Communications

Procedure for Correct Refocusing of the Rotman Lens According to Snell's Law

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Abstract—**A procedure is derived for proper refocusing of the dielectric-filled Rotman lens with beam port locations determined according to Snell's law. The approach provides an alternative lens configuration which may give a wider field of scan, at a given focal length, for lenses fabricated in microstrip or stripline. The presence of grating lobes in the lens is discussed.**

I. Introduction

The techniques for microwave lens design, as described by Rotman and Turner [1], have found wide application in beamforming antenna systems [2]. This approach is well suited to implementation in stripline or microstrip circuitry. In general, the design of a particular strip transmission-line lens is first obtained in air, as per Rotman and Turner, and the dielectric region is then scaled by the inverse square root of the substrate dielectric constant. By the use of a substrate with high dielectric constant, a very compact lens can be obtained.

As an alternative, the array port contour (inner lens contour) can be left unscaled, in which case the beam port locations on the lens focal arc are determined according to Snell's law, i.e.,

$$\sin \beta = \epsilon_r^{-1/2} \sin \alpha,$$

where α is the scan angle of the antenna array, β is the corresponding angle of focus inside the lens, and ϵ_r is the relative dielectric constant in the parallel plate region of the lens. This arrangement provides beam port and array port placements which give improved coupling to the outermost beam ports, particularly for stripline or microstrip lenses used with small arrays. It shall be referred to here as the refracting lens. In this communication, the theoretical performance of the refracting lens is compared with that of the conventional design at wide scan angle and the design equations for correct refocusing of the refracting lens are derived.

II. Comparison of Two Approaches

Fig. 1 shows the array port and beam port curves for two lens designs, one of which comes from simply scaling the design obtained in air to the wavelength in the substrate medium. The other is designed for Snell's law placement of the beam ports and is depicted with the heavier line in the figure. The two are shown superimposed for comparison. The particular design example is for a lens with a focal length of five wavelengths and an array of six elements with half-wavelength element spacing in air. The relative dielectric constant in the lens is chosen to be 2.33 and the beam port curves which are shown in the figure are for ±50 degrees of scan with off-axis points of perfect focus at ±30°. For both designs, the ratio of off-axis to on-axis focal length is $g = 1.06$. Larger values for this parameter give increased phase errors and reduced coupling to the outer beam ports so that the wide-angle performance of the scaled conventional lens is further degraded in comparison with the properly designed refracting lens.

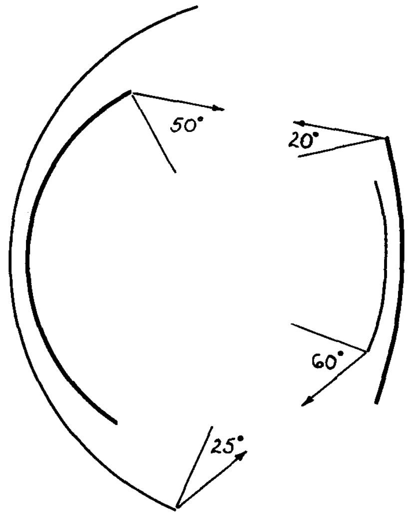

Fig. 1. Comparison of conventional lens and refracting lens with identical parameters. The contours of the refracting lens are drawn with the heavier line.

In order to compare the performance of the two designs at wide scan angles, we shall examine the coupling between the array ports and the outermost beam ports for both cases. The direct-ray amplitude coupling coefficient is given by [3]

$$S_{AB}^0 = \left[\frac{2\pi d_A d_B}{K r_0}\right]^{1/2} \frac{E(\theta_B, d_B) E(\theta_A, d_A)}{\lambda} e^{-jKr_0} \tag{1}$$

where K and λ are the wavenumber and wavelength in the lens parallel plate region, d_A and d_B are the widths of the array port and beam port, $E(\theta, d)$ is the normalized field pattern of a port of width d and angle θ from the port normal, and r_0 is the distance between the array port and beam port. Assuming a uniform field distribution in the port aperture, the normalized radiation pattern is approximately given by [4]

$$E^2(\theta, d) = \left[\frac{\sin(\pi d \sin\theta/\lambda)}{\pi d \sin\theta/\lambda}\right]^2 \cos^2\theta. \tag{2}$$

The approximate look angles between the outermost beam port and the "worst case" array port are shown in Fig. 1 for both design cases. Using these angles in (1) and (2) with beam ports at scan angles 10° apart, it is found that the magnitude of the coupling coefficient for these beam port/array port pairs is 1.36 times (2.7 dB) greater for the refracting lens at a frequency of 10 GHz. In addition to providing improved coupling between the beam ports and array ports, the

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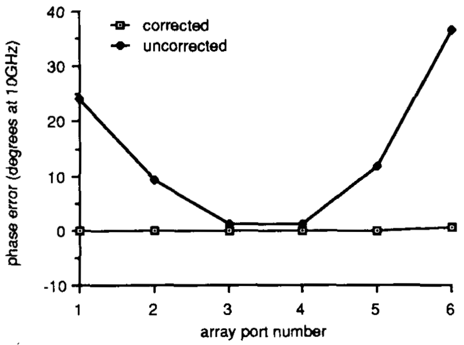

Fig. 2. Plot of phase errors on each ray path for uncorrected refracting lens and correctly refocused refracting lens design example at 50° scan angle.

design equations for the refracting lens yield solutions at wider scan angles than for the scaled conventional lens.

III. Modified Design Equations

The refracting lens with an array port contour or inner lens contour obtained according to the design equations in [1] is not correctly focused. In other words, if the array port contour is obtained strictly according to [1] without scaling to the wavelength in the lens substrate material and with subsequent placement of beam ports according to Snell's law, substantial phase errors will result. This is because the curvature of the lens is typically too large, with respect to wavelength, for Snell's law to be applied directly. We shall refer to the lens design thus obtained as the uncorrected refracting lens design. Fig. 2 shows a plot of phase errors on paths to each array port from the 50° beam port for the uncorrected refracting lens design with the parameters of the previous example. The array ports are numbered sequentially from one end of the array with array port number 6 being the nearest to the beam port. Also plotted in Fig. 2 are the corresponding phase errors for the refracting lens which was designed using the corrected design equations derived here. The corrected refracting lens design gives phase errors much smaller than one degree electrical for this example.

In order to correctly refocus this type of lens, another set of design equations must be obtained, starting from proper modification of the electrical and geometrical constraints which are given in [1, eqs. (1)–(6)]. Definition of the parameters involved is given pictorially in Fig. 3. F_1, F_2, and G are points of perfect focus at angles of $-\beta$, $+\beta$, and 0 degrees, respectively.

Electrical constraints:

$$(\overline{F_1P})\epsilon_r^{1/2} + W + N \sin \alpha = \epsilon_r^{1/2} F + W_0 \tag{3}$$

$$(\overline{F_2P})\epsilon_r^{1/2} + W - N \sin \alpha = \epsilon_r^{1/2} F + W_0 \tag{4}$$

$$(\overline{G_1P})\epsilon_r^{1/2} + W = \epsilon_r^{1/2} G + W_0. \tag{5}$$

Geometrical constraints:

$$(\overline{F_1P})^2 = (F^2 + X^2 + Y^2) + 2\epsilon_r^{-1/2} FX(\epsilon_r - \sin^2 \alpha)^{1/2} - 2\epsilon_r^{-1/2} FY \sin \alpha \tag{6}$$

$$(\overline{F_2P})^2 = (F^2 + X^2 + Y^2) + 2\epsilon_r^{-1/2} FX(\epsilon_r - \sin^2 \alpha)^{1/2} + 2\epsilon_r^{-1/2} FY \sin \alpha \tag{7}$$

$$(\overline{GP})^2 = (G + X)^2 + Y^2. \tag{8}$$

Following Rotman and Turner, a set of parameters, which is normalized with respect to the focal length F, is here defined:

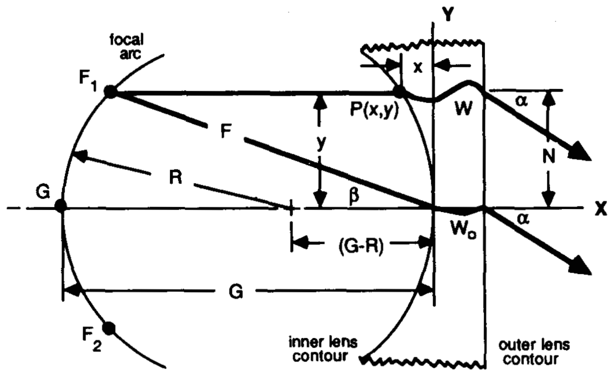

Fig. 3. Microwave lens parameters.

$$\eta = N/F, \ x = X/F, \ y = Y/F,$$

$$w = \frac{W - W_0}{F}, \ g = G/F.$$

Also define,

$$a_0 = (1 - \sin^2 \alpha/\epsilon_r)^{1/2}, \quad b_0 = \sin \alpha.$$

For this case, [1, eqs. (8), (11)] become

$$y\epsilon_r^{1/2} = \eta(\epsilon_r^{1/2} - w) \tag{9}$$

$$\epsilon_r x^2 + \epsilon_r y^2 + 2\epsilon_r gx = w^2 - 2\epsilon_r^{1/2} gw \tag{10}$$

which are the equations to be solved for the (x, y) coordinates of the array ports on the inner lens contour. The following relation is also obtained between the normalized distance η from the lens axis to a given antenna element and the normalized length w of the transmission line connecting the antenna element to an array port,

$$aw^2 + bw + c = 0 \tag{11}$$

where a, b, and c are functions of η. This is the same as [1, eq. (12)] except that the parameters a, b, and c are given, in terms of η, by the following:

$$a = \left[1 - \eta^2 - \frac{(g-1)^2}{(g-a_0)^2}\right] \tag{12}$$

$$b = \left[2g\epsilon_r^{1/2} \frac{(g-1)}{(g-a_0)} - \frac{(g-1)\eta^2 b_0^2}{\epsilon_r^{1/2}(g-a_0)^2} + 2\eta^2\epsilon_r^{1/2} - 2g\epsilon_r^{1/2}\right] \tag{13}$$

$$c = \left[\frac{g\eta^2 b_0^2}{(g-a_0)} - \frac{\eta^4 b_0^4}{4\epsilon_r(g-a_0)^2} - \epsilon_r \eta^2\right]. \tag{14}$$

The choice of the value of g, the ratio of on-axis to off-axis focal length, affects the quality of focusing away from the three points of perfect focus. Rotman and Turner describe a procedure for estimating the optimum value of g for a given choice of the off-axis focus angle α. By substituting β, the off-axis focus angle in the refracting lens, in place of α, [1, eq. (13)] becomes

$$g = 1 + \frac{\beta^2}{2}. \tag{15}$$

This gives a value of g which is close to the optimum for minimizing the overall aberrations of the lens.

The optical aberration of the lens at a given scan angle is obtained from the difference in total path lengths (including cable length) between the central ray, which passes through the origin, and one other ray, with both paths terminating at a given wavefront in free space. The expression for the path length error ΔL is given by

$$\frac{\Delta L}{F}=(h^2+x^2+y^2+2hx\cos\beta+2hy\sin\beta)^{1/2} - h+w+\eta\sin\theta \quad (16)$$

where $h = H/F$ and H is the distance from the origin to the beam port corresponding to scan angle θ, located at angle β on the lens focal arc. Note the similarity of this expression to [1, eq. (14)].

IV. The Grating Lobe "Problem"

If the array port spacing exceeds a half-wavelength in the refracting lens, grating lobes will appear inside the lens when the antenna array is scanned to the outer angles. However, it can be easily shown that grating lobes will not appear in the scan space of the lens. For an antenna array with $\lambda/2$ element spacing, let θ be the scan angle of an incident plane wave. The phase difference δ between adjacent array elements is then given by

$$\delta=-\frac{\lambda}{2}\sin\theta \quad (17)$$

where λ is the free-space wavelength. The effective spacing of the array ports inside the lens is $\epsilon_r^{1/2}\lambda/2$. Be rescaling the problem to air, the angular position of the first grating lobe inside the lens, designated as θ_1, is given by

$$\epsilon_r^{1/2}\frac{\lambda}{2}\sin\theta_1=\lambda+\delta \quad (18)$$

which yields the following relationship between the scan angle θ, and the angular position of the first grating lobe inside the lens:

$$\sin\theta_1=\frac{2}{\epsilon_r^{1/2}}-\frac{\sin\theta}{\epsilon_r^{1/2}}. \quad (19)$$

If θ_{1s} is the corresponding scan angle of the grating lobe, then

$$\sin\theta_{1s}=\epsilon_r^{1/2}\sin\theta_1 \quad (20)$$

which, from (19), gives the following result:

$$\sin\theta_{1s}=2-\sin\theta. \quad (21)$$

The scan angle of the first grating lobe is undefined for array scan angle less than 90 degrees. Thus it is found that grating lobes will not appear in the scan space of the lens. Although they do not appear in the scan space, the presence of grating lobes in the refracting lens constitutes a compromise of effective antenna gain at wide scan angles. This may be an acceptable trade-off for array/lens combinations which require a wide angle of scan if properly terminated beam ports are provided to absorb the grating lobe energy.

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Evaluation of Edge Effects in Slot Arrays Using the Geometrical Theory of Diffraction

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Abstract—**In the design of high-performance slot arrays, the influence of the array edges should not be neglected. The quantitative evaluation of these effects requires the knowledge of the Green's function for a wedge. Upon use of the geometrical theory of diffraction (GTD), an approximate form of this Green's function is computed, which admits a representation in terms of "images." The influence of the edge is modeled as a coupling with a suitable image of the slot, which can be very efficiently computed. Some test cases show that the overall error of this approximation can be neglected since it is comparable with the error due to mechanical tolerances.**

I. Introduction

The design of a waveguide-fed longitudinal shunt slot array has been discussed by various authors. It is well known that an accurate matching of the input impedance of the array can be achieved only when external mutual coupling is taken into account. A procedure which allows a systematic inclusion of such coupling in the design equations has been recently formulated by Elliott [1].

Let us consider an array of N slots whose offsets and lengths are x_n and $2l_n$. Assuming the tangential electric field in the slot given by $V_n^S/w\cos\pi z_n/2l_n\,\hat{x}$ [1], [2], wherein w is the slot width and z_n a local abscissa on the slot, the active admittance Y_n^A of a slot can be expressed recasting [2, eq. (33)] as

$$\frac{1}{Y_n^A}=\frac{1}{Y_n}+\bar{f}_n\cdot\sum_{m\neq n} g_n^m\frac{V_m^S}{V_n^S} \quad (1)$$

wherein Y_n is the self-admittance of a slot and $\bar{f}_n$ a geometric factor depending on (x_n, l_n) as well as on the waveguide dimensions and angular frequency ω.

The coupling coefficient is proportional to the reaction integral between the field of a slot (say n) and the equivalent current distribution of another (say m), and is given by

$$g_n^m=2\pi j\zeta\int_{-l_m}^{l_m}\frac{\mathbf{H}_n(z_m)}{V_n^S}\cdot\left(\hat{z}\cos\frac{\pi z_m}{2l_m}\right)dz_m, \quad (2)$$

wherein ζ is the free-space impedance and $\mathbf{H}_n(z_m)$ is the field due to the slot n measured at the slot m.

The exact expression of $\mathbf{H}_n$ in the presence of one edge is very complicated so that the evaluation of (2) is in general unfeasible. Moreover, no exact expression for $\mathbf{H}_n$ is known when all four edges are included.

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