A Fast Finite Difference Method for Propagation Predictions over Irregular, Inhomogeneous Terrain

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Abstract—Propagation of electromagnetic waves over irregular, inhomogeneous terrain is solved by a finite difference scheme. The method is fast and requires considerably less memory than the integral equation methods. The method requires a storage space of order $O(N)$ and an execution time of order $O(N^2)$. Fields generated by a TE_z line source are represented in an integral form in terms of the field over a flat, constant impedance plane (the incident field), and the field scattered by the terrain irregularities and inhomogeneities. Accurate expressions are provided for the incident field and the Green's function, whose evaluation is otherwise accomplished by the rather time-consuming Sommerfeld's integrals. Measured equation of invariance is used to terminate the computational domain. The sparse matrix generated by the method is inverted by the Ricatti transform. Numerical results are presented for the ground wave as well as for the sky wave. Comparison is made for known geometries to establish the validity and limitations of the method.

I. INTRODUCTION

ELECTROMAGNETIC wave propagation over hilly terrain is important not only in point-to-point communication over land but also in ground-to-air communication. Of late, it has assumed importance in the design of personal communications networks in the context of outdoor propagation. An exact, analytical solution of the problem for general terrain is not possible, and one often resorts to an approximate or a numerical approach. In a previous paper [1], we developed a numerical model for propagation predictions over inhomogeneous, irregular terrain using the magnetic field integral equation. Although the method could include all effects of wave propagation, such as reflection, diffraction, surface wave excitation, and backscattering, a principal limitation of the method was the requirement of large computer resources (CPU time and memory), particularly for electrically large terrain irregularities. For instance, if the integral equation is solved numerically by the method of moments [2], the matrix fill time would be of order $O(N^2)$ and the inversion time of order $O(N^3)$, where N is the total number of unknowns. Since the matrix generated would be dense, the memory requirements would be of order $O(N^2)$. Thus, the method is attractive for small terrain irregularities but is computationally prohibitive for large terrain irregularities.

In this paper, we present a computationally efficient model of wave propagation based on finite differences. It may be noted that this method has no semblance to the one proposed in [3], where one proceeds with the parabolic equation approximation of the Helmholtz equation. Use of parabolic equation approximation precludes backscattering, which can be important sometimes. In contrast, we apply finite differences directly to the Helmholtz equation without introducing any such dubious approximations. Even though one has to handle a larger matrix when dealing with finite differences in contrast to boundary methods such as integral equation methods, the resulting matrix is sparse and often faster to invert than the dense matrix generated via the latter. The matrix fill time is still of order $O(N^2)$, but the inversion time is reduced to a lower order of $O(N)$. Thus, the real advantage of a finite difference scheme would be felt for large problems where the run time is dominated by the inversion time. As the matrix is sparse, substantial savings in memory are also achieved. The memory required is only of order $O(N)$. The method can potentially solve larger problems than is possible with the integral equation methods. The principal difficulty associated with the use of finite differences when applied to open type problems such as those encountered in propagation, antennas, and scattering is the treatment of mesh truncation. We will use a method known as the measured equation of invariance (MEI), originally proposed by Mei *et al.* [4], to contain the computational domain.

The term *scatterer* will be used generically to denote an obstacle in an open environment. Local radiation conditions, such as those proposed in [5] and [6] for terminating the mesh, are accurate only when the truncating boundary is placed in the far field. This would result in a larger matrix size with obvious implications to computational time and memory. Global type radiation conditions [7] can be employed on a tighter terminating boundary to result in a smaller computational domain. However, this will destroy the sparsity of the matrix generated by finite differences and defeats the whole purpose of employing it in the first place. What is needed is a local boundary condition of the type in [5], but applicable very close to the scatterer. Although it is far more complex to find near-field conditions than it is to find far-field ones, this is partially offset by the fact that boundary conditions on a *continuous spatial domain* are not needed when finite methods are used. The MEI method enables one to generate the near-field conditions over a discrete spatial domain.

In this paper we deal only with two-dimensional sources and fields, and with one-dimensional terrain characteristics. It is

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Fig. 1. TE_z line source over irregular, inhomogeneous terrain.

assumed that the terrain is characterized by its local impedance and height over a reference plane, both of which may vary from point to point. In Section II-A, we present a finite difference discretization of the two dimensional Helmholtz equation and present an overview of the current method. To realize the mesh termination conditions with the MEI method described in Section II-D, an accurate representation of the near-zone scattered field is necessary. In Section II-B, we give an integral representation of the scattered field for a TE_z line source over an irregular, impedance surface. The corresponding expressions for the incident field and the Green's functions are presented in Section II-C. Finally in Section III, we present numerical results for both the sky wave and the ground wave and provide comparisons for test geometries.

II. FORMULATION

The geometry considered in the present paper is shown in Fig. 1. The terrain properties, the sources, and the corresponding fields are all invariant with respect to the longitudinal variable z . The transmitting antenna is a transversely polarized electric line source located at (x_o, y_o) . Other symbols indicated in the figure that are needed in Section II-B are defined there. A source such as this will have its electric field confined to the transverse xy plane and is the two-dimensional counterpart of the superposition of a vertical electric dipole and a horizontal electric dipole in three dimensions. The field due to the source may be classified as TE_z , and all components of the field may be expressed in terms of the z -component, H_z , of the magnetic field. It is assumed that all distance variables are normalized with respect to the free-space wavenumber $k_o = \omega\sqrt{\mu_o\epsilon_o}$, where ω is the radian frequency of the wave, ϵ_o is the permittivity, and μ_o the permeability of free space. Accordingly, we set $x := k_o x$, $y := k_o y$, etc. An $e^{j\omega t}$ time dependence is assumed and suppressed. For TE_z polarization, the impedance boundary condition [9] of the form $\hat{n} \times \vec{E} = \eta_o \Delta \hat{n} \times (\hat{n} \times \vec{H})$, relating the tangential components of the electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$ leads to

$$\frac{\partial H_z}{\partial n} = j\Delta H_z \quad (1)$$

where the unit normal $\hat{n}$ points out of the impedance surface, $\eta_o = \sqrt{\mu_o/\epsilon_o}$ is the intrinsic impedance of free space, and Δ is the normalized impedance of the surface. It is to be borne in mind that (1) depicts only an approximate behavior of the true surface fields [9].

Fig. 2. Computational domain for finite differencing. The terrain is bounded at the top by an artificial boundary and at the bottom by the terrain obstacle.

A. Overview of the Present Method

Let ψ denote the z -component of the scattered magnetic field due to a TE_z line source over an inhomogeneous (varying impedance), irregular terrain. We assume that ψ represents scattering only from the irregularities and inhomogeneities in a reference impedance plane. Thus, the scattered field is identically zero when the terrain is flat and has an impedance equal to the reference impedance. The scatterer then consists of those portions of the terrain where 1) the impedance is different from the reference value and 2) the elevation is different from zero. The computational domain consists of a region in space bounded by the terrain at the bottom (also called object or scatterer) and a terminating (i.e., truncating) boundary at the top, as shown in Fig. 2. It is assumed that the object boundary is subdivided into $N - 1$ segments, thereby generating N points on it. The terminating boundary is similarly partitioned into $N - 1$ segments. We generate a structured mesh in the computational domain by adding M interior layers between the object boundary and the terminating boundary, each having N points. A total of $M + 2$ layers are thus generated. Layer numbering is done in ascending order starting from the object boundary (layer number 0) and progressing towards the terminating boundary (layer number $M + 1$). Node numbering is done from left (number 1) to right (number N). We use the notation (x_m^n, y_m^n) , $m = 0, 1, \dots, M + 1$, $n = 1, \dots, N$ to denote the Cartesian coordinates of the n th point on layer number m . Points on the layer immediately following the scatterer are assumed to lie on the local normal. This is to permit easy implementation of the impedance boundary condition of (1).

Within the computational domain, the scattered field ψ satisfies the scalar Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi + \psi = 0. \quad (2)$$

Since the terrain irregularities do not necessarily conform to any standard coordinate system, the mesh is nonorthogonal and traditional finite difference equations are not applicable. We will develop the required finite difference equations similar to the method outlined in [10]. Prompted by the presence of second-order derivatives in the Helmholtz equation, we choose a five-point star mesh centered at $0(X_o, Y_o)$ and surrounded by four neighboring nodes with local coordinates (X_k, Y_k) , $k = 1, \dots, 4$, as shown in Fig. 3(b). The global indices of the nodes 0, 1, 2, 3, and 4 are (m, n) , $(m - 1, n)$, $(m + 1, n)$, $(m, n - 1)$, and $(m, n + 1)$, respectively. Using the notation in the local coordinates that $\psi_k = \psi(X_k, Y_k)$, we

Fig. 3. Computational molecules employed in the paper. (a) At the left end, (b) the central region, and (c) at the right end.

assume a finite difference equation over the mesh of the form

$$\psi_0 + \sum_{k=1}^4 c_k \psi_k = 0 \quad (3)$$

where c_k are unknown complex constants. The coefficients are determined by choosing four linearly independent plane waves (i.e., geometry-independent trial solutions of (2)) traveling along the lines joining the central node to the four neighboring nodes; i.e., we choose

$$\frac{\psi_k}{\psi_0} = e^{-j d_k \cos(\alpha_k - \alpha_l)}, \quad l = 1, \dots, 4 \quad (4)$$

where (d_k, α_k) are the polar coordinates of the neighboring nodes with respect to the central node 0. The linear system of equations resulting from (3) can then be solved for the unknowns c_k . It is interesting to note that the values for the coefficients reduce to standard values when the mesh conforms to a standard coordinate system. The computational domain is open ended at the sides, and one cannot choose a five-point star mesh as above. At the two ends of the domain, a modified mesh as shown in Figs. 3(a) and (c) is used. For the mesh shown in Fig. 3(a), it is possible that the plane waves traveling along directions 3 and 4 become linearly dependent, or nearly so. To avoid this degeneracy, we reverse the direction of the plane wave traveling along 0-3 and solve the linear system. Likewise, to avoid degeneracy with the mesh of Fig. 3(c), we reverse the direction of the plane wave traveling along 0-4 and solve for the coefficients.

At the lower (object) boundary of the computational domain, the impedance boundary condition (1) applies to the total field $H_z = \chi + \psi$, where $\chi = H_{0z}$ is the incident field. For small distances, the solution of (1) along the normal results in

$$\psi_0^n = -\chi_0^n + (\psi_1^n + \chi_1^n) e^{-j \Delta_s^n h_n} \stackrel{\text{def}}{=} -\chi_0^n + (\psi_1^n + \chi_1^n) \beta_n \quad (5)$$

where $h_n = \sqrt{(x_1^n - x_0^n)^2 + (y_1^n - y_0^n)^2}$ is the normal distance between the object and the first interior layer at the n th node, Δ_s^n is the value of the surface impedance Δ_s at the n th node, $\psi_m^n = \psi(x_m^n, y_m^n)$, and $\chi_m^n = \chi(x_m^n, y_m^n)$.

Additional conditions are needed at the artificial, terminating boundary. To simulate free-space, discrete conditions developed by Mei *et al.* [4] are assumed to hold on the boundary. The method proposed by Mei rests on the assumption that the scattered field satisfies some unknown, *continuous* boundary condition near the scatterer. Such an assumption seems plausible in view of the fact that a similar situation does

exist in the far zone [5]. The discrete version enables one to numerically capture it. Accordingly, on the five-point star mesh of Fig. 3(b), formed by layer numbers $M-1$, M , and $M+1$, the scattered field satisfies (in the local coordinates)

$$\psi_0 + \sum_{k=1}^4 a_k \psi_k = 0 \quad (6)$$

where the complex coefficients a_k are different from c_k . They are determined in a fashion similar to the latter except that the trial solutions are no longer plane waves but are dependent on the geometry of the scatterer and the location of the mesh. Such trial solutions can be obtained from the near-zone behavior of the scattered field, and are further discussed in Section II-D. It is important to note that a discrete equation of the form (6) over a five-point computational molecule of Fig. 3 can only capture spatial derivatives up to second order (without the cross terms). This is also true of the computational molecule chosen by Pous in [10]. Equations (3), (5), and (6) are combined to result in a matrix equation of the form

$$\mathbf{A}_1 \Psi^3 + \mathbf{B}_1 \Psi^1 + \mathbf{C}_1 \Psi^2 = \mathbf{F}^1 \quad (7)$$

$$\mathbf{A}_n \Psi^{n-1} + \mathbf{B}_n \Psi^n + \mathbf{C}_n \Psi^{n+1} = \mathbf{F}^n, \quad 2 \leq n \leq N-1 \quad (8)$$

$$\mathbf{A}_N \Psi^{N-1} + \mathbf{B}_N \Psi^N + \mathbf{C}_N \Psi^{N-2} = \mathbf{F}^N \quad (9)$$

where $\Psi^n = [\psi_1^n, \psi_2^n, \dots, \psi_M^n]^T$ is the column vector of unknown values on the M interior layers at the n th node, $\mathbf{A}_n, \mathbf{B}_n, \mathbf{C}_n$ are banded matrixes of order $M \times M$ given by (10)–(12) at the bottom of the next page, and $\mathbf{F}^n$ is the excitation column vector given by

$$\mathbf{F}^n = \begin{bmatrix} -c_1^{2,n} (\beta_n \chi_1^n - \chi_0^n) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (13)$$

In the above matrixes, $c_k^{m,n}$ is the k th finite difference coefficient associated with the node at (x_m^n, y_m^n) and a_k^n is the k th MEI coefficient associated with the node at (x_m^n, y_m^n) . The system of equations defined by (7)–(9) can be solved efficiently by the Ricatti block-by-block elimination technique [8], [16]. The inversion time of the algorithm is only of order $O(N)$. Detailed derivation of the algorithm applicable to the present problem is given in [20] and is not repeated here. After the scattered field is solved in the computational domain, the total field on the scatterer can be recovered from (5). The field at any point can then be determined from the total field on the scatterer by using the integral representation given below. In the next few sections, we supply details of the various steps discussed above.

B. Integral Representation

Fig. 1 shows an electric line source $\vec{J}_0$ located at $O(x_o, y_o)$. It is required to find the total field $(\vec{E}, \vec{H})$ at an arbitrary point $P(x_p, y_p)$ over the irregular, inhomogeneous terrain. It is desirable to express the total field in terms of the scattered field and an incident field, $(\vec{E}_0, \vec{H}_0)$, defined to exist over a flat, constant-impedance plane C_0 of normalized surface

impedance Δ_o . The general relationship between a surface impedance Δ and the local ground constants ($\epsilon_o\epsilon_r, \mu_o, \sigma$) is

$$\Delta = 1/\sqrt{(\epsilon_r - j\sigma_r)} \quad (14)$$

where σ_r is the relative conductivity representing $\sigma/\omega\epsilon_o$, σ being the actual conductivity of earth. The above relationship is used to evaluate not only Δ_o but also Δ_s , which will be encountered below. The field $(\vec{E}, \vec{H})$ is different from $(\vec{E}_0, \vec{H}_0)$ because of 1) the terrain inhomogeneities, or 2) terrain irregularities, or 3) both. Let $(\vec{E}_1, \vec{H}_1)$ be the field due to sources $(\vec{J}_1, \vec{M}_1)$ located at $P(x_p, y_p)$ over the reference impedance plane. The fields 0 and 1 satisfy the impedance boundary condition $\hat{y} \times \vec{E}_0 = \eta_o \Delta_o \hat{y} \times (\hat{y} \times \vec{H}_0)$ on C_0 , where $\hat{y}$ is the unit normal on C_0 . The total fields satisfy the impedance boundary condition $\hat{n} \times \vec{E} = \eta_o \Delta_s \hat{n} \times (\hat{n} \times \vec{H})$ on the terrain surface C , where the impedance Δ_s is, in general, different from Δ_o , and the unit normal $\hat{n}$ points into the upper half space. The surface C is assumed to consist of flat portions C_f , where the impedance Δ_s deviates from Δ_o and/or C_t , where the terrain deviates from being flat. The remaining portion of the terrain surface is coincident with the corresponding portions of C_0 , i.e., it is flat and has a constant impedance Δ_o . It is assumed that the ground deviates from the reference plane only in the positive y direction. Thus, we consider only hilly obstacles and do not address the problem of trenches in a flat plane. Starting from a two-dimensional version of the reciprocity theorem [2], it is shown in [20] that the z component of the total magnetic field, H_z , can be expressed as

$$H_z(P) = H_{0z}(P) - \int_{C=C_f+C_t} H_z(Q) \mathcal{G}(Q, P) d\ell_Q \quad (15)$$

where

$$\mathcal{G}(Q, P) = E_{1\ell}^{(z)}(Q) - \eta_o \Delta_s H_{1z}^{(z)}(Q) \quad (16)$$

is defined as the Green's function at Q due to a z -directed magnetic line source located at P over a plane of impedance Δ_o . The unit tangent $\hat{\ell}$ to the surface is defined such that

$\hat{\ell} \times \hat{n} = \hat{z}$. The subscripts on the fields refer to the respective component being considered. Equation (16) may be used to set up an integral equation for the unknown H_z and solved numerically. However, this will be computationally intensive, as already discussed, and will not be pursued here. In the next section, we present expressions for H_{0z} and $\mathcal{G}(Q, P)$, which allow rapid numerical computation.

C. Incident Magnetic Field and Green's Function

Fig. 4 shows a transversely polarized, electric line source located at (x_o, y_o) over a plane of constant impedance Δ_o . Due to the impedance boundary condition we have $E_{0x} = \eta_o \Delta_o H_{0z}$. The line source carries a total current I_o and has its current moment directed along the unit vector $\hat{i}$, which makes an angle θ_0 with the x axis. The current moment corresponding to the mirror image of the source about a perfectly conducting plane at $y = 0$ points in a direction $\hat{i}'$, which makes an angle $\pi - \theta_0$ with the x axis. Starting from Maxwell's equations, it can be shown that the magnetic field vector potential $\vec{A}$ can be expressed as [20]

$$\vec{A} = \frac{\mu_o I_o}{4j} \left\{ \hat{i} \Pi(x, y; x_o, y_o) + \hat{i}' [\Pi(x, y; x_o, -y_o) - 2\Delta_o \delta \Pi(x, y; x_o, y_o)] \right\} \quad (17)$$

where Π is the free-space Green's function given by

$$\begin{aligned} \Pi(x, y; x_o, y_o) &= \int_{-\infty}^{\infty} \frac{e^{-j[k_x(x-x_o) + \beta|y-y_o|]} dk_x}{\pi\beta} \\ &= H_0^{(2)}[\sqrt{(x-x_o)^2 + (y-y_o)^2}] \end{aligned} \quad (18)$$

and the quantity

$$\delta \Pi(x, y; x_o, y_o) = \int_{-\infty}^{\infty} \frac{e^{-j[\beta(y+y_o) + k_x(x-x_o)]}}{\pi\beta(\beta + \Delta_o)} dk_x \quad (19)$$

arises from the finite conductivity of the plane. The integral defined in (19) is of the so-called Sommerfeld type and

$$\mathbf{A}_n = \begin{pmatrix} c_3^{2,n} & 0 & \cdots & 0 \\ 0 & c_3^{3,n} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_2^n c_3^{M-1,n} - a_3^n c_2^{M-1,n} \end{pmatrix} \quad (10)$$

$$\mathbf{B}_n = \begin{pmatrix} 1 + \beta_n c_1^{2,n} & c_2^{2,n} & 0 & \cdots & 0 \\ c_1^{3,n} & 1 & c_2^{3,n} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_2^n c_1^{M-1,n} - a_1^n c_2^{M-1,n} & a_2^n - c_2^{M-1,n} \end{pmatrix} \quad (11)$$

$$\mathbf{C}_n = \begin{pmatrix} c_4^{2,n} & 0 & \cdots & 0 \\ 0 & c_4^{3,n} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_2^n c_4^{M-1,n} - a_3^n c_4^{M-1,n} \end{pmatrix} \quad (12)$$

Fig. 4. TE_z line source over a flat plane of constant impedance Δ_o .

numerically cumbersome to evaluate directly. To efficiently compute the incident magnetic field and the Green's function, we will express $\delta\Pi$ in terms of complex images similar to the procedure adopted in [12]. It may be worth mentioning that an attempt was made to express $\delta\Pi$ in terms of a *discrete sum* of complex images, as done in [13]. However, the number of complex images was found to be large for small $y + y_o$ and large $|x - x_o|$, and was therefore not considered.

Following [12], we express the spectral function $1/(\beta + \Delta_o)$ in (19) as a Laplace-type integral w.r.t. a transform variable τ . This is done so as to bring the integral w.r.t. k_x to the form of (18), which has a closed form solution. When this is done and the integral w.r.t. k_x is evaluated according to (18), the integral in (19) can be expressed as

$$\delta\Pi(x, y; x_o, y_o) = \int_{\tau=0}^{\infty} e^{-\Delta_o \tau} H_0^{(2)}(D) d\tau \quad (20)$$

where $D = \sqrt{(x - x_o)^2 + (y + y_o - j\tau)^2}$ is the complex distance between the field point (x, y) and a complex image at $(x_o, -y_o + j\tau)$. It is defined so that its real part is greater than zero ($\Re(D) > 0$) to represent an outgoing wave, and its imaginary part is less than zero ($\Im(D) < 0$) for the integral to converge. Even though it is much faster to compute $\delta\Pi$ using (20) than (19), the integration can be slow for certain combinations of Δ_o , $(x - x_o)$, and $(y + y_o)$, where the integrand decays rather slowly on the positive real axis of the complex τ plane. To further aid fast evaluation of the integral (20), we deform the original contour to one over which the asymptotic form of the Hankel function decays most rapidly (steepest descent path). This is obtained by setting $\Re[D(\tau)] = \Re[D(\tau = 0)] = r_2$ on the deformed contour, where $r_2 = \sqrt{(x - x_o)^2 + (y + y_o)^2}$ is the distance of the field point from the mirror image. If $\tau = u + jv$, the steepest descent path through $\tau = 0$ can be obtained as

$$u \rightarrow [0, \infty), \quad v = -(y + y_o) + r_2 \sqrt{\frac{u^2 + (y + y_o)^2}{u^2 + r_2^2}}. \quad (21)$$

On this deformed contour, the distance function behaves as

$$D = r_2 - ju \sqrt{\frac{u^2 + (y + y_o)^2}{u^2 + r_2^2}}. \quad (22)$$

Evaluation of the integral over the steepest descent path permits rapid computation of $\delta\Pi$. The incident magnetic field

could be obtained from the vector potential as

$$H_{0z} = \frac{-k_0 I_o}{4j} \left\{ \sin(\theta_0 - \theta_1) H_1^{(2)}(r_1) + \sin(\theta_0 + \theta_2) H_1^{(2)}(r_2) + 2\Delta_o \left(\sin \theta_0 \frac{\partial}{\partial x} + \cos \theta_0 \frac{\partial}{\partial y} \right) \delta\Pi(x, y; x_o, y_o) \right\} \quad (23)$$

where $r_1 = \sqrt{(x - x_o)^2 + (y - y_o)^2}$ and $\theta_1(\theta_2)$ is the positive angle made with the x axis by the straight line joining the observation point and the source (mirror image) point.

Using a similar analysis, the Green's function (defined through (16)) at the point $Q(x_q, y_q)$ due to a unit voltage, z -directed, magnetic line source located at $P(x_p, y_p)$ can be obtained as

$$\mathcal{G}(Q, P) = \frac{-1}{4j} \left\{ \sin(\theta - \theta_3) H_1^{(2)}(r_3) + \sin(\theta - \theta_4) H_1^{(2)}(r_4) - j\Delta_s [H_0^{(2)}(r_3) + H_0^{(2)}(r_4)] + 2\Delta_o [j\Delta_s + \sin \theta \frac{\partial}{\partial x_q} - \cos \theta \frac{\partial}{\partial y_q}] \delta\Pi(x_q, y_q; x_p, y_p) \right\} \quad (24)$$

where $r_3 = \sqrt{(x_q - x_p)^2 + (y_q - y_p)^2}$, $r_4 = \sqrt{(x_q - x_p)^2 + (y_q + y_p)^2}$, $\theta_3(\theta_4)$ is the positive angle made with the x axis by the straight line joining the observation point Q and the source point P (mirror image), and θ is the angle made with the positive x axis by the unit tangent at Q . Note that the Green's function becomes identically equal to zero when the terrain is flat and $\Delta_s = \Delta_o$.

The far-zone ($r \rightarrow \infty$) fields can be determined using the principal asymptotic forms for the various Hankel functions. For the quantity $\delta\Pi$, far-zone approximation can be obtained by deforming the path of the integral in (19) and evaluating with the saddle point method. The result is

$$\delta\Pi(x, y; x_o, y_o) \sim \sqrt{\frac{2j}{\pi r_2}} \frac{e^{-jr_2}}{(\Delta_o + \sin \theta_2)}. \quad (25)$$

Having provided the integral representation and expressions for various fields, we present in the next section details on the MEI method for terminating the computational domain in the finite difference scheme.

D. MEI Method

As already remarked, the MEI method allows one to generate near-field conditions for the scattered field ψ to simulate a free space at the terminating boundary. In this section, we will describe a procedure for determining the unknown coefficients a_k in (6). Equation (6) states that there exists a linear combination of the scattered field over a small, discrete, spatial domain $\tilde{\rho}_k$ to produce a null field. The coefficients a_k were postulated by Mei *et al.* [4] to be dependent upon the location of the field and geometry of the scatterer but independent of the excitation of the scattered field. The last postulate enables one to determine the coefficients a_k using a finite set of linearly independent scattered fields caused by different excitations. The starting point is an accurate representation of near fields such as (15). The scattered field

is given from (15) as

$$\psi(\vec{\rho}) = H_z(\vec{\rho}) - H_{0z}(\vec{\rho}) = - \int_C H_z(\vec{\rho}') \mathcal{G}(\vec{\rho}', \vec{\rho}) d\ell'. \quad (15')$$

Making use of this in (6) we see that

$$\int_C H_z(\vec{\rho}') \left[\mathcal{G}(\vec{\rho}', \vec{\rho}_0) + \sum_{k=1}^4 a_k \mathcal{G}(\vec{\rho}', \vec{\rho}_k) \right] d\ell' = 0. \quad (26)$$

By choosing various trial functions (or excitations) for H_z , we generate a matrix equation to solve for the unknown coefficients a_k . The specific excitations (which they termed *metrons*) chosen by Mei *et al.* were $H_z = 1, \cos(2\pi s), \sin(2\pi s), \cos(4\pi s)$, and $\sin(4\pi s)$, where s is the normalized arc length, varying between 0 and 1 on the scatterer. We were not able to consistently generate meaningful results using these metrons [20], and thus we considered other choices. A closer look at (26) provides insight as to why the above metrons cannot be expected to work all the time. The coefficients a_k have been postulated to be independent of the excitation (which determines $H_z(\vec{\rho}')$ on the scatterer). This implies that (26) must be true for all choices of $H_z(\vec{\rho}')$, and hence the terms within the bracket be identically zero for all points $\vec{\rho}'$ on the scatterer. The term $\mathcal{G}(\vec{\rho}', \vec{\rho}_k)$ is the field at $\vec{\rho}'$ due to a line source located at $\vec{\rho}_k$. Hence we require that a linear combination of the field due to finitely many of these line sources (five in number) vanish *pointwise* on the scatterer. But this would be difficult to accomplish as there are only a *few* discrete sources (typically four to five, depending on the type of the computational molecule chosen). This would be particularly true for a large scatterer. A more reasonable criterion is to require that the field be minimum in some sense on the scatterer. We choose the coefficients a_k so that integrated square residual

$$R = \int_C \left| \mathcal{G}(\vec{\rho}', \vec{\rho}_0) + \sum_{k=1}^4 a_k \mathcal{G}(\vec{\rho}', \vec{\rho}_k) \right|^2 d\ell' \quad (27)$$

is a minimum on the scatterer. In other words, we determine the coefficients by requiring that $\frac{\partial R}{\partial a_k} = 0$. This position was first taken by Jetvić and Lee [16], who considered the case of a perfectly conducting, circular cylinder to demonstrate the success of the approach. This criterion results in

$$\sum_{k=1}^4 a_k \int_C \mathcal{G}(\vec{\rho}', \vec{\rho}_k) \mathcal{G}^*(\vec{\rho}', \vec{\rho}_n) d\ell' = - \int_C \mathcal{G}(\vec{\rho}', \vec{\rho}_0) \mathcal{G}^*(\vec{\rho}', \vec{\rho}_n) d\ell', \quad n = 1, 2, \dots, K \quad (28)$$

where * denotes complex conjugation. The coefficients a_k can be obtained by solving the linear system defined by (28). We shall label the particular choice of metrons in (28) as the $\mathcal{G}^*$ *metrons*. The $\mathcal{G}^*$ metrons have a physical significance in that they represent incoming fields generated by line sources in an environment compatible with the original problem.

The coefficient matrix in (28) is Hermitian symmetric. The number of integrals required for a five-point stencil, such as

the one shown in Fig. 2, is 14. If every node is utilized in the evaluation of an integral, the total number of flops required to perform the integration for a five-point stencil is $14N$. The number of Green's function evaluations per five-point stencil is $5N$. The total time required to fill the coefficient matrix in (28) for N nodes on the object boundary is then $14N^2 t_f + 5N^2 t_g = O(N^2)$, where t_f and t_g are the respective times required per flop and a single evaluation of the Green's function. Normally, t_g far exceeds t_f , and it is very important to minimize it. For example, if it takes 1 ms for a single evaluation of the Green's function (which, incidentally, is considered very fast), the total time spent in the evaluation of the Green's functions for $N = 1000$ would be 83 min.

The total time needed to fill the matrixes (10) through (13) is also of order $O(N^2)$. The fill time may be reduced approximately by a factor of half by skipping every other point in the evaluation of the integrals. As indicated earlier, the inversion time with the Ricatti transform method is only of order $O(N)$. The overall CPU time of the present method is dominated by the matrix fill time. In the next section, we present numerical results for some test geometries to establish the validity and limitations of the method. More extensive results covering several aspects of numerical experimentation are given in [20]. Here we present only a few representative results.

III. NUMERICAL RESULTS

The entire code was developed in double precision in FORTRAN. The linear equations generated by (3) and (4) for the determination of the coefficients c_k were solved by Gaussian elimination. Due to the generation of a highly ill-conditioned matrix, Gaussian elimination is, however, not suitable for the inversion of the matrix associated with the coefficients a_k . By treating it as a linear least square problem, we have used singular value decomposition (SVD) [18] to construct a minimum norm solution to the matrix equation. The effective rank of the matrix is determined by treating as zero those singular values that are less than a predetermined number R_{cond} times the largest singular value. The condition number κ_2 of the matrix in the 2-norm is the ratio of the highest singular value to the lowest singular value. Of course, if R_{cond} is set less than the infimum of $1/\kappa_2$ over all the nodes on the boundary, SVD would use all the singular vectors and produce the same result as Gaussian elimination for a square matrix. Integration in (28) is performed by the Simpson's rule, making use of either all nodes on the scatterer or every other node. The latter reduces the integration time by a factor of half. The geometry of the scatterer is specified in a discrete form by means of the nodes at which the unknowns are defined. Interpolation with quadratic elements [19] was used to generate a continuous boundary. The code is thus capable of handling a rather arbitrary geometry. Hankel functions with complex arguments in (20) were generated by implementing the algorithm of du Toit [14].

To verify calculations by the complex image approach of Section II-C, we first present results on the computation of the incident magnetic field. Fig. 5 shows the magnitude of

Fig. 5. Magnitude of normalized magnetic field on flat, lossy surface.

the normalized magnetic field due to a vertically polarized line source on the surface of a flat, lossy plane. The ground constants correspond to $\epsilon_r = 15$ and $\sigma_r = 6$. The source is placed at $(0, 0.1\lambda)$. The magnetic field is normalized to the free-space value, which is true of all the results shown in the paper. The upper limit for u in (21) for numerical integration was determined by setting the imaginary part of the complex distance D to 5. Comparison is shown with the calculation based on the Sommerfeld integral given in [1]. Excellent agreement is seen between the two, thus validating the complex image approach. Several other cases have also been successfully tested including the cases of large conductivity, such as $\epsilon_r = 10$ and $\sigma_r = 200$.

We next compare the numerical results for propagation over a semicircular boss on a perfectly conducting plane ($\Delta_o = 0$), for which the exact solution in terms of cylindrical harmonics exists [1]. A vertically polarized source ($\theta_0 = 90^\circ$) is located at $(-50\lambda/3, \lambda/10)$ near a semicircular boss of radius 5λ and center at the origin. The numerical model uses eight layers ($M = 6$, recalling that M denotes the number of interior layers) with an interlayer spacing h of $\lambda/15$. The distance between the outermost layer and the object boundary is $(M + 1)h = 0.47\lambda$. The outer boundary is discretized into roughly 20 segments per wavelength to result in $N = 345$. R_{cond} for SVD was set at 10^{-4} . Fig. 6 shows a good agreement for the magnitude of the surface magnetic field between the numerical and the exact results. The arc length takes a value 0 at the left end of the boss and increases in the clockwise direction. Some ripples are, however, noticed on the numerical solution. Fig. 7 shows comparison of the far-field data between the numerical and exact results. A good agreement between the two is seen. The total computational time needed on a 80486-50 personal computer for the calculation of ground wave data at 345 points and the far-zone data at 361 angles was 2 min 30 s. Almost all of the time was required for filling the various matrixes, with the inversion taking only 1 s.

The accuracy of the solution can be affected by varying R_{cond} , M , N , and h . The single most important factor that

Fig. 6. Surface magnetic field versus normalized arc length on a semicircular boss on a PEC plane. Radius = 5λ .

Fig. 7. Relative pattern versus ϕ (measured from the positive x axis) of vertical source near a semicircular boss on a PEC plane. Radius = 5λ .

influences the numerical solution is the distance between the object and the outer boundaries. Several numerical experiments were carried with respect to the above parameters. Firstly, numerical experimentation revealed that Gaussian elimination, when used for the determination of a_k , yields highly erroneous results a good part of the time. Second, while it is generally true that the accuracy improves when N is increased or when $(M + 1)h$ is increased, difficulty in computing various quantities precisely over a small, discrete, spatial domain tends to obscure this. However, the accuracy of the numerical solution can be generally improved by 1) increasing the separation between the object and the terminating boundaries and 2) concurrently discarding those singular vectors that are corrupted by roundoff errors. The latter is accomplished by decreasing R_{cond} during the SVD solution of the coefficients a_k . In general, it has been found that for a given spatial resolution, the coefficient matrix becomes more ill conditioned (as evidenced by a higher condition number)

Fig. 8. Surface magnetic field versus normalized arc length on a semicircular boss on a PEC plane. Radius = 50λ .

as the separation between the object and outer boundaries gets larger. Consequently, R_{cond} had to be set a lower value in order to achieve meaningful results. For the star mesh chosen in this paper, the following summarize the range of parameter values that produced the best results for the near fields: $M \geq 4$, $h \leq \lambda/15$, $(M+1)h \geq \lambda/4$, number of segments/wavelength ≥ 20 , and $10^{-5} \leq R_{cond} \leq 10^{-3}$. The effect of integration on the numerical solution was also investigated. In one case, integration was performed by making use of all nodes on the boundary, while in the other case every other node was used. As long as a minimum of 20 segments per wavelength was maintained, the two choices produced virtually the same results. The results shown in Fig. 6 were produced by full integration. As expected, the far fields were less sensitive to fine tuning of various parameters.

We have also tested the method on much larger cylinders. Fig. 8 shows the comparison of near-field results for a cylinder of radius 50λ . Fig. 9 shows the far-zone fields. As before, we chose 20 segments per wavelength on the outer boundary, $h = \lambda/15$, $M = 6$, and $R_{cond} = 10^{-4}$. A vertically polarized source was placed at $(-100\lambda, \lambda/10)$. The total arc length of the obstacle was around 157λ , and the total number of unknowns was $N = 3173$. It took 3 h 10 min 52 s to fill the matrixes, 10 s to obtain the near-field solution from it, and 70 s to compute the far fields, all on a 80486-50 PC. Note that the computational time follows the $O(N^2)$ order as discussed previously. Some ripples are seen in the numerical solution, which are believed to be due to roundoff errors. The disagreement in the far fields near the grazing angles in the shadow region is believed to be due to the slight disagreement seen for the near fields in the shadow region. Overall, the agreement for the near and far fields seems very good.

Next, we consider an obstacle that has both concave and convex portions. A good candidate is the case of a Gaussian hill for which results are available in the literature [17], [1].

Fig. 9. Relative pattern versus ϕ of vertical source near a semicircular boss on a PEC plane. Radius = 50λ .

The height of the hill $h_g(x)$ over the $y = 0$ plane is given by

$$h_g(x) = A e^{-[(x-B)/C]^2} \quad (29)$$

where both x and $h_g(x)$ are in wavelengths. We have chosen $A = C = 10\lambda/3$, $B = 0$. The obstacle is taken as the portion of the Gaussian hill where the height exceeds $\lambda/100$. For the chosen parameters, this occurs for $|x| \lesssim 8\lambda$. The total arc length of the obstacle is about 18λ . The constitutive parameters of the earth are taken as $\epsilon_r = 10$, $\sigma = 10 \text{ m S/m}$ at a frequency of 1 MHz. The latter corresponds to $\sigma_r = 180$. A vertically polarized source is located at $(-50\lambda/3, \lambda/100)$ to the left of the hill. We choose 20 segments per wavelength, $M = 8$, $h = \lambda/15$, and $R_{cond} = 10^{-3}$. The total number of unknowns on the obstacle is 361. In [17], the solution is obtained via a Volterra-type integral equation starting from a parabolic approximation of the Helmholtz equation. Fig. 10 compares the numerical solution of the present method with that of [17]. (It may be worth mentioning that for this particular geometry, the original metrons suggested by Mei *et al.* produced highly erroneous results). It is seen that the two agree fairly well with each other. The increased field strength on the illuminated side is due to focusing by the concave portion of the hill, which the method correctly predicts. No such increased field strength is present in the incident field.

It is interesting to compare the merits of the present method *vis-à-vis* the method of [1], which solves the problem using a magnetic field integral equation and boundary element method. It took 4 min 47 s on a 80486-50 PC to produce the results for the ground wave data shown in Fig. 10 as well as to generate sky wave data at 361 angles. Identical results were obtained with half integration, which took only 3 min. This is in contrast to the method of [1], which took 75 min for the same problem. Of course, [1] uses the free-space Green's function, in contrast to the half-space Green's function employed here. The number of unknowns in [1] was 725 (obtained with 12 nodes per wavelength). For the same number of unknowns, the present method, being of order $O(N^2)$, would take about 19 min.

Fig. 10. Surface magnetic field versus normalized horizontal distance over lossy, Gaussian hill.

Fig. 11. Surface magnetic field versus normalized horizontal distance over lossy, Gaussian hill. Exact Green's function versus approximate Green's function.

This is still faster by a factor of about four compared to [1]. If half integration is used, the method is faster by a factor of about eight. The savings in memory in the present method are tremendous. For the problem at hand, the method of [1] with 725 unknowns requires a storage space of at least 8.4 Mb, compared to only 254 kB with the present method.

To speed up the calculation of the Green's function we have assumed $\Delta_o = 0$ in (24). This amounts to assuming that the flat terrain outside the obstacle is made up of a perfect conductor. Since the fill time is the eventual bottleneck of the method, this results in substantial savings in time. For example, it took 3 h 10 min on a 80486-50 PC to solve the problem of Gaussian hill using the exact Green's function and half integration, compared to only 3 min with the approximate Green's function! The results were identical except near the right end of the hill, where the two differed only slightly. One would expect the use of approximate Green's function to be the worst when Δ_o is significantly different from zero. To demonstrate the magnitudes of errors involved, we consider the case of highly lossy earth with $\epsilon_r = 2$ and $\sigma = 0.056m$ S/m at 1 MHz ($\sigma_r = 1$). A Gaussian hill of type (29) with $A = \lambda$, $B = 0$, and $C = 0.76\lambda$ is considered. A vertically polarized source is placed at $(-3\lambda, \lambda/10)$. Twenty segments per wavelength, $M = 4$, $h = 0.05\lambda$ and $Rcond = 10^{-5}$ were chosen, and integration was performed utilizing all the nodes. Fig. 11 shows the calculations with the exact and the approximate Green's functions. The field that would exist on flat earth is also shown for comparison. Calculations with the approximate Green's function took 34 s, while those with the exact Green's function took 46 min 40 s. However, it is seen that the results do not differ much from each other, justifying the use of the approximate Green's function.

Finally, to demonstrate the utility of the method in studying terrain effects, we present results on the effect of relative location of the source with respect to an obstacle. A vertically polarized source is placed near a Gaussian hill (29) having $A = 1.2\lambda$, $B = 0$, and $C = \lambda$. The total arc length of the

obstacle is 5.1λ . The ground constants are $\epsilon_r = 15$, $\sigma_r = 15$. Four interior layers with 20 segments per wavelength, and $h = 0.05\lambda$ were chosen in the numerical model. In one case, the source is placed at the bottom of the hill at a horizontal distance of 4.5λ from the peak. In the second case the source is placed at the top of the hill ($x_o = 0$). Fig. 12 shows the normalized surface field on the hill. Once again there is focusing on the illuminated side of the hill when the source is at the bottom. The field for the source at the top of the hill is symmetric as expected. Note that the field in the shadow region of the hill for $x_o = -4.5\lambda$ is as strong as the corresponding field for $x_o = 0$. There is no apparent advantage of siting the antenna at the top of the hill to receive the ground wave. However, the radiation pattern of the source from $\phi = 0$ to 90° is significantly affected by the presence of the hill. This can be seen from Fig. 13, which shows the relative pattern for the two source locations. Irrespective of the source location, the radiation pattern has a deep minimum near grazing angles that is characteristic of vertical antennas over lossy earth [21].

IV. SUMMARY

A fast, finite difference method that includes all aspects of wave phenomena such as reflection, refraction, diffraction, and backscattering is presented for predicting two-dimensional propagation over inhomogeneous, irregular terrain. The terrain is characterized by its elevation and impedance, the latter being dependent on the ground constants of the earth. Both of these may vary with distance. The terrain topology data is specified at discrete points. Interpolation using quadratic elements is done to define a continuous geometry. The computational domain for the problem consists of the area bounded by the terrain at the bottom and a truncating boundary at the top. To simulate free space on the truncating boundary, a discrete, near-field boundary condition of Mei type, derivable from an integral representation of the fields, is imposed. Green's function for half-space is used to reduce the number of

Fig. 12. Effect of source location on the ground wave.

Fig. 13. Effect of source location on the sky-wave pattern. Angle shown is ϕ .

unknowns. Unknowns are distributed on the terrain only where its elevation is nonzero and/or where its impedance differs from a reference value. Finite difference coefficients valid for an irregular, nonorthogonal mesh are presented. Accurate expressions are provided for the Green's function and incident fields over a constant-impedance, flat plane. These expressions permit rapid field computation, as they do not involve the troublesome Sommerfeld integrals. Results are presented for the ground wave as well as for the sky wave.

The truncating boundary can be in the near-field of the obstacle—as near as a $\lambda/2$ away from it, though reasonable results were obtained with a spacing of only $\lambda/4$. The method works both for convex and concave geometries. Good results have been obtained with a node density of about 20 per wavelength, at least four interior layers, and an interlayer spacing of about $\lambda/15$. The metrons used for the determination of the Mei coefficients are proportional to the complex conjugate of the Green's function and fully accommodate the environment of the problem. Singular value decomposition, which permits

filtering of the space spanned by the corrupted singular vectors, is used to solve for the coefficients.

The method is attractive for large terrain obstacles where other methods tend to be slow. The overall computational time of the method is of order $O(N^2)$ and is dominated by the matrix filltime. Storage requirement is of order $O(N)$, where N is the total number of unknowns (nodes). As an example, for a terrain obstacle extending over 18 wavelengths, the number of nodes on the boundary is around 361. To compute groundwave data at all of these nodes as well as to compute the far fields at 361 angles, the method takes around 3 min on a 80486-50 PC. The total storage required for the problem is around 254 kB. If the obstacle extends over 180 wavelengths, it would take around 300 min and require a storage space of around 2.5 MB. At a frequency of 1 MHz, the latter case corresponds to an obstacle having an arc length of 54 km. At a frequency of 1 GHz it corresponds to an arc length of 54 m.

Certain spurious oscillations of the field have been observed in some regions of the shadow region. This is attributed to the fact that it is difficult to calculate near fields precisely over an electrically small, discrete domain. It is speculated that they can be reduced by choosing a computational molecule that accommodates higher order derivatives. This is more so because the one-sided difference must be employed at the ends of the computational domain. More accurate and efficient schemes of evaluating the near fields and the Green's function will have to be made to further decrease roundoff errors and reduce the computational time before the method can be applied to the 3-D case.

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