MUTUAL COUPLING BETWEEN HORIZONTAL EARTH RETURN CONDUCTORS USING ACTUAL ROUTING PARAMETERS

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Abstract Formula for calculating mutual coupling between diversely routed, horizontal earth-return conductors used in the measurement of grounding system impedance is derived by using the complex ground plane concept used to approximate the Carson Equation. Each potential conductor section is approximated with equal length segments that parallel one of the current conductor sections. By repeating the parallel segmentation of each potential conductor section with each subsequent current conductor section and summing the segment mutuals, overall mutual between diversely routed conductors is obtained. This process is useful for calculating mutual to the potential conductor and for comparing possible potential conductor routings so as to minimize the overall mutual coupling.

KEYWORDS: MUTUAL IMPEDANCE, FINITE LENGTH CONDUCTORS, EARTH RETURN, GROUNDING IMPEDANCE, GROUNDING SYSTEMS

INTRODUCTION

Impedance measurements of grounding grids and grounding systems require the use of temporarily installed conductors lying on ground, out-of-service transmission lines, or a combination of the two for connecting between the test grid and remote current and potential electrodes [1].

Because of constraints imposed by terrain, right-of-aways, roads, energized lines, and private property boundaries, it is unlikely, except for the smallest grids, that test leads (or, transmission lines) will be routed in either a parallel or angled simple straight line manner. As described in Ref. [1], mutual coupling from the current carrying conductor to the conductor used to measure grounding potential rise could introduce significant errors in the grounding impedance measurement. Moreover, accuracy of measurement for low impedance grounding systems will require keeping the length of closely paralleled sections of current and potential conductors to a minimum.

Presently, mutual coupling from current to potential, for complex routing, can only be estimated from straight line approximations of their overall routing. In this paper we extend the segmented method for mutual coupling between angled conductors (using the complex ground return plane [1]), to diverse routings of horizontal conductors. Then, mutual impedance to the potential conductor, for a given routing of the current conductor, can be minimized by comparing mutual couplings for several accessible potential conductor routings.

Mutuals calculated with formulas presented here are accurate for the direct component between conductors. Overall mutuals, as affected by the image component, fall below Carson for the wider spacing of very long parallel conductors [1].

90 WH 057-0 PWRD A paper recommended and approved by the IEEE Power System Instrumentation & Measurements Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1990 Winter Meeting, Atlanta, Georgia, February 4 - 8, 1990. Manuscript submitted August 24, 1989 made available for printing January 16, 1990.

Figure 1 ROUTING COORDINATES FOR EACH CURRENT AND POTENTIAL CONDUCTOR SECTION

CURRENT				POTENTIAL			
LOC	A _r	B _r	h _c	Loc	a _i	b _i	h _p
C ₁	0	0	0	P ₁	0	0	0
C ₂	0	0	10	P ₂	0	-250	0
C ₃	0	1000	10	P ₃	300	-250	0
C ₄	400	1500	10	P ₄	0	500	0
C ₅	400	10000	10	P ₅	300	500	0
C ₆	700	10000	10	P ₆	2000	650	0
C ₇	700	10000	0				

X,Y,Z coordinates in meter

Formulas used to determine the mutual impedance from image conductors will require the evaluation of logarithms and square roots of complex numbers.

CONDUCTOR ROUTINGS

Current and Potential conductor routings, shown in Fig. 1, demonstrates the diverse routings of a out-of-service transmission line connecting between the test grid and the remote grid which acts as the current electrode, and three possible routings of a test conductor, lying on the earth, which would connect between the test grid and a ground rod used as the remote potential electrode.

The relationship from any current section to each potential section may be parallel, angular and/or right angles. However, the mutual coupling analysis is aided by using a XYZ coordinate system to define change of direction locations of each current and potential section and their respective heights. The X-Y plane lies on the earth's surface and coordinate Z = h_c or h_p are current and potential conductor heights, respectively.

Equations (9, 12 and 14 or 15, 16 and 18) of the Appendix use the X-Y coordinates that define the routing of current and potential conductors to determine the mutual impedance between horizontal earth return conductors. Equations (22, 24 and 25) of the Appendix are used for angled conductors that can be defined by their length, horizontal angle, and when both originate from a common vertical axis.

Figure 2 **COMPARISON OF MUTUAL IMPEDANCE FOR THREE POTENTIAL CONDUCTOR ROUTINGS OF FIG. 1 WITH AN ANGLED ROUTING**

I = P₁ to P₄ to P₅ to P₆
 II = P₁ to P₂ to P₃ to P₅ to P₆
 III = P₁ to P₂ to P₃ to P₆
 ANGLE = CUR: C₂ to C₆. POT: P₁ to P₆
 f = 60 Hz C = 10024.5 m P = 2103 m $\theta = 68^\circ$

MUTUAL IMPEDANCE VERSUS POTENTIAL ROUTINGS

Mutual impedance between the transmission line current conductor and each of the three potential conductor routings (I, II, III) of Fig. 1 are compared in Fig. 2 with a fictitious straight line angled routing from C₂ to C₆ for the current and from P₁ to P₆ for the potential. Their mutual impedance components, R_M and X_M , calculated with Eqs. (9, 12 and 14 or 15, 16 and 18) of the Appendix are plotted in Fig. 2 against homogeneous earth resistivities of 1 to 100,000 $\Omega\cdot m$.

Potential conductor routings, I and II, which lie closer to the current conductor have mutual components that are larger than for the fictitious angled routing. Routing III which is farthest from the current conductor has components that fall below the angled case. The magnitude of mutual impedance for routings II and III are reduced by the negative mutual coupling to the conductor section P₁ to P₂. Although there are many constraints that limit conductor routings, routing selection for minimum mutual coupling can be aided by prior study.

Figure 3 **MUTUAL IMPEDANCE BETWEEN ANGLED CONDUCTORS**
 f = 60 Hz, C = P = 1 km, $h_c = 10$ m, $h_p = 0$ m

MUTUAL IMPEDANCE BETWEEN ANGLED CONDUCTORS

When it is possible to route test conductors radially from the grid in a straight line, mutual coupling between angle conductors can be determined with Eqs. (22, 24 and 25) of the Appendix. Fig. 3 shows the mutual impedance component variations of two 1 km long conductors for angle orientations $0^\circ < \theta < 180^\circ$ and six homogeneous earth resistivities, 1 to 100,000 $\Omega\cdot m$. As earth resistivity increases the resistive component, R_M , decreases and the reactive component, X_M , increases. Preferred angle is 90° , however, orientations from 60° to 120° will have very small mutual impedance components. Orientation angles 120° to 180° have much smaller mutual components than those of 0° to 60° .

Equations (22, 24 and 25) of the Appendix give results that agree with the segmented summation method of Reference [1] Eqs. [19, 20 and 21] and the segmented summation method of this paper, Eqs. (9, 12 and 14 or 15, 16 and 18) of the Appendix.

CONCLUSIONS

1. Mutual impedance between horizontal earth return conductors with any orientation can be determined by the segmented method and Eqs. (9, 12 and 14 or 15, 16 and 18) of the Appendix.
2. Formulae presented in the paper are based on homogeneous earth, however, approximating heterogeneous earth is not critical as large variations in earth resistivity result in smaller changes in the resistive and reactive components of mutual impedance.
3. The segmented method for diversely routed conductors is adaptable to the programmable hand calculator.
4. Negative mutual impedance of an initial potential section oriented 180° with an initial current section is useful for reducing overall mutual coupling.

REFERENCES

- [1] E. J. Rogers and J. F. White, "Mutual Coupling between Finite Lengths of Parallel or Angled Horizontal Earth Return Conductors" *IEEE Transactions on Power Delivery*, Vol. 4, No. 1, Jan. 1989, p.p. 103-113.
- [2] F.W. Grover, "Inductance Calculations: Working Formulas and Tables", Dover Publishing Co., N.Y., N.Y., 1946, p.p. 8 and 55-57.

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APPENDIX

A 1 MUTUAL COUPLING FOR DIVERSE CONDUCTOR ROUTINGS

The Coordinate System Determining the mutual coupling between diversely routed conductors, such as Fig. 1, requires establishing a reference location for a X-Y coordinate system. Any convenient location on the grid can be selected and the X-Y coordinates determined for the measuring point on the grid, the end points of each current section, and the end points of each potential section. In the Fig. 1 example, the measuring point on the grid is used as the reference origin and the X-axis is oriented East to West with the Y-axis North to South.

Mutual impedance between any horizontal earth return current-section to any horizontal earth return potential-section is

Figure A.1 **PARALLEL SEGMENT APPROXIMATION OF THE POTENTIAL CONDUCTOR SECTION**

Coordinates:

Any Cur. Section r-s: (A_r, B_r, h_C) to (A_s, B_s, h_C)

Any Pot. Section i-j: (a_i, b_i, h_P) to (a_j, b_j, h_P) n = 3

found by approximating the potential-section with equal length segments that parallel a particular current-section. Overall mutual impedance between all current sections to all potential sections is found by summing these segment mutuals.

Segmenting the Potential Section Figure A.1 shows the orientation of one section of the Current conductor and one section of the Potential conductor on the coordinate system. From the number of parallel segments that are used to approximate the Potential Section and the end coordinates of both sections it is possible to calculate the end coordinates of each parallel segment. The above example uses n = 3 or three parallel segments to approximate the Potential Section. Coordinates for each parallel segment are determined with Eqs. (1, 2, 3, 4).

$$d_k = a_i + \frac{k-1}{n} (a_j - a_i) - \frac{B_s - B_r}{2n} G(i,j,r,s) \quad (1)$$

$$f_k = a_i + \frac{k}{n} (a_j - a_i) - \frac{B_s - B_r}{2n} G(i,j,r,s) \quad (2)$$

$$e_k = b_i + \frac{k-1}{n} (b_j - b_i) - \frac{A_s - A_r}{2n} G(i,j,r,s) \quad (3)$$

$$g_k = b_i + \frac{k}{n} (b_j - b_i) - \frac{A_s - A_r}{2n} G(i,j,r,s) \quad (4)$$

$$G(i,j,r,s) = \frac{(b_j - b_i)(A_s - A_r) - (a_j - a_i)(B_s - B_r)}{C_{rs}^2}$$

$$j = i + 1 \text{ and } i = 1, 2, 3, \dots, I \quad k = 1, 2, 3, \dots, n$$

$$C_{rs} = \sqrt{(A_s - A_r)^2 + (B_s - B_r)^2}$$

When a potential section parallels a current section, only one segment is required (n = 1) for the potential section to determine mutual impedances between these sections. When a potential section is at right angles with a current section mutual impedance between them is zero (use n = 1 or skip calculation). And, when a potential section has an angular orientation with a current section use $10 \leq n \leq 50$. The larger "n" being required for orientations less than 135° and more than 45°.

In the Fig. 1 example, P₁ to P₂, P₁ to P₄, and P₃ to P₅ parallel C₂ to C₃ and C₄ to C₅. P₂ to P₃ and P₄ to P₅ parallel C₅ to C₆. P₂ to P₃ and P₄ to P₅ are at right angles with C₂ to C₃ and C₄ to C₅. Only current sections which are at an angle with potential sections and only potential sections which are at an angle with current sections require a "n" of 10 or more (C₃ to C₄, P₃ to P₆ and P₅ to P₆).

Figure A.2 **MUTUAL COUPLING BETWEEN THE CURRENT SECTION AND A PARALLEL SEGMENT**

X-Y plane is at the earth surface with the positive direction of the Z-axis vertical.

Current Section: (A_r, B_r, h_c) to (A_s, B_s, h_c)

Potential Segment: (d_k, e_k, h_p) to (f_k, g_k, h_p)

The Potential Segment parallels the Current Section

Direct Mutual Coupling from a Current Conductor Section to a Parallel Potential Segment The voltage induced in the Potential Segment of Fig. A.2 due to current I in the parallel Current Section is given by the Neuman Integral [2] with the condition that the current section does not parallel the Y-axis. $A_s \neq A_r$:

$$V_{Md} = j \frac{\omega \mu_0 I}{4\pi} \int_{d_k, e_k}^{f_k, g_k} \int_{A_r, B_r}^{A_s, B_s} \frac{dZ_c dZ_p}{r_{CPD}} \quad (5)$$

Where:

$$r_{CPD} = \sqrt{(X_c - X_p)^2 + (Y_c - Y_p)^2 + h_A^2}$$

$$h_A = |h_c - h_p| \quad j = \sqrt{-1}$$

$$\omega = 2\pi f, \quad f \text{ in Hz} \quad \mu_0 = 4\pi / 10^7$$

Then mutual impedance in ohms with X,Y,Z coordinates in meters,

$$Z_{Md} = \frac{V_{Md}}{I} = j \frac{\omega \mu_0}{4\pi} M_d \quad (6)$$

and,

$$M_d = \int_{d_k, e_k}^{f_k, g_k} \int_{A_r, B_r}^{A_s, B_s} \frac{dZ_c dZ_p}{\sqrt{(X_c - X_p)^2 + (Y_c - Y_p)^2 + h_A^2}} \quad (7)$$

This integral can be converted to X-axis integration with the following relationships:

$$Y_c = m_r X_c + k_c \quad Y_p = m_r X_p + k_p$$

$$m_r = \frac{B_s - B_r}{A_s - A_r} \quad k_p = b_i - m_r a_i$$

$$k_c = B_r - m_r A_r \quad dZ_p = dX_p \sqrt{1 + m_r^2}$$

$$dZ_c = dX_c \sqrt{1 + m_r^2}$$

X-axis integration for (7) is,

$$M_d = \int_{d_k}^{f_k} \int_{A_r}^{A_s} \frac{dX_c dX_p}{\sqrt{c X_c^2 - b X_c + a}} \quad (8)$$

Where:

$$a = (1 + m_r^2) X_p^2 - 2 m_r (k_c - k_p) X_p + (k_c - k_p)^2 + h_A^2$$

$$b = 2 (X_p (1 + m_r^2) + m_r (k_c - k_p))$$

$$c = 1 + m_r^2$$

Integration of (7) is completed for variable, X_c and limits A_s and A_r substituted. Then a , b and c are substituted and the resulting equation is integrated for variable X_p . Limits f_k and d_k are substituted and the final equation for M_d is obtained.

$$M_d = - (G_1 + m_r G_0) \ln R_1$$

$$+ (G_2 + m_r G_0) \ln R_2$$

$$+ (G_3 + m_r G_0) \ln R_3$$

$$- (G_4 + m_r G_0) \ln R_4 + S_1 - S_2 - S_3 + S_4 \quad (9)$$

Where:

R = number of Current Sections

$$m_r = \frac{B_s - B_r}{A_s - A_r} \quad s = r + 1 \text{ and } r = 1, 2, 3, \dots, R$$

$$G_0 = \frac{(B_r - e_k)(A_s - A_r) - (B_s - B_r)(A_r - d_k)}{C_{rs}}$$

$$G_1 = \frac{A_s - f_k}{A_s - A_r} C_{rs} \quad R_1 = G_1 + m_r G_0 + S_1$$

$$G_2 = \frac{A_s - d_k}{A_s - A_r} C_{rs} \quad R_2 = G_2 + m_r G_0 + S_2$$

$$G_3 = \frac{A_r - f_k}{A_s - A_r} C_{rs} \quad R_3 = G_3 + m_r G_0 + S_3$$

$$G_4 = \frac{A_r - d_k}{A_s - A_r} C_{rs} \quad R_4 = G_4 + m_r G_0 + S_4$$

$$S_1 = \sqrt{(G_1 + m_r G_0)^2 + G_0^2 + h_A^2}$$

$$S_2 = \sqrt{(G_2 + m_r G_0)^2 + G_0^2 + h_A^2}$$

$$S_3 = \sqrt{(G_3 + m_r G_0)^2 + G_0^2 + h_A^2}$$

$$S_4 = \sqrt{(G_4 + m_r G_0)^2 + G_0^2 + h_A^2}$$

$$h_A = |h_c - h_p| \quad A_s \neq A_r$$

Mutual Coupling from Image to Parallel Segment In a manner similar to that used for the direct mutual, the mutual coupling to the potential segment from the current section image is determined by solving the Neuman Integral,

$$V_{Mi} = j \frac{\omega \mu_0 I}{4\pi} \int_{d_k, e_k}^{f_k, g_k} \int_{A_r, B_r}^{A_s, B_s} \frac{dZ_c dZ_p}{r_{CPi}} \quad (10)$$

Where:

$$r_{CPI} = \sqrt{(X_C - X_P)^2 + (Y_C - Y_P)^2 + I_0^2}$$

$$I_0 = \sqrt{h_B^2 + 2 d h_B - j (2 d^2 + 2 d h_B)}$$

$$h_B = h_C + h_P \quad j = \sqrt{-1}$$

$$d = \frac{\sqrt{10}}{2\pi} 10^3 \sqrt{\frac{\rho}{f}}, \text{ m} \quad \omega = 2\pi f, \text{ f in Hz}$$

$$\mu_0 = 4\pi / 10^7 \quad \rho = \text{earth resistivity, } \Omega\text{-m}$$

Then mutual impedance in ohms with X,Y,Z coordinates in meters,

$$Z_{Mi} = \frac{V_{Mi}}{I} = j \frac{\omega \mu_0}{4\pi} M_i \quad (11)$$

and,

$$M_i = - (G_1 + m_r G_0) \ln X_1 \\ + (G_2 + m_r G_0) \ln X_2 \\ + (G_3 + m_r G_0) \ln X_3 \\ - (G_4 + m_r G_0) \ln X_4 + T_1 - T_2 - T_3 + T_4 \quad (12)$$

Where: G_0, G_1, G_2, G_3 , and G_4 are as for the direct mutual case,

$$T_1 = \sqrt{(G_1 + m_r G_0)^2 + G_0^2 + I_A^2}$$

$$T_2 = \sqrt{(G_2 + m_r G_0)^2 + G_0^2 + I_A^2}$$

$$T_3 = \sqrt{(G_3 + m_r G_0)^2 + G_0^2 + I_A^2}$$

$$T_4 = \sqrt{(G_4 + m_r G_0)^2 + G_0^2 + I_A^2}$$

$$I_0 = \sqrt{h_B^2 + 2 d h_B - j (2 d^2 + 2 d h_B)}$$

$$X_1 = G_1 + m_r G_0 + T_1 \quad X_3 = G_3 + m_r G_0 + T_3$$

$$X_2 = G_2 + m_r G_0 + T_2 \quad X_4 = G_4 + m_r G_0 + T_4$$

$$h_B = h_C + h_P \quad A_S = A_r$$

Summation of Section Mutuals Overall mutual impedance between all current sections to all potential sections is determined by the summation of these section mutuals using Eqs. (9 and 12):

$$Z_{CP} = R_M + j X_M \quad (13)$$

$$Z_{CP} = j \frac{\omega \mu_0}{4\pi} \sum_{r=1}^R \sum_{j=1}^I \sum_{k=1}^n (M_d - M_i) \quad (14)$$

Where:

R = number of Current Sections $r = 1, 2, 3, \dots, R$

I = number of Potential Section $i = 1, 2, 3, \dots, I$

n = number of Segments used to approximate a Potential Section and $k = 1, 2, 3, \dots, n$

$$A_S = A_r$$

A.2 MUTUAL COUPLING WHEN THE CURRENT SECTION PARALLELS THE Y-AXIS

When a Current Section parallels the Y-axis ($A_S = A_r$), formulas (9), and (12) become indeterminate. The formula for mutual coupling between this Current Section and each Potential Segment is as follows:

For Direct Mutual

$$M_{dY} = + G_5 \ln R_5 - G_6 \ln R_6 - G_7 \ln R_7 \\ + G_8 \ln R_8 - S_5 + S_6 + S_7 - S_8 \quad (15)$$

Where:

$$G_5 = B_r - g_k \quad R_5 = G_5 + S_5$$

$$G_6 = B_r - e_k \quad R_6 = G_6 + S_6$$

$$G_7 = B_s - g_k \quad R_7 = G_7 + S_7$$

$$G_8 = B_s - e_k \quad R_8 = G_8 + S_8$$

$$S_5 = \sqrt{(B_r - g_k)^2 + (A_r - d_k)^2 + h_A^2}$$

$$S_6 = \sqrt{(B_r - e_k)^2 + (A_r - d_k)^2 + h_A^2}$$

$$S_7 = \sqrt{(B_s - g_k)^2 + (A_r - d_k)^2 + h_A^2}$$

$$S_8 = \sqrt{(B_s - e_k)^2 + (A_r - d_k)^2 + h_A^2}$$

$$h_A = |h_C - h_P| \quad A_S = A_r$$

For Image Mutual

$$M_{iY} = + G_5 \ln X_5 - G_6 \ln X_6 - G_7 \ln X_7 \\ + G_8 \ln X_8 - T_5 + T_6 + T_7 - T_8 \quad (16)$$

$$X_5 = G_5 + T_5 \quad X_7 = G_7 + T_7$$

$$X_6 = G_6 + T_6 \quad X_8 = G_8 + T_8$$

$$T_5 = \sqrt{(B_r - g_k)^2 + (A_r - d_k)^2 + I_0^2}$$

$$T_6 = \sqrt{(B_r - e_k)^2 + (A_r - d_k)^2 + I_0^2}$$

$$T_7 = \sqrt{(B_s - g_k)^2 + (A_r - d_k)^2 + I_0^2}$$

$$T_8 = \sqrt{(B_s - e_k)^2 + (A_r - d_k)^2 + I_0^2}$$

$$I_0 = \sqrt{h_B^2 + 2 d h_B - j (2 d^2 + 2 d h_B)}$$

$$h_B = h_C + h_P \quad \text{"d" as before}$$

For d_k, e_k, f_k and g_k see Eqs. (1, 2, 3, 4)

Summation of Section Mutuals Then using Eqs. (15 and 16) mutual from a Current Section that parallels the Y-axis to all Potential Sections:

$$Z_{CP} = R_{MY} + j X_{MY} \quad (17)$$

$$Z_{CP} = j \frac{\omega \mu_0}{4\pi} \sum_{i=1}^I \sum_{k=1}^n (M_{dY} - M_{iY}) \quad (18)$$

Where:

I = number of Potential Section $i = 1, 2, 3, \dots, I$

n = number of Segments used to approximate a Potential Segment and $k = 1, 2, 3, \dots, n$

$A_S = A_R$

Figure A.3 **MUTUAL COUPLING BETWEEN HORIZONTAL EARTH RETURN ANGLED CONDUCTORS**

A.3 **MUTUAL COUPLING BETWEEN HORIZONTAL EARTH RETURN ANGLED CONDUCTORS**

Mutual coupling between horizontal earth return angled conductors was determined in [1] by using the parallel segmented method of approximating the potential conductor. A closed form formula for the angled conductor mutual can be derived by solving the Neuman Integral. In Fig. A.3 conductor P is at angle θ with conductor C which is conducting current, I. The voltage directly induced in P by current I in C is given by the Neuman Integral, [2].

$$V_{AMd} = j \frac{\omega \mu_0 I}{4\pi} \cos \theta \int_0^P \int_0^C \frac{dc dp}{r_{dA}} \quad (19)$$

Where:

$$r_{dA} = \sqrt{c^2 + p^2 - 2cp \cos \theta + h_A^2}$$

$$h_A = |h_C - h_P|$$

$$\omega = 2\pi f, \quad f \text{ in Hz}$$

$$\mu_0 = 4\pi / 10^7$$

θ = Horizontal Angle between C and P

Direct Angled Mutual The direct angled mutual impedance in ohms with C and P in meters,

$$Z_{AMd} = \frac{V_{AMd}}{I} = j \frac{\omega \mu_0 \cos \theta}{4\pi} M_{Ad} \quad (20)$$

And,

$$M_{Ad} = \int_0^P \int_0^C \frac{dc dp}{\sqrt{c^2 + p^2 - 2cp \cos \theta + h_A^2}} \quad (21)$$

The integral solution from Grover [2] is modified by

$\tanh^{-1} A = \frac{1}{2} \ln \frac{1+A}{1-A}$ and changing symbols to C, P, h_A and θ is,

$$M_{Ad} = C \ln \frac{D_1 + D_2 + P}{D_1 + D_2 - P} + P \ln \frac{D_1 + D_2 + C}{D_1 + D_2 - C} + \frac{h_A}{\sin \theta} \left[-\tan^{-1} \frac{\cos \theta}{\sin \theta} + \tan^{-1} \frac{h_A \cos \theta}{D_2 \sin \theta} + \tan^{-1} \frac{h_A \cos \theta}{D_3 \sin \theta} - \tan^{-1} \frac{h_A^2 \cos \theta + C P \sin^2 \theta}{D_1 h_A \sin \theta} \right] \quad (22)$$

Where:

$$D_1 = \sqrt{C^2 + p^2 - 2CP \cos \theta + h_A^2}$$

$$D_2 = \sqrt{C^2 + h_A^2} \quad D_3 = \sqrt{p^2 + h_A^2}$$

$$h_A = |h_C - h_P| \quad \tan^{-1} = \text{arc tangent}$$

Image Angled Mutual With procedures similar to those used for the direct angled, mutual coupling from the Current Image Conductor,

$$Z_{AMi} = j \frac{\omega \mu_0 \cos \theta}{4\pi} M_{Ai} \quad (23)$$

$$M_{Ai} = C \ln \frac{I_1 + I_2 + P}{I_1 + I_2 - P} + P \ln \frac{I_1 + I_2 + C}{I_1 + I_2 - C}$$

$$+ \frac{I_0}{\sin \theta} \left[-\tan^{-1} \frac{\cos \theta}{\sin \theta} + \tan^{-1} \frac{I_0 \cos \theta}{I_2 \sin \theta} \right.$$

$$+ \tan^{-1} \frac{I_0 \cos \theta}{I_3 \sin \theta}$$

$$\left. - \tan^{-1} \frac{I_0^2 \cos \theta + C P \sin^2 \theta}{I_1 I_0 \sin \theta} \right] \quad (24)$$

Where:

$$I_1 = \sqrt{C^2 + p^2 - 2CP \cos \theta + I_0^2}$$

$$I_2 = \sqrt{C^2 + I_0^2}$$

$$I_3 = \sqrt{p^2 + I_0^2}$$

$$I_0 = \sqrt{h_B^2 + 2d h_B - j(2d^2 + 2d h_B)}$$

$$h_B = h_C + h_P \quad d = \frac{\sqrt{10}}{2\pi} 10^3 \sqrt{\frac{p}{f}}$$

Total Angled Mutual Impedance Total Mutual Impedance Between Angled Conductors using Eqs. (22) and (24),

$$Z_{AM} = j \frac{\omega \mu_0 \cos \theta}{4\pi} (M_{Ad} - M_{Ai}) \quad (25)$$

$$\omega = 2\pi f, \quad f \text{ in Hz}$$

$$\mu_0 = 4\pi / 10^7$$

$$0^\circ < \theta < 180^\circ$$

A.4 **MUTUAL COUPLING BETWEEN VERTICAL CONDUCTORS**

Mutual coupling between vertical conductors is usually negligible in comparison with mutual coupling between horizontal earth return conductors. In the Fig. 1 example there is no vertical mutual coupling as the potential conductor is laying on earth. In

Figure A.4 **MUTUAL COUPLING BETWEEN VERTICAL CONDUCTORS**

Coordinates:

CUR: A_r, B_r, h_r to A_r, B_r, h_s POT: a_i, b_i, h_i to a_i, b_i, h_j

Fig. A.5 mutual coupling between two 15 m vertical conductors is shown for separations of 5 to 200 m. Both the resistive and reactive components could be neglected for impedance measurements.

Mutual coupling between the vertical conductors of Fig. A.4 as derived with the Neuman Integral results in the following mutual impedance equations:

Direct Vertical Mutual

$$Z_{Vd} = j \frac{\omega \mu_0}{4\pi} M_{Vd}$$

$$M_{Vd} = -h_1 \ln W_1 + h_2 \ln W_2 + h_3 \ln W_3 - h_4 \ln W_4 + V_1 - V_2 - V_3 + V_4$$

Where:

$$\begin{aligned} W_1 &= h_1 + V_1 & h_1 &= h_s - h_j \\ W_2 &= h_2 + V_2 & h_2 &= h_s - h_i \\ W_3 &= h_3 + V_3 & h_3 &= h_r - h_j \\ W_4 &= h_4 + V_4 & h_4 &= h_r - h_i \end{aligned}$$

$$V_1 = \sqrt{h_1^2 + (A_r - a_i)^2 + (B_r - b_i)^2}$$

$$V_2 = \sqrt{h_2^2 + (A_r - a_i)^2 + (B_r - b_i)^2}$$

$$V_3 = \sqrt{h_3^2 + (A_r - a_i)^2 + (B_r - b_i)^2}$$

$$V_4 = \sqrt{h_4^2 + (A_r - a_i)^2 + (B_r - b_i)^2}$$

$$\omega = 2\pi f, \quad f \text{ in Hz} \quad \mu_0 = 4\pi / 10^7$$

Image Vertical Mutual

$$Z_{Vi} = j \frac{\omega \mu_0}{4\pi} M_{Vi}$$

$$M_{Vi} = +h_5 \ln W_5 - h_6 \ln W_6 - h_7 \ln W_7 + h_8 \ln W_8 - V_5 + V_6 + V_7 - V_8 + (d - j d) \ln \left(\frac{W_5 W_8}{W_6 W_7} \right)$$

Where:

$$h_5 = h_s + d - j d + V_5 \quad h_5 = h_s - h_j$$

$$A_s = A_r$$

$$B_s = B_r$$

$$a_j = a_i$$

$$b_j = b_i$$

(26)

(27)

(28)

(29)

$$W_6 = h_6 + d - j d + V_6$$

$$h_6 = h_s - h_i$$

$$B_s = B_r$$

$$W_7 = h_7 + d - j d + V_7$$

$$h_7 = h_r - h_j$$

$$a_j = a_i$$

$$W_8 = h_8 + d - j d + V_8$$

$$h_8 = h_r - h_i$$

$$b_j = b_i$$

$$V_5 = \sqrt{U_5^2 + (A_r - a_i)^2 + (B_r - b_i)^2}$$

$$V_6 = \sqrt{U_6^2 + (A_r - a_i)^2 + (B_r - b_i)^2}$$

$$V_7 = \sqrt{U_7^2 + (A_r - a_i)^2 + (B_r - b_i)^2}$$

$$V_8 = \sqrt{U_8^2 + (A_r - a_i)^2 + (B_r - b_i)^2}$$

$$U_5^2 = h_5^2 + 2 d h_5 - j (2 d^2 + 2 d h_5)$$

$$U_6^2 = h_6^2 + 2 d h_6 - j (2 d^2 + 2 d h_6)$$

$$U_7^2 = h_7^2 + 2 d h_7 - j (2 d^2 + 2 d h_7)$$

$$U_8^2 = h_8^2 + 2 d h_8 - j (2 d^2 + 2 d h_8)$$

$$\omega = 2\pi f, \quad f \text{ in Hz} \quad \mu_0 = 4\pi / 10^7$$

$$d = \frac{\sqrt{10}}{2\pi} 10^3 \sqrt{\frac{\rho}{f}} \quad \rho \text{ in } \Omega\text{-m}$$

Overall Vertical Mutual from Eqs. (27) and (29)

$$Z_V = j \frac{\omega \mu_0}{4\pi} (M_{Vd} - M_{Vi})$$

(30)

$$\omega = 2\pi f, \quad f \text{ in Hz} \quad \mu_0 = 4\pi / 10^7$$

Figure A.5 **MUTUAL COUPLING BETWEEN LINE CONNECTIONS AT THE SUBSTATION DEAD-END TOWER**

$$\begin{aligned} \text{Spacing:} \quad & f = 60 \text{ Hz} & h_s = h_j = 15 \text{ m} \\ \#1 = 5 \text{ m} & \#3 = 20 \text{ m} & \#5 = 100 \text{ m} \\ \#2 = 10 \text{ m} & \#4 = 50 \text{ m} & \#6 = 200 \text{ m} \end{aligned}$$

Discussion

Jacques Fortin (Hydro-Quebec): The author's paper extends the understanding and application of formulas for determining mutual impedance between diversely routed earth return conductors used to measure grounding grid impedance. As the grounding impedance measurement could be in error from mutual coupling, current test conductor to potential test conductor, correction of measured impedance may be required. Additional measurement errors can be introduced in the potential conductor by mutual coupling from current flow in overhead neutrals, buried neutrals, overhead ground wires (that are bonded to the substation ground grid and towers), water piping (that connect to the neutral conductors), and buried bare shield conductors, such as contained in urban areas. Are the author's methods applicable to these situations?

On page three of the paper the authors list conditions requiring segmentation of the potential sections. Would the authors indicate the tests used to determine if a potential section is parallel at right angles or angular with a current section? At the end of the page, do the list of angles sections requiring a "n" of 10 or more should be: (C_2C_3, P_3P_6, P_3P_6) , $(C_3C_4, P_1P_2, P_1P_4, P_2P_3, P_4P_5, P_5P_6, P_3P_6)$, (C_4C_5, P_1P_5, P_3P_6) , (C_5C_6, P_3P_6, P_3P_6) ?

Is there any advantage to selecting the coordinate system orientation so as to parallel either a current or potential section?

At the meeting the authors presented a comparison between ANSI/IEEE Standard 367-1987 and the Segmented Methods of the paper to calculated mutual impedance between a power line and a telephone line given in the standard. This material is of general interest and would be a valuable addition to the paper.

Manuscript received March 1, 1990.

CLOSURE

E.J. ROGERS AND J.F. WHITE. The authors appreciate the comments, questions, and interest of the discussor.

Mr Fortin has raised the very practical problem of errors introduced in the measurement of grounding potential rise by test current flowing in grounding conductors extending externally from the grid. Mutual impedance formulas require the exact knowledge of conductor routing relative to the potential conductor and the current distribution in these conductors. In urban areas that contain water piping, distribution neutrals and buried shield conductors, all tied to the grid, it is unlikely that these requirements can be entirely satisfied. Grounding measurement in these urban cases will require special procedures such as: current distribution measurements, potential conductor routing so as to minimize this mutual coupling, step and touch measurements near external circuits that conduct significant portions of the fault current to earth, and locating the remote potential electrode 30 m or more from these external earth return circuits. In the case of the overhead ground wire bonded to grids and towers, it is possible to make mutual corrections. Line sections, in this paper are defined as the straight portions of the line that do not change direction.

Figure C-1 TESTS TO DETERMINE A POTENTIAL SECTION ORIENTATION WITH A CURRENT SECTION

Fig. C-1 summarizes condition tests used to determine whether a potential section parallels, is at right angles, or angular with the current section. The authors are in-agreement with the list of angled sections of Fig. 1 requiring a "n" of 10 or more.

Formulas (9, 12, and 14) for calculating mutual impedance between one current section and each potential section and the test program of Fig. C-1 could be simplified and Formulas (15, 16, and 18) eliminated, if the coordinate system were to be oriented to agree with a current section. However, mutual impedance to potential sections calculated with each subsequent current section will require translation and rotation of the coordinate system axis so as to agree with each new current section. All potential section coordinates would change in each new coordinate system. Mr. Fortin's suggestion if incorporated in a calculator program, could result in fewer programs steps and possibly less computation time.

At the recommendation of Mr. Fortin we are including the comparison between the Segmented Method (057) and the ANSI/IEEE Standard 367-1987 Method (367) of calculating mutual impedance due to nonuniform exposure [3].

In this example mutual impedance is calculated between a power line and telecommunication line shown in Fig. C-2. Figure C-2 is a copy of Fig. 35 page 80 in Standard 367. The telephone line is composed of 10 sections extending from A to O and back to P. The power line is one straight section. Although not stipulated in 367 for this comparison, the power line is assumed to start 500 m before "A" and extended 500 m past "O". With this qualification, the power line will approach the infinite line approximation. Because of additional end-effects, a shorter power line would reduce the 057 calculated mutual coupling. Further, the power line height is taken as 15 m and the telephone line height is taken as 7.5 m, which results in a height difference of 7.5 m, as specified by 367.

The 367 method [3] is based on: (1) The Carson formula for mutual impedance between infinitely long conductors, (2) Uniform earth, (3) Approximating an angled potential section with one or more parallel segments with their overall parallel separations equal to the average of the angled separation, (4) Neglecting end-effects of finite length conductors. Items (1) and (4) will result in higher mutual impedance component values and item (3) smaller mutual reactive component, $\bar{X}_M$, values.

The segmented method [1] is based on: (1) The complex plane approximation for Carson, (2) Uniform earth, (3) Accounting for the end-effects between finite length conductors, using both direct and image conductors, as derived with the Neuman Integral, (4) Approximating angled potential sections with 10 or more parallel segments, (5) Conductor heights above earth are utilized in the 057 method.

Figure C-3 compares the cumulative mutual impedance between the power line and telecommunication line of Fig. C-2 calculated with 367 (Refer to page 85 Fig. 38 [3]) and 057. The 057 inductive component $\bar{X}_M$ falls below 367 up to near the line cross under point. The telecommunication line is approximated by 367 with 3 parallel segments in the H-L angled section. This results in a lower estimated $\bar{X}_M$ and the 057 cumulative curve is above 367 after 1575 m. With 057 all angled sections are approximated with 10 parallel segments. $\bar{R}_M$ calculated with 057 is well below the 367 cumulative curve.

Figure C-4 is the mutual impedance components, $\bar{R}_M$ and $\bar{X}_M$, calculated for each telecommunication line section. Fig. C-4 shows the differences between 057 and 367. Table 1 summarizes numerical values of the mutual impedance components calculated with 057 and 367 by line sections. The largest difference in the $\bar{X}_M$ component occurs in the G-M section near where the telecommunication line crosses under the power line.

Figure C-2 **NONUNIFORM EXPOSURE BETWEEN A POWER LINE AND A TELECOMMUNICATION LINE**
Taken from ANSI/IEEE Standard 367-1987 page 80

Figure C-3 **CUMULATIVE MUTUAL IMPEDANCE COMPONENTS FOR FIG. C-1**
 $f = 60 \text{ Hz}$ $\rho = 1000 \Omega\text{-m}$ $h_C = 15 \text{ m}$ $h_P = 7.5 \text{ m}$

Table 1 **COMPARISON OF THE MUTUAL IMPEDANCE COMPONENTS CALCULATED BY 367 AND 057 FOR EACH LINE SECTION**
 $f = 60 \text{ Hz}$ $\rho = 1000 \Omega\text{-m}$ $h_C = 15 \text{ m}$ $h_P = 7.5 \text{ m}$

LINE SECTION	SECTION LENGTH m	R _M Ω		X _M Ω	
		367	057-0	367	057-0
A-B	300	.018	.0085	.050	.041
B-C	300	.018	.0090	.059	.051
C-D	300	.018	.0094	.068	.062
D-E	200	.012	.0064	.051	.046
E-F	100	.006	.0032	.030	.029
F-G	225	.014	.0073	.080	.076
G-M	600	.028	.0194	.182	.240
M-N	225	.014	.0071	.080	.075
N-O	675	.041	.0199	.207	.183
O-P	450	-.027	-.0130	-.114	-.097

Figure C-4 **MUTUAL IMPEDANCE COMPONENTS FOR EACH LINE SECTION OF FIG. C-1**
 $f = 60 \text{ Hz}$ $\rho = 1000 \Omega\text{-m}$ $h_C = 15 \text{ m}$ $h_P = 7.5 \text{ m}$

Figure C-5 **OVERALL MUTUAL IMPEDANCE COMPONENTS FOR FIG. C-1 VERSUS EARTH RESISTIVITY**
 $f = 60 \text{ Hz}$ $h_C = 15 \text{ m}$ $h_P = 7.5 \text{ m}$

Figure C-5 shows the overall mutual coupling between the power line and the telecommunication line for earth resistivities 1 to 100.00 $\Omega\text{-m}$ as calculated by 057. Earth resistivity variations along the power line could be approximated by two or more resistivity zones.

REFERENCES

- ANSI/IEEE Standard 367-1987, IEEE Recommended Practice for Determining the Electric Power Station Ground Potential Rise and Induced Voltage from a Power Fault.