Note that for $0 \leq \delta \leq \pi$, only the TM_z mode, with a zero cutoff frequency, may propagate. The corresponding ranges for d/λ_0 with typical values of dielectric substrate relative permittivities are:

$$0 \leq d/\lambda_0 \leq 0.456, \quad \text{for } \epsilon_r = 2.2$$

$$0 \leq d/\lambda_0 \leq 0.168, \quad \text{for } \epsilon_r = 9.8.$$

Thus, for applications within the microwave range or below and with commonly used dielectric substrates, the d/λ_0 ratio remains in the ranges given above, such that only one surface wave mode may propagate.

The power P^S contained in the surface wave may be determined by the Poynting vector integration in a cylindrical surface of a large radius and an infinite height. For this, the E_z component of the electric field has to be obtained. From one of the Maxwell's equations:

$$\nabla \times \vec{H} = -i\omega\epsilon_0 \vec{E}. \quad (13)$$

Neglecting terms that vary with distance as fast as $r^{-3/2}$, E_z^S may be expressed by

$$E_z^S = \frac{-\lambda}{\omega\epsilon_0} H_\phi^S. \quad (14)$$

Thus P^S is obtained from

$$P^S = \frac{\lambda}{2\omega\epsilon_0} \int_0^{2\pi} d\phi \int_0^\infty r dz |H_\phi^S|^2. \quad (15)$$

The power contained in the space wave is obtained from the integration of the Poynting vector over a spherical surface with a sufficiently large radius R , such that the far-field expressions may be used. Thus

$$P^R = \frac{377}{2} \int_0^{2\pi} d\phi \int_0^\pi d\theta R^2 \sin \theta [|H_\phi^R|^2 + |H_\theta^R|^2]. \quad (16)$$

IV. NUMERICAL RESULTS

Since a lossless dielectric substrate has been assumed, the total power excited by the antenna is the power contained in the surface wave P^S added to the power contained in the space wave P^R .

The radiation efficiency of the space wave is thus defined by

$$\eta = \frac{P^R}{P^R + P^S}.$$

Results for η as a function of d/λ_0 , and for $\epsilon_r = 2.2$, are shown in Fig. 3. The dashed lines are used to indicate the region where there is deterioration of the assumption $d \ll \lambda_d$. The a and b parameters were chosen in order to have the condition (3) satisfied for the various propagating modes. Note from the figure that the efficiency decreases with an increase of d/λ_0 for the TM_{11} and TM_{12} modes. The efficiency of the TM_{13} mode has a tendency to approach a maximum of 100 percent as d/λ_0 is increased. Results for η as a function of d/λ_0 , and for $\epsilon_r = 9.8$ have indicated that the efficiency remains nearly 100% when $d/\lambda_d \leq 0.1$ for the three previously mentioned TM modes. The TM_{12} mode has a wider band than the TM_{11} mode [8]. Thus, it is a convenient mode to be used with very thin dielectric substrates. However, if the dielectric substrate thickness is not a

Fig. 3. Radiation efficiency of a microstrip ring antenna operating in the modes: TM_{11} ($a = 0.07\lambda_0$), TM_{12} ($a = 0.35\lambda_0$), and TM_{13} ($a = 0.68\lambda_0$), with $b = 2a$ and $\epsilon_r = 2.2$.

critical parameter, the antenna may be operated in the TM_{13} mode, near its maximum efficiency.

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Self- and Mutual Impedances of Traveling-Wave Linear Antennas

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Abstract—A traveling-wave distribution of current may be produced on a conventional linear antenna, such as a dipole, through resistive loading. The self- and mutual impedances of parallel traveling-wave linear antennas are measured and compared with those of the corresponding conventional linear antennas. A discussion of the coupling effects on the input impedance of an array element is presented, and it is shown that these effects are much less pronounced for traveling-wave antennas, which may be advantageous for some array applications.

Manuscript received August 19, 1988; revised November 10, 1988.

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IEEE Log Number 8929247.

Fig. 1. Coupled traveling-wave monopole antennas.

It has been previously shown that an essentially traveling-wave distribution of current can be produced on a linear antenna by inserting a resistance of suitable magnitude one-quarter wavelength from the end of the antenna. An extensive investigation of the traveling-wave linear antenna has been conducted and its properties have been compared with those of the conventional standing-wave linear antenna [1]. Here the behavior of coupled traveling-wave antennas is investigated and compared with that of conventional linear antennas in order to determine if there are any advantages in using a traveling-wave antenna as an array element. The measurements were made on coupled monopole antennas over a ground plane, as is shown in Fig. 1.

Consider the case of two parallel, identical antennas. The self-impedance Z_{S1} of antenna 1 in the presence of antenna 2 is defined as the input impedance of antenna 1 when the input terminals of antenna 2 are open-circuited. The mutual impedance Z_{12} of antenna 1 in the presence of antenna 2 is defined as the negative of the ratio of the voltage induced across the open terminals of antenna 2, to the input current of antenna 1. Both the self- and mutual impedances of identical, parallel antennas are a function of the distribution of current on the antennas and the spacing between them.

The measured self- and mutual impedances [2] of standing- and traveling-wave dipole antennas of half-lengths $h = \lambda/2$, $5\lambda/8$, and $3\lambda/4$ are shown in Figs. 2, 3, and 4. Note that all curves exhibit the same type of behavior. The curves of self-resistance and -reactance oscillate about the input resistances and reactances of the corresponding isolated antennas. As the spacing b between the antennas is increased, the self-impedance of the coupled antenna approaches that of the isolated antenna; the mutual resistance and reactance oscillate

about zero ohms, and as is expected, tend to zero as the spacing is increased.

The performance of an antenna array can be significantly affected by the presence of mutual coupling. In order to illustrate this effect a two-element array is analyzed. Consider the set of linear equations that characterize this array.

$$V_1 = Z_{S1} I_1 + Z_{12} I_2 \quad (1)$$

$$V_2 = Z_{12} I_1 + Z_{S1} I_2 \quad (2)$$

$$\frac{V_2}{V_1} = \frac{|V_2|}{|V_1|} e^{j(\theta_2 - \theta_1)} \quad (3)$$

$$= K e^{j\theta} \quad (4)$$

where V_1 and V_2 are the driving voltages, θ_1 and θ_2 are their respective phases, and I_1 and I_2 are the input currents. The input impedance of antenna 1 is then

$$I_1 = Z_{S1} + Z_{12} \frac{I_2}{I_1} \quad (5)$$

It can be shown

$$Z_{in} = \frac{Z_{S1}^2 - Z_{12}^2}{Z_{S1} - Z_{12} K e^{j\theta}} \quad (6)$$

This expression has the form

$$w = \frac{az + b}{cz + d} \quad (7)$$

Fig. 2. Self- and mutual impedances of traveling-wave and standing-wave dipoles ($h = \lambda/2$).

Fig. 3. Self- and mutual impedances of traveling-wave and standing-wave dipoles ($h = 5\lambda/8$).

Fig. 4. Self- and mutual impedances of traveling-wave and standing-wave dipoles ($h = 3\lambda/2$).

where

$$a = 0$$

$$b = Z_{S1}^2 - Z_{12}^2$$

$$c = Z_{12}$$

$$d = Z_{S1}$$

$$z = Ke^{j\theta}$$

Equation (7) represents a bilinear fractional transformation of z into w . One of its properties is that it transforms circles into circles. Since $z = Ke^{j\theta}$ is a circle, then the locus of points of Z_{in} as a function of θ must lie on a circle. It can be shown that the center of the circle is

$$Z_{cen} = \frac{(Z_{S1}^2 - Z_{12}^2)Z_{S1}^*}{|Z_{S1}|^2 - |Z_{12}|^2} \quad (8)$$

with radius

$$r = \frac{|Z_{S1}^2 - Z_{12}^2| |Z_{12}|}{|Z_{S1}|^2 - |Z_{12}|^2} \quad (9)$$

For electronically scanned arrays the direction of the main beam is changed by varying the phase difference θ between the driving voltages of adjacent antenna elements. In order to maintain the same input current on each antenna, as θ is varied, one must correct for the change in input impedance that occurs. This effect tends to complicate the task of driving each antenna properly. It is therefore desirable to reduce the variation of Z_{in} with θ . This can be accomplished by keeping the radius r small.

The normalized input impedances of pairs of traveling-wave

Fig. 5. Circles of input impedance as a function of scan angle for coupled antennas.

antennas having half-lengths of $h = \lambda/2$, $5\lambda/8$, and $3\lambda/4$, and spaced 0.5λ apart, and driven with equal voltages having phase differences, which vary from 0° through 360° , are plotted on a Smith chart as shown in Fig. 5. Note that the variation of the input impedance as a function of phase difference is very small. For comparison, the

normalized input impedances of standing-wave dipoles having the same half-lengths are also shown in Fig. 5; note the large variation of the input impedance for this type antenna. This behavior indicates that the mutual coupling between antenna elements having a traveling-wave distribution of current is weaker than that for standing-wave antennas. For more than two elements, the mutual impedance will still be smaller for traveling-wave elements as compared to standing-wave elements; however, the locus of points of the input impedance as a function of the phase difference between array elements will no longer be a circle.

Admittedly, there is a price to be paid for this reduction in the mutual coupling; about 50 percent of the input energy is dissipated in the resistive load [1]. However, there are applications, particularly for receiving arrays, for which this trade-off is warranted.

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Radiation Patterns of a Paraboloidal Reflector Fed by a Pyramidal Horn with Lossy Walls

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Abstract—A reduction of 7 dB in cross polarization in pyramidal horns is achieved by covering the walls of the horn with a dissipative material. The corresponding loss in co-polar gain is found to be less than 1 dB. The effect of dissipative material is to reduce wall surface currents and hence cross polarization.

INTRODUCTION

Experimental studies indicated that far-field cross polarization in pyramidal horns is brought about by surface currents on the walls normal to the direction of polarization [1], [2]. This suggests that if radiation from the walls is attenuated, then far-field cross polarization can be reduced. Attenuation of fields radiated by the walls can be achieved by coating the inner surface of the horn walls with some absorbing material or by placing absorbing slabs against the interior surface of the walls.

The presence of the absorbing material within the horn cavity is expected to reduce not only the cross-polarized fields but also the co-polarized ones. The effect on the co-polar field is, however, much less than that on the cross-polarized field. Besides loss in co-polar gain, one also expects the co-polar radiation pattern to be affected due to possible changes in aperture field distribution as dictated by the boundary conditions at the lossy walls [3].

It is the purpose of this communication to report some recent measurements carried out on a small pyramidal horn, to verify the above theory, and to study the effect of absorbing the walls on the co-polar gain and radiation pattern.

Manuscript received January 4, 1988; revised March 22, 1988.

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IEEE Log Number 8929250.

Fig. 1. Measured co-polar radiation patterns. (a) *H*-plane. (b) *E*-plane. --- without absorbing slabs, ••• with absorbing slabs.

MEASUREMENTS

The co-polar and cross-polar radiation patterns of a parabolic reflector fed by a pyramidal horn were measured with the primary feed first unloaded with absorbing material and then with the walls covered with thin slabs of dielectric coated with graphite powder.

The dimensions of reflector and primary feed are

reflector:	diameter	= 31 cm
	focal length	= 11.5 cm
horn:	4 cm × 4 cm	
slab thickness	= 0.55 mm.	

The reflector under test was operated in the receive mode at a frequency of 10 GHz. For the transmitting antenna an 8 × 8 cm pyramidal horn was used. Measurements were carried out with the transmitter and receiver 12 m apart and 5 m above the ground.

RESULTS

The co-polar and cross-polar radiation patterns of the reflector with no absorbing slabs inserted in the feed are shown in Figs. 1 and 2, respectively, by the dotted curves. In the same figures are shown the co-polar and cross-polar patterns after the absorbing slabs have been introduced. It is seen that the loss in co-polar gain is about 0.7 dB. The absorbing slabs reduce beamwidth only slightly. On the other hand, the peak of the first sidelobe in the *H*-plane is decreased by 1 dB. In general absorbing slabs improve the co-polar pattern slightly with a small loss in gain. *E*-plane measurements were limited to within 13° off-axis by the mechanism of the turntable. The change in this pattern due to absorbing slabs is not significant as seen from Fig. 1(b).

The cross-polar patterns shown in Fig. 2 are the patterns in a plane containing maximum cross polarization. Due to asymmetry introduced by the feeder waveguide, this plane is shifted from the usual 45° plane. It is seen that the improvement in the first cross polar lobe due to absorbing walls is about 7 dB. The peak of the second lobe is not affected, however. The overall improvement in peak cross polarization is thus 6 dB.

FURTHER COMMENTS

While cross polarization in pyramidal horns is believed, by the present authors, to result from radiation by the walls normal to the main polarization and therefore absorbing slabs should be placed