Fuzzy Logic Applications to Multisensor-Multitarget Correlation

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A consistent tactical picture requires data fusion technology to combine and propagate information received from diverse objects and usually vague situations. The information may be contained in two types of data; *numerical* data received from sensor measurements, and *linguistic* data obtained from human operators and domain experts. In real world situations, the numerical data may be noisy, inconsistent, and incomplete, and the linguistic information is imprecise and vague. To deal with these two types of data simultaneously, fuzzy sets and fuzzy logic provide a methodology to obtain an approximate but consistent tactical picture in a timely manner for very complex or ill-defined engineering problems.

A functional paradigm for fuzzy data fusion is presented. It consists of four basic elements: 1) *fuzzification* of crisp elements, 2) *fuzzy knowledge base* derived from numerical input/output relations and humans, 3) fuzzy *inference mechanism* based on a class of fuzzy logic; 4) *defuzzification* of fuzzy outputs into crisp outputs for use by a plant. For real-time practical systems, the on-line determination of a fuzzy membership function from a given set of crisp inputs is vital. To this end, a methodology for estimating an optimal membership function from crisp input data has been implemented. This is based on the possibility/probability consistency principle as proposed by L. A. Zadeh. A relationship between the fuzzy membership function and the confidence level of statistical input data has been developed and it serves as a design parameter for fuzzification. This technique has been applied to a two-dimensional multisensor-multitarget tracking system. Fuzzy system performance evaluations have been presented. With simulated data in the laboratory environment, the simulation has been performed to evaluate the Mission Avionics Sensor Synergism (MASS) Systems. These results show better performance for the data correlation function using the fuzzy logic techniques.

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I. INTRODUCTION

Fuzzy logic designs have been successfully applied to a large number of industrial control systems. These include the control of trains, cement kiln, traffic lights, power systems, and air conditioners. A fuzzy controller handles adjustments and variations the way a person thinks and it can set speed at low, medium, or high. In these systems, computers make decisions like humans.

For successful execution of operational tactics, a consistent picture plays a crucial role. However, data fusion technology is the key to a consistent tactical picture. To develop a consistent tactical picture, we have to deal with very diverse objects and imperfect information. By this we mean information that is incomplete, inconsistent, inaccurate, imprecise, uncertain, unreliable, or some combination of these. However, in algorithmic formulations, the data fusion technology usually requires mathematically precise descriptions of these processes and objects to fuse the information. To highlight the gap between the reality and the mathematical abstraction, A. Einstein states, "So far the laws of mathematics refer to reality, they are not certain. And so far they are certain, they do not refer to reality." In this sense, only "approximate" representations can work closely with real world situations. We deal here with approximate reasoning to fuse imperfect information.

Fuzzy logic provides a framework and flexibility to couple human judgment with standard mathematical tools. Mathematics is among the most explicit of human cognitive processes whereas judgment, on the other hand, requires the exercise of human intelligence. Fuzzy system techniques can provide approximate solutions, in a simple and robust way, to engineering problems which are too complex or ill-defined to yield analytical solutions, whose boundaries are not crisp, and where data are incomplete, inconsistent, imprecise and ambiguous. For real world problems, we deal with two types of uncertainty: probabilistic uncertainty and possibilistic (vague and imprecise) uncertainty. We usually receive information in two forms, numerical data received from sensor measurements, and linguistic information in two forms, numerical data received from sensor measurements, and linguistic information from human operators and experts. Since the sensors employ simplified mathematical models, and provide measurements based on past operations, these sensor measurements cannot accurately cover future simulations. On the other hand, when human beings express their skill by linguistic rules, some information is lost in communication. As a result, neither of the two is enough for representing and solving real-world engineering problems. Success usually comes to those who apply technology to their best advantage. Thus, the idea is to use both numerical and linguistic information using fuzzy logic to generate approximate solutions for complex engineering problems.

Both fuzzy imprecision and probabilistic uncertainty are intrinsic features of operational systems. In a multisensor tracking system [16], we deal with uncertainty. This uncertainty may be caused by both random variation of measurement data as well as inaccuracy in data recording (fuzzy data). Thus, we have two types of variables associated with the tracking processes: random variable and fuzzy variable. A fuzzy variable is associated with a possibility distribution in much the same manner as a random variable with a probability distribution. However, in general, a variable (data) may be both fuzzy and probabilistic at the same time. And this variable may be associated with a possibility distribution and a probability distribution with the weak connection between the two expressed as the possibility/probability consistency principle [19].

In tracking multiple targets there can be an uncertainty associated with measurements as well as the origin of the measurements when there is clutter or the false-alarm rate is high. An uncertainty may also be associated with the tracking system when multiple targets are in the same neighborhood. Further, a multisensor tracking system can be associated with an additional uncertainty contributed by the following processes.

1) Data alignment. This is one of the major problems in integrating sensors into a multisensor system. Measurements may require conversion to a common coordinate and time base, and unit adjustments.

2) Different dimensions. Sensor measurements may have a different number of spatial measuring dimensions (such as IRST-to-ESM (AZ,EL) and (EL)). This gives rise to an uncertainty in dimension.

3) Different sensor characteristics (such as target viewing angles, measurement accuracies, sensor resolutions, and field of view). Difference in any of these further complicate the problem of associating measurements.

4) Object/attribute correlation. A variety of sensors allows the measurements of target attributes (target type, size). A correlation of these gives rise to a certain uncertainty.

5) Data/object correlation. Correlating kinematic measurements (range, range-rate, azimuth) received from different sensors with multiple tracks gives rise to missed correlation (Type I errors) and incorrect correlation (Type II errors).

6) Object positional, kinematic, and attribute estimation. This involves predicting different sensor observations, updating and predicting position/kinematic/attribute estimates, and managing sensors. All these processes associated with a multisensor tracking system contribute to the uncertainty of tracking targets.

In practical applications of fuzzy systems, we need to know the membership of a fuzzy variable. The membership grades are obtained either subjectively or as the values of a function of those particular events. The main focus in this researc is to demonstrate the feasibility of applying fuzzy logic techniques to correlation problem solving. In this work, we generate membership functions for stochastic variables characterizing multisensor tracking problems [16], develop fuzzy rules for multisensor data association, and finally, defuzzify the fuzzy results obtained for use by a tracking system. Two examples demonstrate the feasibility of using fuzzy logic for solving data association problems in a multisensor-multitarget tracking.

II. FUZZY SETS AND FUZZY LOGIC SYSTEMS

At the heart of fuzzy logic is the fuzzy set. Here we define fuzzy sets and fuzzy logic, and their interrelationship.

A. Fuzzy Sets

A fuzzy system is a class of objects in which there is no sharp boundary between those objects that belong to the class and those do not. Membership function in a fuzzy set is a matter of degree. In addition, an element may also be a member of more than one set [5]. In a classic Boolean logic, an element either belongs to the set or does not; there are only two possible states (1 or 0). Despite the imprecise boundaries of a fuzzy set, a set F can be defined *precisely* by associating with each objects x, a number between 0 and 1 which represents its grade of membership in F. Each fuzzy set represents a linguistic term of some linguistic variable. A linguistic variable is defined as a variable whose values are sentences in a natural language. For example, "age" is a linguistic variable; this variable can only take linguistic values. The linguistic values for the linguistic variable, "age," may be very young, young, old, very old, etc. These linguistic terms play a key role in human communication. Here the emphasis is not on *measuring* the contents of the information, but rather on defining the *meaning* of a linguistic value by its possibility distribution [19].

We provide some definitions in mathematical forms.

A universe of discourse U is a collection of objects which can be discrete or continuous. A fuzzy set F in U characterizes a membership function $\mu_j : U_0 >$ $[0,1]$. This is labeled by a linguistic term, where a linguistic term is a word such as "young," "old," etc. For example, let U be the values of "age" of a person. Then we define four fuzzy sets in U, namely, "Very Young," "Young," "Old," and "Very Old." Fig. 1 presents a graphic representation of the membership function for the linguistic variable "age". Clearly this provides smooth transitions and overlaps among the

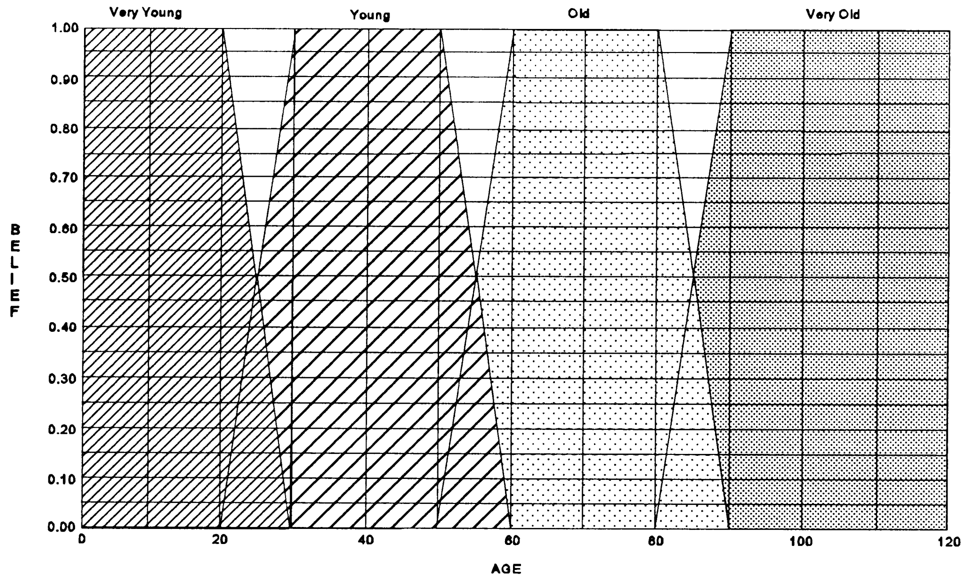

Fig. 1. Fuzzy membership function.

fuzzy sets conforming to a natural situation in a real world.

The mathematical framework of theory of fuzzy sets provides a natural basis for fuzzy logic, as well as for the theory of possibility.

B. Fuzzy Logic

Basically, fuzzy logic is a generalization of binary logic. Fuzzy sets, membership functions, and IF–THEN rules are three primary elements of a fuzzy logic system. In the IF–THEN rules (which are logical statements), linguistic terms are used to incorporate vagueness and ambiguity. This logical inferencing using fuzzy sets is known as fuzzy logic. There are two main categories of fuzzy implication inference rules in approximate reasoning; the generalized modus popens (GMP), and the generalized modus tollens (GMT) [8]. The forward data-driven inference (GMP) plays an important role in fuzzy control systems. The backward goal-driven inference (GMT) can play a powerful role in special situations.

III. BASIC ELEMENTS OF A FUZZY SYSTEM

A fuzzy system contains four basic elements; fuzzification interface, fuzzy knowledge-base, fuzzy inference engine, and defuzzification. Fig. 2 provides a functional paradigm for fuzzy systems. These elements are described below.

A. Fuzzification Interface

The fuzzification interface transforms each numerical data received from sensor measurements into fuzzy variables. The number of fuzzy sets defined in the input discourse and their specific membership functions define the fuzzification interface design [5]. Fig. 1 defines a fuzzification interface for the linguistic variable age. This maps the crisp value of the age of a person into four fuzzy sets which define four linguistic values (very young, young, old, very old). Let the age of a person be 55 years, then the outputs of this fuzzy interface related to the belief result in $\mu_{\text{very-young}(55)=0}$, $\mu_{\text{young}(55)=0.5}$, $\mu_{\text{old}(55)=0.5}$, and $\mu_{\text{very-old}(55)=0}$.

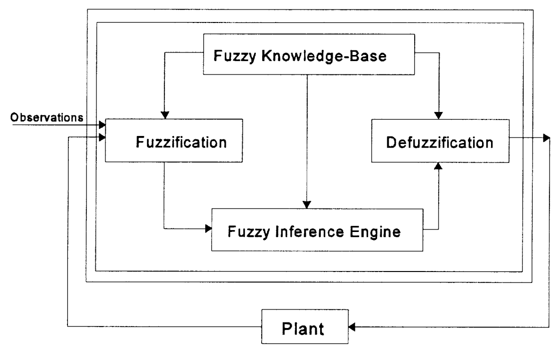

Fig. 2. Functional paradigm for fuzzy systems.

It is a fact of life that much of the evidence on which human decisions are based is both fuzzy and granular [18]. With this motivation, the fuzzification of numerical data from sensor measurements needs dividing an optimal (or empirical) membership X into a number of fuzzy subsets (or term sets). This may also be considered a fuzzy partitioning of a fuzzy set X. The main requirement is to come up with fuzzy subsets that ensure a uniform activation of the original membership function of a fuzzy set X [11].

B. Fuzzy Knowledge-Base

This contains IF–THEN rules and simple fuzzy statements, and provides a methodology to represent human knowledge. We specify the meaning of fuzzy rules using linguistic terms defined by their membership functions (Fig. 1).

C. Fuzzy Inference Engine

The fuzzy inference engine employs a particular kind of fuzzy logic. It stimulates human decision making procedure, and employs fuzzy knowledge-base and fuzzy input to generate fuzzy decisions (output).

There are two common methods to perform fuzzy logic inferences; the max-min method and the max-product method. In the max-min method, the final output membership function for each output is the union of the fuzzy sets assigned to that output, and the degree of membership values are clipped at the degree of membership for the corresponding premise. In the max-product inference method, the final output membership function for each output is the union of the fuzzy sets assigned to that output in a conclusion, and their degree of membership values are scaled to peak at the degree of membership for the corresponding premise. The max-min interference method is explained in Appendix A.

D. Defuzzification Interface

All fuzzy logic inference methods results in fuzzy values for all output information. The defuzzification interface transforms the fuzzy output into crisp (nonfuzzy) data for use by a plant. There are several defuzzification methods including the centroid method and the height method [5, 8]. The centroid method (also known as the center of gravity method) is the most common in use. This method selects the output value corresponding to the centroid (center of gravity) of the output membership function as the crisp value for an output. Appendix A illustrates its application.

IV. FUZZY MEMBERSHIP FUNCTION ESTIMATION APPROACH

The determination of a fuzzy membership function is most crucial in applying a fuzzy system design methodology to engineering problems. For real-time practical systems, the on-line generation of membership functions is vital. There is no general method available for determining membership functions from a given set of crisp input. Dubois and Prade [5] have surveyed a number of techniques to generate membership functions in specific cases. In many cases, membership functions are determined subjectively. However, it is worth noting that a membership function may be subjective, but not arbitrary. And subjective judgments are not additive. Further the grades of membership are "meaning representations" of linguistic terms of linguistic variables which are essential key elements of any natural language in human communication and reasoning [18]. Lai and Hwang [7] have classified all existing membership functions into four broad categories as follows.

1) Membership functions based on heuristic determination. In this category, we have Zadeh's unimodal function, Dimitru and Luban's power function, and Svarowski's sin function.

2) Membership functions based on reliability concerns with respect to the particular problem. This contains Zimmermann's linear function, Tanaka, Uejima and Asai's symmetric triangular functions, Hannan's piecewise linear function, Sakawa and Yumine's exponential and hyperbolic inverse functions, Dimitru and Luban's function, and Dubois and Prade's L-R fuzzy number.

3) Membership functions based on more theoretical demands. This includes Civanlar and Trussel's function, and Svarovski's function.

4) Membership functions as a model for human concepts such as Hersh and Caramazza's function, Zimmermann and Zysno's function.

In practice, a system designer may also use empirical or statistical distributions to design optimum membership functions. To solve multisensor tracking problems, there are several statistical techniques including maximum likelihood estimation and Kalman filtering. Since multisensor tracking systems employ statistical inputs, the membership function estimation based on statistical input data distributions shall be appropriate. Zadeh [19–21] has provided a mechanism, called a possibility probability consistency principle which is described below.

A. Possibility/Probability Consistency Principle

The concepts of possibility and probability are inherent in human thinking. These underlie the human ability to reason in approximate terms. Clearly, this demands developing a better understanding of the interrelationship and interplay between possibility and probability. It will enhance our ability to develop machines which can simulate human reasoning to attain goals defined imprecisely in an uncertain and vague environment. It may be noted that a high degree of possibility does not necessarily imply a high degree of probability, nor does a low degree of probability imply a low degree of possibility. However, if an event is bound to be impossible, it is bound to be improbable, but not vice-versa [6, 18]. This heuristic relationship between possibility and probability may be stated as the possibility probability consistency principle.

For any union D of disjoint intervals, the associated possibility and probability distributions

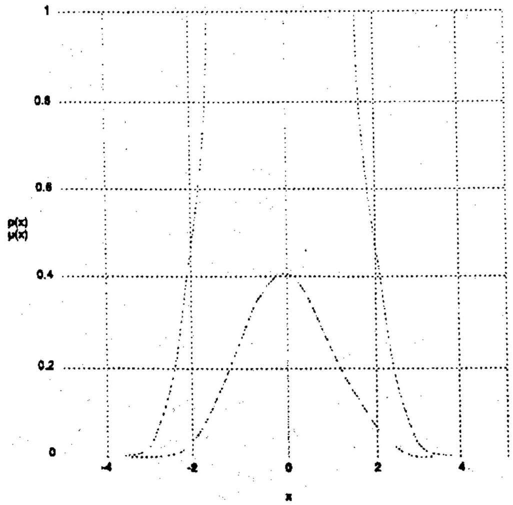

Fig. 3. Membership function for Gaussian pdf.

can be written as

$$\pi(D) = \sup_{X \in D} h(x) / \sup h$$

and

$$\Pr(D) = \int_D h(x)\,dx \Big/ \int_{-\infty}^{\infty} h(x)\,dx.$$

The consistency principle states that the degree of possibility of an event is greater than or equal to its degree of probability [19]. Mathematically, this can be written [5] as

$$\sup_{X \in D} h(x) / \sup h \geq \int_D h(x)\,dx \Big/ \int_{-\infty}^{\infty} h(x)\,dx$$

for any set D on the real line.

It is noteworthy that this principle is not a precise law rather it is an approximate formalization of the heuristic observation that a lessening of the possibility of an event tends to lessen its probability, but not conversely. Further, Civanlar and Trussel [2] have shown that, for every probability density function there exists a lower bound for the confidence level of the statistical input data over which the optimal membership function satisfies the consistency principle. Sudkamp [15] provides probability possibility transformations, and Delgado and Moral [4] discuss the concept of the consistency principle.

B. Optimal Membership Generation

The possibility probability consistency principle can be applied to generate optimal membership functions. In this work, we generate membership functions for tracking data based on their probability density functions (pdf). In this way, we relate the fuzzy membership functions to physical properties of tracking systems. The optimal membership function can be shown [2] to be

$$\mu(x) = \begin{Bmatrix} \lambda p(x) & \text{if} \quad \lambda p(x) < 1 \\ 1 & \text{if} \quad \lambda p(x) \geq 1 \end{Bmatrix}$$

where $p(x)$ is the pdf, and p is a constant satisfying the equation

$$\lambda \int_{\lambda p(x) < 1} p^2(\eta)\,d\eta + \int_{\lambda p(x) \geq 1} p(\eta)\,d\eta - C = 0$$

where C is a confidence level of the statistical data used, and it serves as a design parameter. This mathematical equation provides a basis to generate an optimal fuzzy membership function. We demonstrate its applications below.

V. FUZZY MEMBERSHIP FUNCTION DETERMINATION

Given a value of confidence level of statistical data and its pdf, we can apply the consistency principle

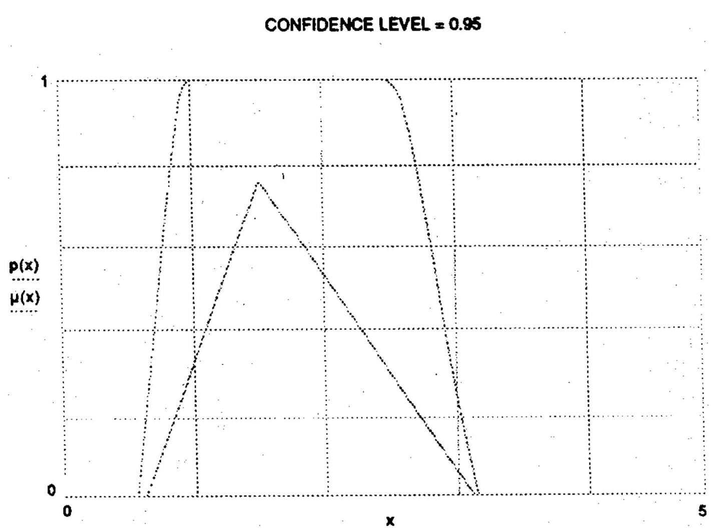

Fig. 4. Membership function for triangular pdf.

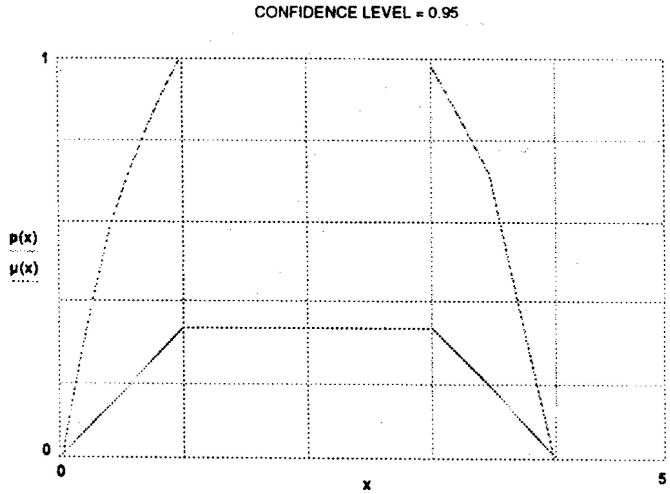

Fig. 5. Membership function for trapezoidal pdf.

to obtain an optimal membership function. The determination of an optimal membership function requires the solution of the equations given in Section IVB. Further, Mathcad 4.0 from MathSoft Inc. has been used to solve the equation for various values of the confidence level C. Here we present membership functions for five types of pdfs, namely, Gaussian, triangular, trapezoidal, histogram, and chi-square.

1) *Gaussian*: We can plot membership functions for the Gaussian pdf with different confidence levels. Fig. 3 shows this for 95% confidence level.

2) *Triangular*: A triangular pdf is very important in ocean surveillance. Fig. 4 presents a membership function corresponding to 95% confidence level.

3) *Trapezoidal*: Membership function for trapezoidal pdf for a confidence level of 95% is shown in Fig. 5.

4) *Histogram*: In practice, the pdf may take the form of a histogram. Fig. 6 shows a fuzzy membership function.

5) *Chi-square*: In tracking applications, a discriminant is formed using chi-square variables. Fig. 7 presents a fuzzy membership function for a chi-square variable.

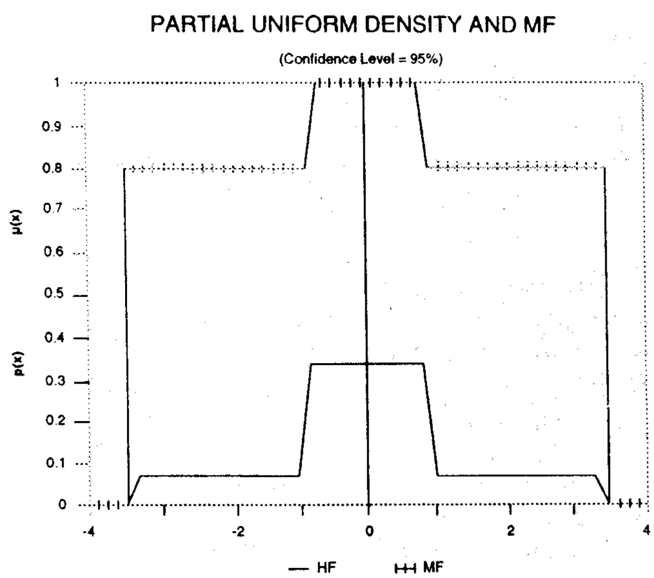

Fig. 6. Partial uniform density and MF.

VI. INFORMATION GRANULARITY AND FRAME OF COGNITION

Once a fuzzy membership function of a variable of interest is obtained, one can use linguistic labels (say, small, medium, big) that act as elastic constraints over a given universe of discourse and thus identify some of its regions as compatible to the highest degree with these constraints. These linguistic labels are referred to as *information granules*. Here the information is granular in the sense that the data points within a granule have to be dealt with as a whole rather than individually [18]. As a result this concept of information granularity provides a basis for construction of a general theory of evidence in which the evidence is allowed to be fuzzy in nature.

A set of information granules defined for a certain linguistic variable X over a given universe of discourse constitutes a *frame of cognition* of this variable [12]. Sometimes the frame of cognition is also referred to as a *fuzzy partition*. A frame of cognition may contain several linguistic labels. By adjusting the granularity of the labels one can easily implement the principle of incompatibility. This principle asserts that high precision is incompatible with high complexity [20]. The main features of this frame are specificity and robustness. Fig. 8 depicts different specificity. The courser the fuzzy set, the lower its granularity. The more precise the fuzzy set, the higher the value attained for its specificity. The maximum specificity corresponds to a pointwise assessment of the value of a variable.

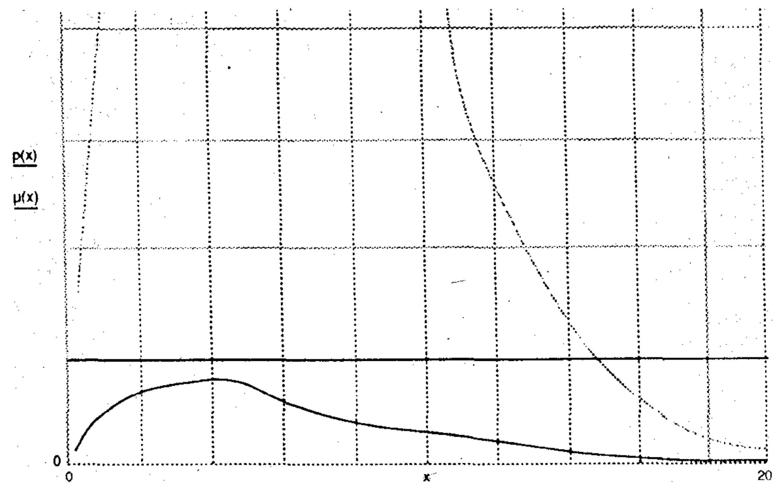

Fig. 7. Fuzzy membership function for chi-square variable.

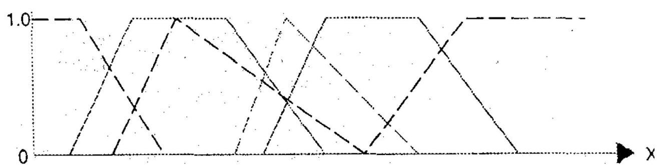

Fig. 8. Two frames of cognition with different specificity.

A. Reconstruction Problem

Given a piece of fuzzy information (such as a membership function of a fuzzy variable) how does one approximate this membership with uniform activation using granules? Basically this is a reconstruction problem. This deals with the selection of the linguistic labels and their distribution. This has a primodial impact on the performance of the reconstruct.

In a frame of cognition, the linguistic variable must satisfy the following two basic conditions.

1) Coverage. A frame covers a fuzzy set X when any element of X belongs to at least one label of the frame.

2) Semantic Soundness. This states that each semantic label is a unimodal and normal fuzzy set and they are also sufficiently disjoint [10]. To specify the semantics of a certain linguistic term, we provide a modal (typical) value of the considered term along with the lower and upper bounds. The main requirement is to come up with a collection of fuzzy labels of uniform activation. This requirement converts into a construction of a frame of cognition with uniform entropy that subsequently means that, on average, the linguistic labels are "activated" to the same extent.

The construction of the fuzzy partition (frame of cognition) driven by the criterion of equal entropy leads to an optimization problem [11] which is complex and computation-intensive. To overcome this problem, we use the richness of fuzzy sets to generate a suboptimal solution. First, derive a composite membership function from a given probability density function. Next, approximate this membership function with a number of submembership functions as closely as possible for a desired accuracy of the result. This approach is discussed below.

B. Motivation for Approximate Solutions

As per L. A. Zadeh, the key motivation for applying fuzzy sets to solving complex problems is to emulate an activity of a human being involved in solving such problems. Fuzzy algorithms do not model processes but model only the decision making procedures. The objective is to generate an *approximate* (suboptimal) solution to a complex problem which is simple, efficient, and effective [20]. A human approach to solving a complex problem is to structure the knowledge about it in terms of some general concepts and afterwards to reveal essential relationships between them. Some psychological findings suggest that 7 ± 2 linguistic terms as an upper limit for the cardinality of the fuzzy partitioning when perceived in the sense of a basic vocabulary of linguistic terms [12]. Fuzzy system techniques are characterized by approximate rather than categorical reasoning, acceptance of imprecise and incomplete information, and use of linguistic variables. It may be emphasized that an approximate solution obtained using fuzzy sets is a *suboptimal* solution based on precise mathematical relationships, not on some heuristic considerations.

With this in mind, we apply an approximate method to reconstruct a given membership function with various information granules. The level of precision as contrasted to generality can easily be modified by changing the number of linguistic labels and modifying their parameters. However, to ensure noninteraction of control rules, the fuzzy sets of control must not be too precise and cover the

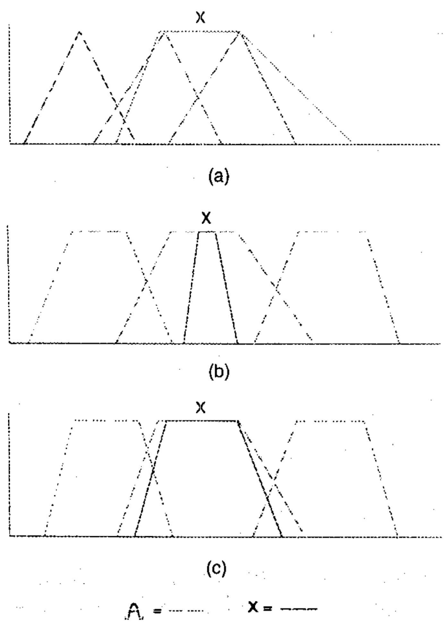

Fig. 9. Information granularity of $\mathcal{A}$ and X.

entire space of control. An approximate method of reconstructing a fuzzy variable is to distribute the supports of the fuzzy sets according to the modal values of the pdf, $p(x)$, of the considered variable and subsequently equalize the highest entropy values by replacing the original membership function by their trapezoidal counterparts. The linguistic terms like positive high and negative high need not be very specific in the cognition frame. On the contrary, the description of the area close to zero should be defined in more detail to make the control actions more specific and assure enough sensitivity in the generated control actions [11]. Figs. 9(a) through 9(c) present various levels of information granularity of the set $\mathcal{A}$ and a given membership function X.

In Fig. 9(a) the granularity of $\mathcal{A}$ is too high (linguistic labels are too specific) to handle the granularity of X. Fig. 9(b) shows that the granularity of $\mathcal{A}$ is too low. In Fig. 9(c) the granularity becomes completely justifiable owing to the input data that becomes less precise. For purely numerical data, the granularity of $\mathcal{A}$ does not play a significant role.

As becomes obvious later during applications of fuzzy techniques, information granularity enhances our ability to implement the principle of incompatibility and efficiently express the tradeoffs existing between achievable level of precision and relevancy.

VII. MULTISENSOR TRACKING

There are three main components in a multisensor tracking system: *data alignment*, *data correlation*, and *trajectory estimation*. We concentrate on the data association component of a multisensor tracking process. The data correlation may suffer from two problems, namely, missed correlation (Type I errors) and incorrect correlation (Type II errors).

Classic algorithmic methods for data correlation (data association) base their approaches on the following: 1) *nearest neighbor* that includes single-hypothesis techniques and multiple-hypothesis techniques [14], and 2) *all-neighbor* that includes branch-and-bound techniques [9] and joint-probabilistic-data-association [1].

Nonalgorithmic (approximate) methods include fuzzy logic, neural networks, and knowledge-based techniques. The *trajectory estimation* may employ any of the available techniques including Kalman filtering and least-squares.

In practice, signals from targets of interest are invariably accompanied by clutter, interference, multipath, and other distractions which make data correlation the heart of the tracking problem. The data correlation can be performed at three levels: report-to-report, report-to-track, and track-to-track. The report-to-report correlation represents correlation at the basic level. However, this is very computation-intensive. In a practical multisensor tracking system, one can deploy track-to-track correlation to initialize the tracker, and thereafter use report-to-track correlation methodology.

VIII. APPLICATIONS

Two examples are presented to demonstrate the feasibility of using fuzzy logic for data correlation in multisensor tracking. The input variables are position and speed errors. These are assumed to have Gaussian pdfs. The optimal membership of a Gaussian pdf is shown in Fig. 3 (which is trapezoidal). To represent this as a frame of cognition, we approximate this using a number of fuzzy labels as discussed in Section VI. Figs. 8 and 9 show the frames of cognitions. The output of the correlator is a decision variable. A decision functional is defined on the basis of an innovation function [16]. Thus the pdf of a correlation variable is the pdf of a chi-square variable given in Section V5. Further, as discussed in Section IIID, here we employ the centroid method to convert the fuzzy outputs to the crisp data in these two examples.

A. Example 1

A target is moving with a constant acceleration. Given: $a = 0.5$ ft/s^2 (acceleration), and $\Delta T = 1$ s (sampling interval). The initial conditions: position = 5 ft, and speed = 9 ft/s. The target motion is characterized by the following recursive equations:

$$\text{Speed} = \text{speed} + a * \Delta T$$

$$\text{Position} = \text{position} + \text{speed} * \Delta T.$$

We have two independent sets of measurements; target position and target speed. Let Mposition

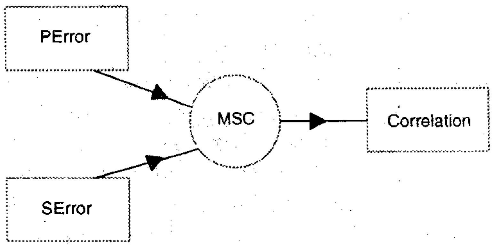

Fig. 10. Block diagram for correlation decision.

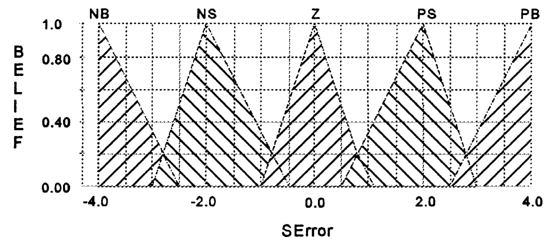

Fig. 12. SError—membership function.

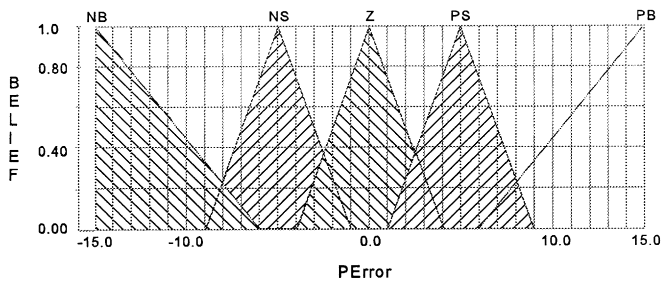

Fig. 11. PError—membership function.

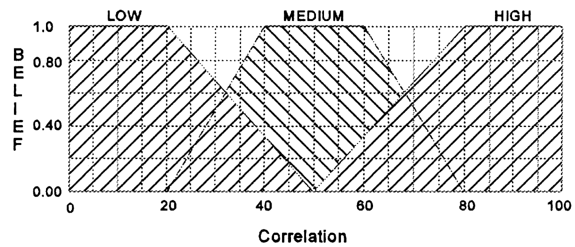

Fig. 13. Correlation membership function.

and Mspeed denote the position and the speed measurements. Define the position-difference and speed-difference errors as follows.

PError = Mposition – Predicted Position

SError = Mspeed – Predicted Speed

Mean(PError) = 0, Standard Deviation (PError) = 3.76 ft

Mean(SError) = 0, Standard Deviation (SError) = 0.835 ft/s

True positions are also our predicted values.

Fig. 10 provides the functional block diagram for correlation decision. Here, the MSC denotes a fuzzy variable and embodies all the fuzzy rules and fuzzy statements. The output variable correlation represents the fuzzy decision for data association. We can relate Fig. 10 to Fig. 2 in the following ways.

PError and SError represent crisp inputs to the fuzzification interface. The MSC embodies the fuzzy knowledge base, and the correlation represents the output of the defuzzification interface. A fuzzy tool provides the fuzzy inference engine for a chosen class of fuzzy logic.

Based upon the methodology discussed in Section VI, we define five fuzzy sets in the PError discourse, and their specific membership functions in the fuzzy interface (Fig. 11). The following symbols denote these membership functions:

NB = Very Low NS = Low Z = Medium

PS = High PB = Very High

The SError discourse also has five fuzzy sets (Fig. 12). The correlation discourse has only three membership functions (Fig. 13), and these memberships represent Low, Medium, and High. The fuzzy variable, MSC, has 25 IF–THEN rules. Table I gives the interrelationships.

A sample of these rules are provided below.

RULE-1:

IF PError is NB AND SError is NB THEN Correlation = LOW.

RULE-2:

IF PError is NB AND SError is NS THEN Correlation = LOW.

TABLE I
Rule Base

PError \ SError	VERY-LOW	LOW	MEDIUM	HIGH	VERY-HIGH
Very-Low	LOW	LOW	LOW	MEDIUM	MEDIUM
Low	LOW	LOW	MEDIUM	MEDIUM	MEDIUM
Medium	LOW	MEDIUM	MEDIUM	MEDIUM	HIGH
High	MEDIUM	MEDIUM	MEDIUM	HIGH	HIGH
Very-High	MEDIUM	MEDIUM	HIGH	HIGH	HIGH

TABLE II
Correlation Results

SPEED	POSITION	Mposition	Mspeed	PError	SError	CORRELATION
9.5	14.5	15.1	9.0	0.6	-0.5	51.94
10.0	24.5	26.2	9.5	1.7	-0.5	50.0
10.5	35.0	29.5	11.0	-5.2	0.5	50.0
11.0	46.0	43.5	10.5	-2.5	-0.5	50.0
11.5	57.5	55.4	11.2	-2.1	-0.3	50.0
12.0	69.5	76.2	12.8	6.7	0.8	50.0
12.5	82.0	81.0	11.5	-1.0	-1.0	69.13
13.0	95.0	90.2	12.0	-4.8	-1.0	50.0
13.5	108.5	114.0	15.0	-5.5	1.5	18.57
14.0	122.5	123.6	15.0	1.1	1.0	81.43

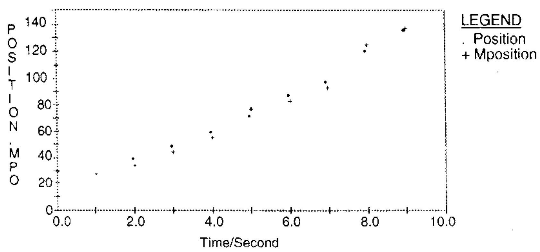

Fig. 14. Target trajectory.

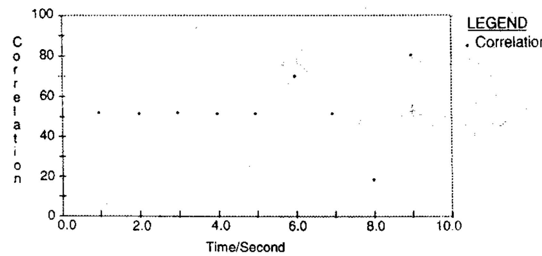

Fig. 16. Correlation results (%).

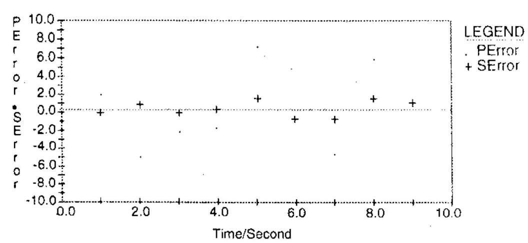

Fig. 15. Position errors and speed errors.

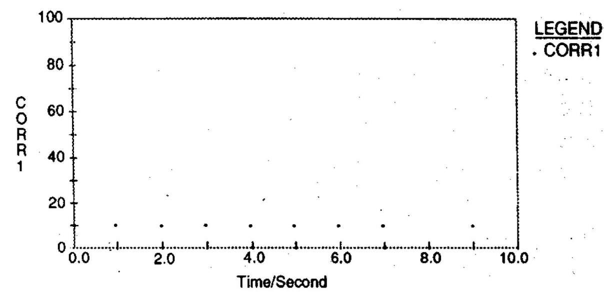

Fig. 17. Binary decisions for CORR1.

RULE-3:
IF PError is NB AND SError is PS THEN Correlation = MED.
RULE-4:
IF PError is NB AND SError is Z THEN Correlation = MED.
RULE-5:
IF PError is NB AND SError is PB THEN Correlation = MED.

The true target positions and speeds were fuzzified to provide measurement data for simulation. The simulation was run for 10 s. The correlation represents the defuzzification outputs. The correlation results are provided in Table II.

Fig. 14 depicts the target trajectory along with its sampled position. The distribution of position and speed errors are shown in Fig. 15. The correlation results in terms of the grades are plotted in Fig. 16.

However, for practical applications, these must be converted into numbers. One method is to use binary decision. One criterion implemented here is that a particular measurement should be correlated if the grade of correlation >= 50%. Fig. 17 presents the result.

The correlation results show that all the measurements except the ninth one should be associated.

B. Example 2

Two targets are crossing each other are shown in Fig. 19. The initial conditions are as follows:

Target 1	*Target 2*
Position1 = 9 ft Position2 = 100 ft Speed1 = 5 ft/s $a = 0.5$ ft/s^2 (constant acceleration) $\Delta T = 1$ s (sampling interval) Their trajectories are governed by Speed1 = Speed1 + $a * \Delta T$ Position1 = Position1 + Speed1 $* \Delta T$ Speed2 = Speed2 + $a * \Delta T$ Position2 = Position2 + Speed2 $* \Delta T$	Speed2 = 4 ft/s

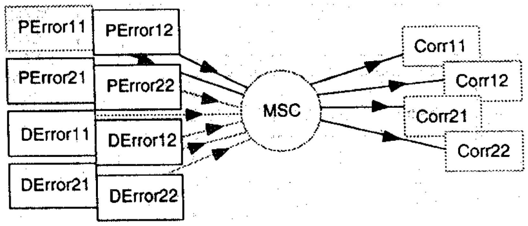

Fig. 18. Block diagram for two targets crossing.

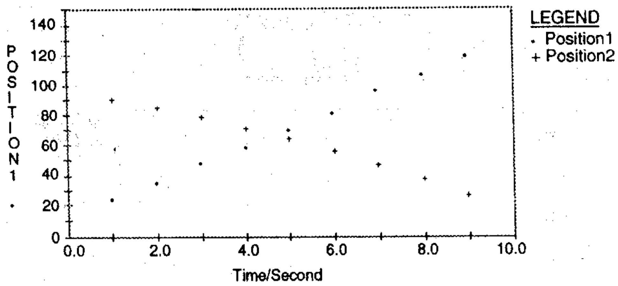

Fig. 19. Target trajectories.

We have four sets of independent measurements for the two target positions and speeds: Mposition1, Mspeed1, Mposition2, and Mspeed2. Define the errors as follows.

PError11 = Mposition1 – Position1
DError11 = Mspeed1 – Speed1
PError12 = Mposition1 – Position2
DError12 = Mspeed1 – Speed2
PError21 = Mposition2 – Position1
DError21 = Mspeed2 – Speed1
PError22 = Mposition2 – Position2
DError22 = Mspeed2 – Speed2
Mean(PError11) = 0
STD(PError11) = 3.76 ft
Mean(DError11) = 0
STD(DError11) = 0.835 ft/s
Mean(PError22) = 0
STD(PError22) = 3.571 ft
Mean(DError22) = 0
STD(DError22) = 0.975 ft/s.

The errors corresponding to the target and the measurement are shown in Tables III and IV.

Relating the entries of Table IV to the corresponding variables of Table III, a fuzzy correlation variable, Corr11, represents the correlation between fuzzy variables PError11 and DError11; Corr22 between PError22 and SError22; and a fuzzy cross-correlation variable Corr12 represents correlation between fuzzy variables PError12 and DError12; and Corr21 between PError21 and DError21. Fig. 18 shows the data association scheme. Relating this figure to the fuzzy system paradigm (Fig. 2), we have the following.

PError11, PError12, DError11, DError12, PError21, PError22, DError21, and DError22 represent the observations (crisp inputs) to the fuzzification interface. The fuzzy variables Corr11, Corr12, Corr21 and Corr22 are the outputs of the defuzzification interface. The MSC embodies the fuzzy knowledge. A fuzzy tool provides the fuzzy inference engine.

We have applied 100 IF–THEN rules to this correlation decision problem. The fuzzy correlation results are denoted in Table V.

Figs. 20–23 show the plots of fuzzy correlation membership functions. The Y-axis represents the grades of fuzzy correlation variables.

This shows that there is no cross-correlation between measurements PError12 and DError12.

This figure shows that there exists one cross-correlation between PError21 and DError21. But

TABLE III
Track and Measurement Errors

MEASUREMENTS	TRACKS NO. 1	NO. 2
No. 1	PError11 DError11	PError12 DError12
No. 2	PError21 DError21	PError22 DError22

TABLE IV
Target and Measurement Errors

MEASUREMENTS	TARGETS NO. 1	NO. 2
No. 1	Corr11	Corr12
No. 2	Corr21	Corr22

TABLE V
Fuzzy Correlation Results

	TIME (SECONDS)									
CORRELATION	1	2	3	4	5	6	7	8	9	10
Corr11	51.9	50.0	50.0	50.0	50.0	50.0	69.1	50.0	18.57	81.4
Corr12	0	0	0	0	0	0	0	0	0	0
Corr21	0	0	0	0	0	0	18.57	0	0	0
Corr22	50.0	50.0	50.0	50.0	28.0	50.0	66.92	18.57	62.45	28.0

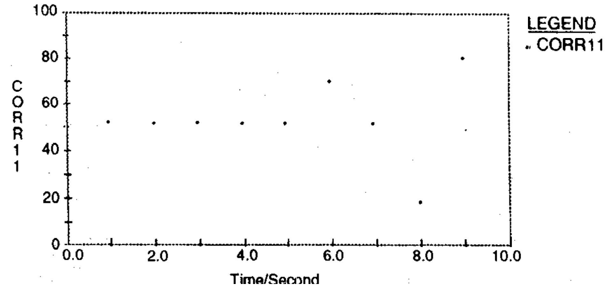

Fig. 20. Plot correlation component for CORR11.

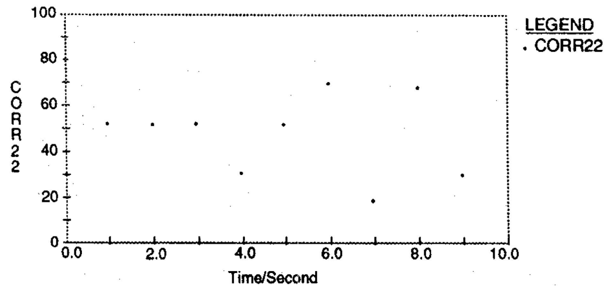

Fig. 21. Plot correlation component for CORR22.

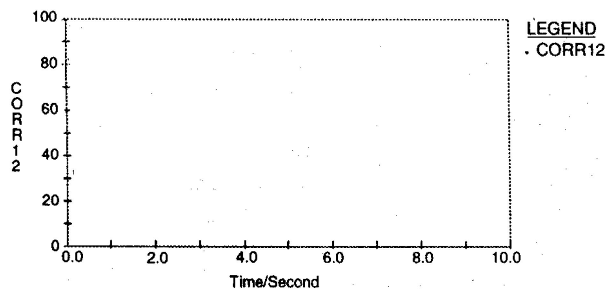

Fig. 22. Plot correlation component for CORR12.

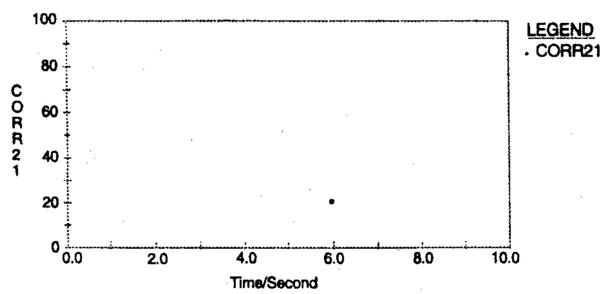

Fig. 23. Plot correlation component for CORR21.

the value of this grade is low. The combined correlation results for all the measurements are shown in Fig. 24.

The corresponding binary correlation results are based on 50% or more grades in the correlation membership functions and are denoted by: CORR11, CORR12, CORR21, and CORR22. These are shown in Figs. 25–28.

Fig. 25 suggests that the eighth measurement of the pair, Mposition1 and Mspeed1, should not be associated.

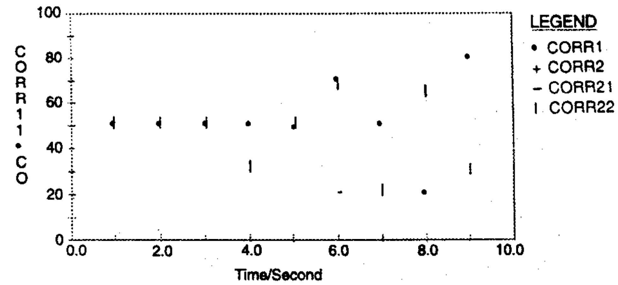

Fig. 24. Combined correlation results for CORR11, CORR12, CORR21, CORR22.

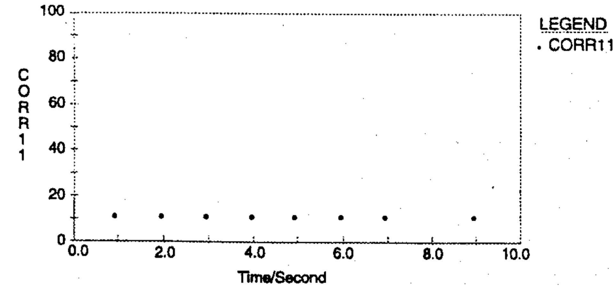

Fig. 25. Binary decision for CORR11.

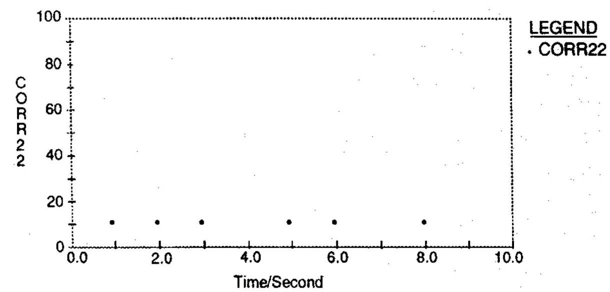

Fig. 26. Binary decision for CORR22.

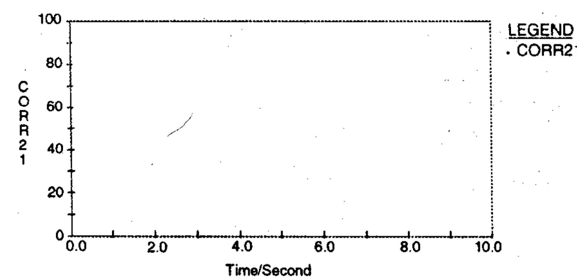

Fig. 27. Binary decision for CORR12.

Fig. 26 suggests further that the fourth and seventh measurements of the pair, Mposition2 and Mspeed2, should not be associated.

Based on our decision criterion, the binary decisions for CORR12 and CORR21 are zeroes. The overall binary decisions for correlation individual measurements with tracks are shown in Fig. 29.

This represents an overall decision made by the fuzzy correlator as shown in Fig. 18. The eighth

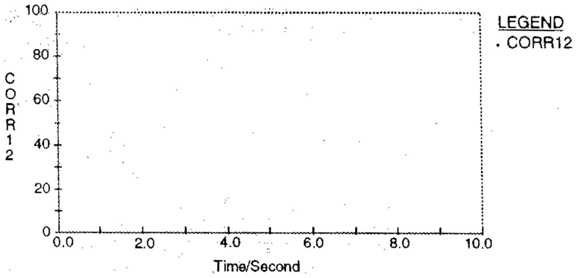

Fig. 28. Binary decision for CORR21.

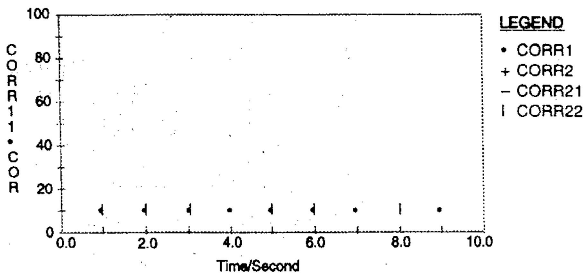

Fig. 29. Overall binary decisions for CORR11, CORR12, CORR21, CORR22.

measurement of the pair, Mposition1 and Speed1; and the fourth and seventh measurements of the pair, Mposition2 and Mspeed2, should not be associated. The cross-correlation represented in Fig. 23 did not affect the result because it is below the binary threshold set out for the test.

IX. FUZZY SYSTEM PERFORMANCE EVALUATION

Since a fuzzy logic technique generates an approximate solution, an evaluation criterion for its performance is shown in Table VI for Example 1 whether this approximate solution is acceptable. In practice, it is only possible to judge the effectiveness of fuzzy logic techniques, when a sufficient number of functioning devices or solutions for complex problems have been constructed on the basis of fuzzy logic theory.

A. Arbitrary Evaluation Criteria

Two evaluation criteria for data association are defined below

Criterion 1: PError $\Leftarrow 1\sigma_p$ AND SError $\Leftarrow 1\sigma_s$

Criterion 2: PError $\Leftarrow 2\sigma_p$ AND SError $\Leftarrow 2\sigma_s$.

Table VI shows that the fuzzy system performs 70% with respect to the first criterion, and 90% with respect to the second for Example 1.

Table VII shows that the fuzzy system performs 65% with respect to the first criterion, and 80% with

TABLE VI
Performance Evaluation (Example 1)

OBSERVATION NO.	FUZZY SYSTEM DECISION (YES OR NO)	ARBITRARY CRITERIA (YES OR NO)	
		CRITERION	CRITERION
1	Y	Y	Y
2	Y	Y	Y
3	Y	N	Y
4	Y	Y	Y
5	Y	Y	Y
6	Y	N	Y
7	Y	Y	Y
8	Y	N	Y
9	N	N	Y
10	Y	Y	Y

TABLE VII
Performance Evaluation (Example 1)

OBS. NO.	MEAS NO. 1	CRIT NO. 1	CRIT NO. 2	MEAS NO. 2	CRIT NO. 1	CRIT NO. 2
1	Y	Y	Y	Y	Y	Y
2	Y	Y	Y	Y	N	Y
3	Y	N	Y	Y	Y	Y
4	Y	Y	Y	Y	Y	Y
5	Y	Y	Y	N	Y	Y
6	Y	N	Y	Y	Y	Y
7	Y	Y	Y	Y	N	Y
8	Y	N	Y	N	Y	Y
9	N	N	Y	Y	Y	Y
10	Y	Y	Y	N	N	Y

TABLE VIII
Performance Evaluation (%)

CRITERION	EXAMPLE NO.	EXAMPLE
1	70	65
2	90	80

respect to the second for Example 2. After the system has been developed, it requires tuning of the system response. A self-tuning technique based on fuzzy meta-rules provides one such methodology [3].

Table VIII summarizes the performance evaluation results. The number of fuzzy rules and their impact on performance is discussed in Section X.

B. Evaluation Using Mission Avionics Sensor Synergism System

The Mission Avionics Sensor Synergism (MASS) laboratory consists of the hardware and software necessary to provide real-time simulations of a Tactical Coordinator (TACCO) station in a Navy surveillance aircraft involved in ASUW, Surface-Subsurface Surveillance Coordination (SSSC) or ASW missions. The TACCO is supplied with data from simulated aircraft avionic systems. Sensor contact reports are supplied directly to the TACCO, to sensor trackers, and to MASS correlation functions. Sensor tracker and correlation function outputs are also supplied to the TACCO for evaluation and comparison with conventionally derived target tracks. The laboratory is capable of storing mission scenarios onto what are known as extraction files. Extraction files from both the laboratory and the aircraft are able to be "replayed" for post-mission evaluation and development. Data from a flight tested Multisensor Multitarget Correlation (MSMTC) system named MASS [9] were used to test fuzzy logic techniques on track-to-track correlation processes. The data were generated via sophisticated laboratory simulations.

This analysis focused on the standardized squared difference, which is a statistic computed from the parameters of two tracks and is assumed to be a chi-square distributed random variable. This statistic is normally compared with a threshold based on the chi-square pdf with the appropriate degrees of freedom. A heuristic examination of values of this statistic for a specific case of correlation processing indicated that a less stringent threshold would be reasonable. Transferring the correlation problem from a "probability domain" to a "possibility domain" is exactly the basis of fuzzy logic [6]. The heuristic threshold improved performance in two examined cases, suggesting that a formal fuzzy logic algorithm for automating threshold selection would benefit track-to-track correlation processing.

MULTI-SENSOR MULTI-TARGET ASSOCIATIONS
FOR EXTRACTION FILE A

Eleven Targets Constant Velocity

TARGET NUMBER

	1	2	3	4	5	6	7	8	9	10	11
Radar	X				X	X		X	X	X	X
IRDS		X		X		X	X		X	X	X
Acoustic			X	X			X	X	X		X
ESM	X	X	X				X	X		X	X

Fig. 30. Case A target/sensor distribution.

MULTI-SENSOR MULTI-TARGET ASSOCIATIONS
FOR EXTRACTION FILE B

Eleven Targets Constant Velocity

TARGET NUMBER

	1	2	3	4	5	6	7	8	9	10	11
Radar	X				X	X		X	X	X	X
IRDS		X		X		X	X		X	X	X
Acoustic			X	X	X		X	X	X		X
ESM	X	X	X				X	X		X	X

Fig. 31. Case B target/sensor distribution.

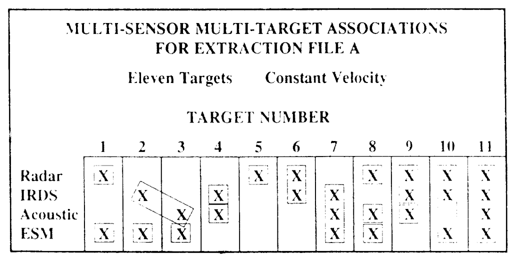

Fig. 32. Case A associations from classic technique.

The Track Association function was isolated and reproduced on a PC to enable efficient experimentation with a set of extraction files. Two cases of Track Association, one from each of two extraction files, were examined. The two cases were each run using classic techniques and then using fuzzy logic techniques. We refer to the extraction files as Extraction File A (containing Case A) and Extraction File B (containing Case B). Figs. 30 and 31 show the cases' tactical scenarios.

The only difference between the two cases is that target 5 is monitored by acoustics sensors in Case B but not in Case A.

1) *Case A*: MINYAN denotes a composite track. This uses kinematic data to perform multisensor multitarget correlation. For this scenario Track Association evokes four passes. In the first pass, the seven radar tracks are established as the initial MINYAN tracks. In the second pass, the function attempts to combine the seven IRDS tracks with the initial MINYAN tracks. Ideally after the second pass, the IRDS tracks on targets 6, 9, 10, and 11 would be combined with the appropriate MINYAN tracks and the other IRDS tracks would establish three new MINYAN tracks. After four passes, the twenty-seven sensor tracks should be correctly combined into eleven MINYAN tracks.

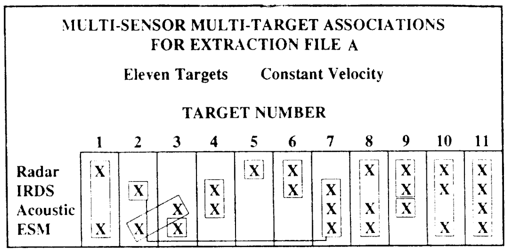

Fig. 33. Case A associations from fuzzy logic technique.

STANDARDIZED SQUARED DIFFERENCE VALUES
CASE A SECOND RUN

MINYAN Track Target Number

		1	4	5	6	7	8	9	10	11
	3	57875	420	1083	1903	934	6147	7879	2377	3924
Acoustic	4	79589	14	646	5054	1104	8110	7169	2765	2077
Track	7	49451	2933	2649	3044	8	685	2184	554	3458
Target	8	33208	3692	4065	1065	92	44	2015	1576	6627
Number	9	60252	2568	10404	32767	13394	12126	104	12182	2433
	11	52710	381	6945	25924	2886	3559	2147	6160	4

Fig. 34. Case A, run 2, pass 3 values for standardized square difference.

Fig. 32 shows the results of Track Association when the thresholds to test the hypothesis that two tracks could represent the same target is set at levels dictated by classic probability theory.

The process results in seventeen MINYAN tracks, one false correlation, five failures to correlate, and two denied opportunities to correlate due to the false correlations.

Examining the values of the parameter D, described in Section III, revealed a heuristically identifiable cluster containing values for track pairs where most were correctly matched. The threshold for this cluster was arbitrarily set at 100 and Case A was rerun. The measure of agreement $E(J,K)$ was set equal to $D/100$ for all sensors. Fig. 33 shows results.

Results were twelve MINYAN tracks, two false correlations, one failure to correlate, and one denied opportunity to correlate due to false correlations.

Fig. 34 presents the values of D computed in the third pass of the second run of Case A. At this point, six acoustic tracks were being compared with nine MINYAN tracks.

2) *Case B*: Fig. 35 shows results of running Case B with the classic threshold. Results are sixteen MINYAN tracks, one false correlation, four failures to correlate, and two denied opportunities to correlate due to the false correlations.

Fig. 36 shows results of running Case B with the threshold set at 100. Results are eleven MINYAN tracks, two false correlations, no failures to correlate, and two denied opportunities to correlate due to false correlations.

X. CONCLUSIONS/REMARKS

It is important to note that a fuzzy logic approach generates an approximate solution to the problem. Despite the fact that the crisp output to a fuzzy system

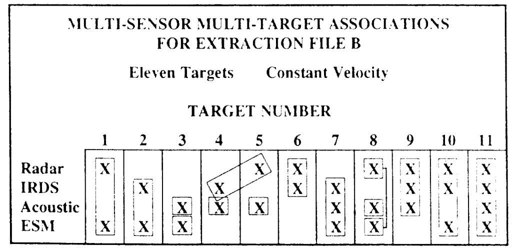

Fig. 35. Case B associations from classic technique.

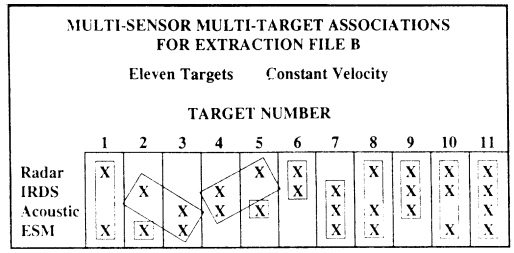

Fig. 36. Case B associations from fuzzy logic technique.

is arrived at via precise mathematics, the outputs are still approximate and subject to the accuracy of the rule set and membership functions. However, it has been shown that fuzzy systems with product inference, centroid defuzzification, and Gaussian membership functions are capable of approximating any real continuous function on a compact set to arbitrary accuracy [17]. Remember, fuzzy algorithms model the decision maker, not the process. As a result, fuzzy logic is totally intuitive. We must verify whether it makes sense with this system. This says *only applications* matter.

A technique for generating on-line membership functions for input and output variables based on their pdfs has been presented. The key feature of a fuzzy system technique based on fuzzy logic is its ability to combine information from different classes of variables (like pressure, temperature,...) using fuzzy inference operations. Thus, it solves complex problems using inexact inputs received from diverse sensors and provides approximate solutions. The examples presented here have taken into account only kinematic data and have not considered attributes which are critical elements for data association. Further, this fuzzy approach puts equal emphasis on both kinematic and attribute data.

Example 1 employs 25 rules and example 2 100 rules. On the surface, it seems that the number of rules is increasing exponentially like any classic combinatorial problem. Fuzzy techniques are highly flexible in creating rules giving much latitude in reducing the rule base. However, *precision is costly*. General guidelines for creating rules are as follows.

1) Use attribute data in conjunction with kinematic data. This may greatly reduce the number of rules

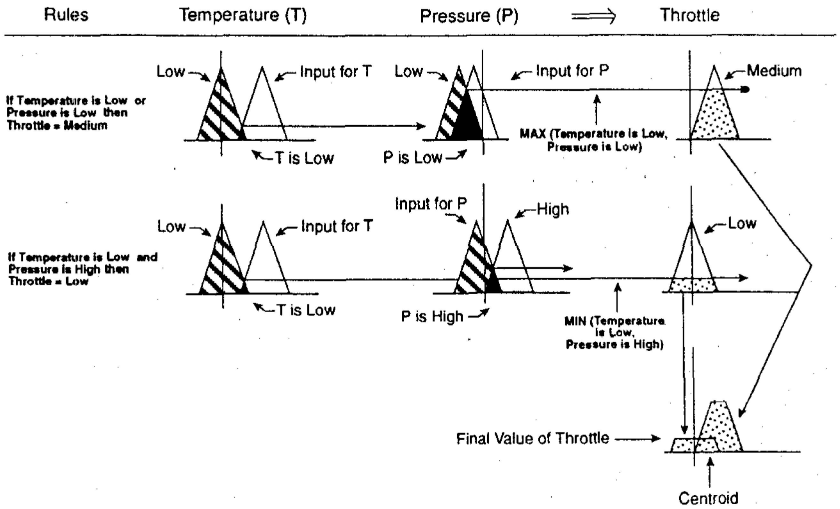

Fig. 37. Max-Min inference method.

required to achieve the same accuracy for the decision result.

2) Where precise outputs are required, define more input and output terms. This results in a large number of rules.

3) Where imprecision can be tolerated, define fewer input and output terms. Hence, it results in a smaller number of rules.

Therefore, we need to assess a degree of imprecision, that the system can tolerate. In addition, fine tuning can be achieved by adjusting relative width of a membership function as well as the relative shapes and sizes of the output membership function. To overcome this problem further, we can also apply a hierarchical rule structure [13] and self-tuning using meta-rules [3].

In this paper, a fuzzy logic technique that can be applied to resolve data association problems in multisensor multitarget tracking has been explored. The normal distance measure has not been used in the usual manner, but the fuzzy logic technique has fuzzified the distance measures for use by the fuzzy knowledge-base. The fact of the matter is that here we do not *measure* the contents of the information, but rather we emphasize on the *meaning* of a linguistic value defined by its possibility distribution. Performance evaluation has been done with two criteria. Of course, fuzzy logic is not the best approach for every control problem. This is the first fuzzy technique approach to data association problems. With simulated data in the laboratory environment, the simulation has been performed to evaluate the MASS system as reported in Section IXB. These results show better performance for the data correlation function using the fuzzy logic techniques.

XI. FUTURE PLANS

1) Incorporate attributes into the fuzzy knowledge-base. Solve data association problems using both kinematic and attribute information present in the fuzzy knowledge-base.

2) Develop, test and evaluate a fuzzy logic algorithm that formalizes threshold selection.

3) Apply the above fuzzy logic methodology to solve tracking problems for both the components of data association and of trajectory estimation in real time. Run the simulation with real data in the laboratory environment.

4) Compare and evaluate the performance of the fuzzy logic technique with 1) multiple-hypothesis techniques, and 2) branch-and-bound techniques for multisensor multitarget tracking problems in a dense clutter environment.

5) Integrate this nonalgorithmic methodology into a navy platform enabling the platform to handle a large number of diverse targets in a dense clutter ocean environment.

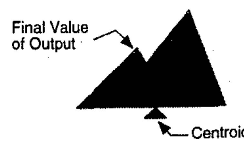

Fig. 38. Centroid defuzzification method.

APPENDIX A. MAX-MIN INFERENCE METHOD

In the max-min inference method, the final output membership function for each output is the union of the fuzzy sets assigned to that output in a conclusion after clipping their degree of membership values at the degree of membership for the corresponding premise, as shown in Fig. 37.

The centroid defuzzification method picks the output value corresponding to the centroid (center of gravity) of the output membership function as the crisp value for an output. If one were to draw the output membership function on a piece of cardboard and cut it out, the center of gravity would be the crisp value at which the cardboard balances on a razor blade, as shown in Fig. 38.

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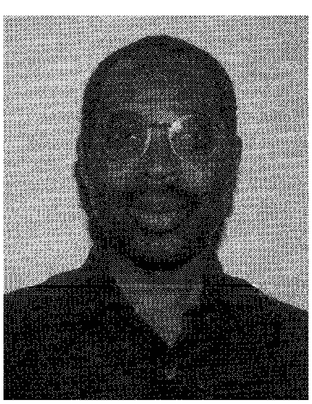

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