations were made for the scattering of a plane wave by a perfectly conducting cylinder. The results were seen to be consistent with UTD, both in terms of the magnitude and direction of the rays. By looking at these rays in the regions where they are accurately defined (away from the scattering cylinder), the quantitative agreement was excellent. However, close to the cylinder there were numerical differences of a few decibels, since then we were "comparing apples to oranges." That is, the method presented in this paper allows a description of the "amount of field" propagating as a function of direction, without making the ray approximation $\lambda \rightarrow 0$. Thus the method presented here should be especially useful in regions where the ray approximation is not valid, such as very

TABLE II
FIELD STRENGTHS FOR $\lambda = 0.25$, $\alpha = 4$ AND $k/\Delta z = 40$

Polarization	Path	Position		Field Strength		Difference (dB)
		x	z	NUM	UTD	
E	S	1	2.5	0.0376	0.0130	4.6
E	L	1	2.5	0.00019	0.00041	-3.3
H	S	1	2.5	0.2466	0.1329	2.7
H	L	1	2.5	0.0239	0.0184	1.1
H	S	1	3.0	0.2541	0.1639	1.9
H	L	1	3.0	0.0371	0.0302	0.9
H	S	1	3.5	0.2591	0.1864	1.4
H	L	1	3.5	0.0497	0.0419	0.7
H	S	1	4.0	0.2629	0.2028	1.1
H	L	1	4.0	0.0609	0.0527	0.6
H	S	1	4.5	0.2663	0.2143	0.9
H	L	1	4.5	0.0708	0.0626	0.5
H	L	2	5.769	0.0586	0.0554	0.2

Fig. 9. Polar plot of $|F(x|x')|$ for H-pol, $x' = 1$, $z = 2.5$, $\lambda = 0.25$, $k_1/\Delta z = 40$, and $\alpha = 4$.

Fig. 10. Logarithmic scale plot of data in Fig. 9.

close to scatterers and in cavities. It has the further advantage of being applicable whenever numerical values of the field are available. The application of the type of graph described in this paper should be useful both for understanding basic

physical effects and for designing structures that interact with radiation.

REFERENCES

- [1] F. D. Tappert, "The parabolic approximation method," in *Wave Propagation and Underwater Acoustics*, Lecture Notes in Physics, no. 70. New York: Springer Verlag, 1977, pp. 224-280.
- [2] P. D. Scully and D. Lee, Eds., "Recent progress in the development and application of the parabolic equation," Naval Underwater Systems Center, CT, TD7145, 1984.
- [3] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals*. New York: McGraw-Hill, 1965.
- [4] J. B. Keller and D. W. McLaughlin, "The Feynman integral," *Amer. Math. Month.*, vol. 82, pp. 451-465, 1975.
- [5] A. P. Vinogradov and V. S. Filinov, "Monte Carlo methods for computation of complex-valued functions and Feynman path integrals," *Sov. Phys. Dokl.*, vol. 26, Nov. 1981.
- [6] P. C. Clemmow, *The Plane Wave Spectrum Representation of Electromagnetic Fields*. New York: Pergamon, 1966.
- [7] J. A. Arnaud, *Beam and Fiber Optics*. New York: Academic, 1976, p. 50.
- [8] P. H. Pathak, "An asymptotic analysis of the scattering of plane waves by a smooth convex cylinder," *Radio Sci.*, vol. 14, no. 3, pp. 419-435, 1979.

Francis X. Canning was born in Summit, NJ, on September 8, 1950. He received the A.B. degree in mathematics and physics from Dartmouth College, Hanover, NH, and the M.S. and Ph.D. degrees in physics from the University of Massachusetts at Amherst in 1971, 1977, and 1982, respectively.

From 1982 to 1987, he was at the Fuse and Sensors Department of the Naval Weapons Center at China Lake, CA, where he first worked in atmospheric effects on radar propagation, and later on radar scattering. In August 1987, he moved to

the Electromagnetics Department of the Rockwell International Science Center in Thousand Oaks, CA.