THE ANALYTICAL MONTE CARLO METHOD FOR APPROXIMATING
THE DISTRIBUTION OF A PLANT'S ELECTRICAL GENERATION

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Abstract - The planning of a fuel supply strategy for an electric power generation plant must consider the fact that the amount of fuel burned during a given period is random. For an all thermal system, the method described here provides an efficient way to approximate the distribution of a plant's generation (and, therefore, of its fuel burn) as a function of uncertainty in load forecasts and the plant's performance. The moments of the distribution of the plant's generation are estimated by combining the statistical method of the generation of system moments with probabilistic production costing in a way that models, analytically, a Monte Carlo process. The resulting algorithm, referred to as the Analytical Monte Carlo (AMC) method, is evaluated by comparing its estimates of the moments of each plant's generation with that obtained using a Monte Carlo model. The results indicate the method yields practical estimates for evaluating uncertainty that may be used in long-term studies.

INTRODUCTION

Modern production costing algorithms produce estimates of the expected generation of a plant. This does not provide any information as to how that generation may vary about that expected value. When such information is needed, a series of scenarios must be considered or a Monte Carlo model used. The form of the distribution of a plant's generation is often needed by decisionmakers in order to estimate the risk involved in a particular strategy. The AMC method, to be discussed below, will provide estimates of the moments of a plant's generation distribution, including expected generation, the variance in the generation, and the skewness of this distribution. The method (referred to in statistics as the method for the generation of system moments) consists of expanding the function which gives the energy served by a plant, as a function of the load and plant outage distributions, by a multivariable Taylor series. Expressions for the moments of the distribution of the plant's generation are derived from the multivariable Taylor series representation.

The motivation for the AMC method is provided by the Monte Carlo simulation of a power system in which the units forced outages and deratings are represented as distributions of equivalent forced outage rates (EFOR). It is generally the case that actual forced outage rate distribution of plants is not known but that EFORs for various periods of time are known. The very calculation of an EFOR implies the assumption of

a two-state model for the unit or plant under consideration. It is assumed that each unit's EFOR distribution is known. In a Monte Carlo model, as each unit is dispatched, an EFOR is chosen from the EFOR distribution to represent the unit's outages for the particular iteration. For a plant with several units that are assumed to have independent but identical distributions, the EFOR of each unit is chosen independently. As each unit is convolved into the equivalent load duration curve (ELDC), its generation is computed and added into the total for the plant's generation. After many iterations, the individual estimates are used to compute the mean, variance, and other moments of the plant's generation distribution. The AMC method uses analytical techniques to estimate the moments of the plant's generation distribution.

Before describing the general method, the EFOR model, which is derived by modeling a Monte Carlo process, is described. The general method is followed by the derivation of expressions for application of the method to production costing. Finally, examples are given comparing Monte Carlo results with AMC estimates.

The EFOR Model

The part of the Monte Carlo process to be considered in detail is the using of one EFOR selected from the EFOR distribution to represent the plant for one Monte Carlo iteration. The chosen EFOR is used in the Monte Carlo iteration to model the unit as a two-state unit.

The probability distribution of the equivalent load arising from this unit is then given by

$$h(x) = \begin{cases} 1 - f & \text{if } x = 0 \\ f & \text{if } x = c \end{cases}$$

where c is the unit's capacity and f is the chosen EFOR.

The expected load resulting from this unit is just the expected value of h or $U = cf$.

The variance of this load is the variance of h or $V = c^2 f(1 - f)$.

The third moment of this load is the third moment of h or $T = c^3 f(1 - f)(1 - 2f)$.

Now recall that the particular EFOR value f was chosen from a distribution of the EFOR for the unit. Thus, U , V , and T may be considered functions of the random variable f . As such, the first three moments of U , V , and T may be computed. Let $\mu_1(x)$, $\mu_2(x)$, and $\mu_3(x)$ denote the first, second, and third moments of x , respectively. Then

$$\mu_1(U) = \mu_1(cf) = c\mu_1(f)$$

$$\mu_2(U) = c^2\mu_2(f)$$

$$\mu_3(U) = c^3\mu_3(f)$$

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$$\begin{aligned}
\mu_1(V) &= \mu_1[c^2 f(1-f)] = c^2 \mu_1[f(1-f)] \\
\mu_2(V) &= \mu_1[(V - \mu_1(V))^2] \\
&= c^4 \mu_1[\{f(1-f) - \mu_1[f(1-f)]\}^2] \\
\mu_3(V) &= c^6 \mu_1[\{f(1-f) - \mu_1[f(1-f)]\}^3] \\
\mu_1(T) &= \mu_1[c^3 f(1-f)(1-2f)] = c^3 \mu_1[f(1-f)(1-2f)] \\
\mu_2(T) &= \mu_1[(c^3 f(1-f)(1-2f) - \mu_1(T))^2] \\
&= c^6 \mu_1[\{f(1-f)(1-2f) - \mu_1[f(1-f)(1-2f)]\}^2] \\
\mu_3(T) &= c^9 \mu_1[\{f(1-f)(1-2f) - \mu_1[f(1-f)(1-2f)]\}^3]
\end{aligned}$$

These expressions for the first three moments of the distributions of U, V, and T, are the functions of f that yield the mean, variance, and third moment of the units equivalent load distribution. In the usual cumulant method [2], the quantities $\mu_1(U)$, $\mu_1(V)$, and $\mu_1(T)$ are used in the estimating of the expected generation of the unit in question (and, of course, influence the units following it in the loading order). The distributions of U, V, and T may be used as part of a probabilistic (as opposed to a Monte Carlo) model for estimating the moments of the distribution of a unit's and, therefore, of a plant's, generation.

Of course, the expressions shown above are particular to the two-state model chosen to represent the units. There is no apparent reason that multi-state models could not be used just as well as an analogue to a Monte Carlo model using multi-state models.

For application to a plant, one reasons as follows. Since it is assumed that the units composing a plant have identical EFOR distributions and that unit outages are independent of any other unit's outage, it follows, from the property of cumulants (the cumulants of the sum of independent random variables are equal to the sum of the cumulants of those random variables), that for a station composed of n units

$$\begin{aligned}
\mu_1(nX) &= n\mu_1(X) \\
\mu_2(nX) &= n\mu_2(X) \\
\mu_3(nX) &= n\mu_3(X)
\end{aligned}$$

where X is either U, V, or T.

As will be seen later, these relationships and the moments of U, V, and T allow the AMC method to represent the EFOR distribution used in the Monte Carlo method.

The General Method

A general discussion of estimating the moments of a function of several random variables is presented in this section. A particular realization applicable to production costing is presented in the following section. The general development follows Hahn and Shapiro [1].

Let $x_1, x_2, \dots, x_n$ denote random variables with $\mu_1(x_i), \mu_2(x_i), \mu_3(x_i), \dots, \mu_n(x_i)$, $i = 1, 2, \dots, n$ known. Let Z be given by a function h of $x_1, x_2, \dots, x_n$, i.e., $Z = h(x_1, x_2, \dots, x_n)$.

The object is to approximate $\mu_1(Z)$, $\mu_2(Z)$, and $\mu_3(Z)$ based on the component variable values of $\mu_1(x_i), \mu_2(x_i)$ and $\mu_3(x_i)$, and the function relating Z to the component variables. The method for the generation of system moments consists of using a multivariable Taylor series to expand $h(x_1, x_2, \dots, x_n)$ about the point at which each of the component variables takes on its expected value, i.e., $(\mu_1(x_1), \mu_2(x_1), \dots, \mu_2(x_n))$. Approximations for $\mu_1(Z)$, $\mu_2(Z)$, and $\mu_3(Z)$ are obtained by taking expected values in the resulting expression.

The expansion of h in a multivariable Taylor series, retaining terms up to second order, about the point $(\mu_1(x_1), \mu_2(x_2), \dots, \mu_n(x_n))$ yields the following approximation (since the series is truncated, these are approximate rather than equalities).

$$\begin{aligned}
h(x_1, x_2, \dots, x_n) &= h(\mu_1(x_1), \mu_2(x_2), \dots, \mu_n(x_n)) \\
&+ \sum_{i=1}^n \frac{\partial h}{\partial x_i} [x_i - \mu_1(x_i)] \\
&+ 1/2 \left\{ \sum_{i=1}^n \frac{\partial^2 h}{\partial x_i^2} [x_i - \mu_1(x_i)]^2 \right. \\
&\left. + \sum_{i < j} \frac{\partial^2 h}{\partial x_i \partial x_j} [x_i - \mu_1(x_i)][x_j - \mu_1(x_j)] \right\}
\end{aligned}$$

Where the partial derivatives are assumed to be evaluated at the point $(\mu_1(x_1), \mu_2(x_2), \dots, \mu_n(x_n))$. Assuming that the component variables are uncorrelated and retaining terms up to second order, the results are (see Ref. 1 for complete derivations)

$$\begin{aligned}
\mu_1(Z) &= h(\mu_1(x_1), \mu_2(x_2), \dots, \mu_n(x_n)) + 1/2 \sum_{i=1}^n \frac{\partial^2 h}{\partial x_i^2} \mu_2(x_i) \\
\mu_2(Z) &= \sum_{i=1}^n \left(\frac{\partial h}{\partial x_i} \right)^2 \mu_2(x_i) + \sum_{i=1}^n \left(\frac{\partial h}{\partial x_i} \right) \left(\frac{\partial^2 h}{\partial x_i^2} \right) \mu_3(x_i) \\
\mu_3(Z) &= \sum_{i=1}^n \left(\frac{\partial^3 h}{\partial x_i^3} \right) \mu_3(x_i)
\end{aligned}$$

Hahn and Shapiro give $\mu_4(Z)$, as well as expressions for $\mu_i(Z)$, $i = 1, 2, 3, 4$ in case the component variables are correlated; however, only the above will be used in this paper.

Application to Production Costing

To apply this approach in production costing a differentiable function which yields the energy produced by a plant is needed. One such function arises in the "tails" method of production-costing technique [2]. In that technique, the unserved energy is evaluated before and after a plant is loaded under the ELDC. The reduction in unserved energy is the expected total energy served by the plant.

The unserved energy (UE), using a truncated Gram-Charlier series as in Ref. 2, is given by

$$UE(X, U, V, T) = \sqrt{V}[N(Z) + N(Z)\frac{T(X-U)}{6V^2} - \frac{Z}{Z} \int N(y)dy]P$$

where X is the installed capacity, $Z = \frac{X-U}{\sqrt{V}}$, P is

length of time period, and $N(Z) = \frac{1}{\sqrt{2\pi}} e^{-Z^2/2}$ (the normal density function). From this, a function yielding the expected energy produced by a plant may be written. The following variables will be used.

X_1 - installed capacity before loading plant of interest

U_1 - mean of ELDC before loading plant

V_1 - variance of ELDC before loading plant

T_1 - third moment of ELDC before loading plant

$Z_1 = (X_1 - U_1)/\sqrt{V_1}$

X_2 - station capacity

U_2 - mean of station outage load distribution

V_2 - variance of station outage load distribution

T_2 - third moment of station outage load distribution

$Z_2 = (X_2 + X_1 - U_1 - U_2)/\sqrt{V_1 + V_2}$

Then the energy served by the plant is given by the function

$$E[X_1, U_1, V_1, T_1, X_2, U_2, V_2, T_2] = UE(X_1, U_1, V_1, T_1) - UE(X_1 + X_2, U_1 + U_2, V_1 + V_2, T_1 + T_2)$$

$$\begin{aligned} &= \sqrt{V_1}[N(Z_1) - Z_1 \int N(y)dy + N(Z_1)\frac{T_1(X_1 - U_1)}{6V_1^2}]P \\ &\quad - \sqrt{V_1 + V_2}[N(Z_2) - Z_2 \int N(y)dy \\ &\quad + N(Z_2)\frac{(T_1 + T_2)(X_1 + X_2 - U_1 - U_2)}{6(V_1 + V_2)^2}]P \end{aligned}$$

The derivation of the moments of the random variables U_2 , V_2 , and T_2 has already been described. In a somewhat similar manner the variables U_1 , V_1 , and T_1 can be considered initially to be random variables either with $\mu_1(U_1)$, $\mu_1(V_1)$, and $\mu_1(T_1)$, $i = 2, 3$ equal zero (in case there is no uncertainty in the load forecast) or with these defined to represent the uncertainty in the load forecast. One way to define these would be to take a set of load forecasts with probability of occurrence of each known, to calculate the moments for each forecast, and then to calculate the moments of the moments. This technique will be illustrated in the sequel. It may also be possible to prepare load forecasts with the distributions of U_1 , V_1 , and T_1 being the output of the load forecasting process. It appears that weather uncertainty could be easily included in this set up. It seems that the weather deviation may leave the $\mu_1(U_1)$, $\mu_1(V_1)$, and

$\mu_1(T_1)$ unchanged but should alter the higher moments of these parameters. In any case, U_1 , V_1 , and T_1 for the initial load are considered to be uncorrelated random variables with known moments.

To apply the results of the multivariable Taylor series, the partial derivatives of the function E must be found. The necessary derivatives are listed below.

$$\begin{aligned} \frac{\partial E}{\partial U_2} &= - \left[\int N(y)dy + (T_1 + T_2)N(Z_2)(Z_2^2 - 1) \right] / \\ &\quad Z_2 \\ &\quad (6(V_1 + V_2)^{3/2})P \end{aligned}$$

$$\frac{\partial E}{\partial U_1} = \left[\int N(y)dy + T_1 N(Z_1)(Z_1^2 - 1) / (6V_1^{3/2}) \right] P + \frac{\partial E}{\partial U_2}$$

$$\begin{aligned} \frac{\partial E}{\partial V_2} &= \frac{-N(Z_2)}{2\sqrt{V_1 + V_2}} [1 + (T_1 + T_2)Z_2(Z_2^2 - 3)] / \\ &\quad (6(V_1 + V_2)^{3/2})P \end{aligned}$$

$$\frac{\partial E}{\partial V_1} = \frac{N(Z_1)}{2\sqrt{V_1}} [1 + T_1 Z_1(Z_1^2 - 3) / (6V_1^{3/2})] P + \frac{\partial E}{\partial V_2}$$

$$\frac{\partial E}{\partial T_2} = \frac{-N(Z_2)Z_2}{6(V_1 + V_2)^{3/2}} P$$

$$\frac{\partial E}{\partial T_1} = \frac{N(Z_1)Z_1}{6V_1^{3/2}} P + \frac{\partial E}{\partial T_2}$$

$$\frac{\partial^2 E}{\partial^2 U_2^2} = -N(Z_2) \left\{ \frac{1}{\sqrt{V_1 + V_2}} + \frac{Z_2(T_1 + T_2)}{6(V_1 + V_2)^{3/2}} [Z_2^2 - 3] \right\} P$$

$$\frac{\partial^2 E}{\partial U_1^2} = N(Z_1) \left\{ \frac{1}{\sqrt{V_1}} + Z_1 T_1 (Z_1^2 - 3) / (6V_1^{3/2}) \right\} P + \frac{\partial^2 E}{\partial U_2^2}$$

$$\frac{\partial^2 E}{\partial V_2^2} = \frac{-N(Z_2)}{4(V_1 + V_2)^{3/2}}$$

$$[-1 + (T_1 + T_2)Z_2(-7Z_2^2 + 15)] / \{6(V_1 + V_2)^{3/2}\}$$

$$+ Z_2^2 \{1 + (T_1 + T_2)Z_2(Z_2^2 - 3) / \{6(V_1 + V_2)^{3/2}\}\} P$$

$$\frac{\partial^2 E}{\partial V_1^2} = \frac{N(Z_1)}{4V_1^{3/2}} [-1 + T_1 Z_1(-7Z_1^2 + 15)] / \{6V_1^{3/2}\}$$

$$+ Z_1^2 \{1 + T_1 Z_1(Z_1^2 - 3) / (6V_1^{3/2})\} P + \frac{\partial^2 E}{\partial V_2^2}$$

$$\frac{\partial^2 E}{\partial T_2^2} = 0$$

$$\frac{\partial^2 E}{\partial T_1^2} = 0$$

With these partials known, the moments of the distribution of the energy generated by the plants can be estimated. The detailed expressions follow.

$$\begin{aligned}\mu_1(E) = & E[X_1, \mu_1(U_1), \mu_1(V_1), \mu_1(T_1), \\ & X_2, \mu_1(U_2), \mu_1(V_2), \mu_1(T_2)] \\ & + 1/2[\frac{\partial^2 E}{\partial U_1^2} \mu_2(U_1) + \frac{\partial^2 E}{\partial V_1^2} \mu_2(V_1) \\ & + \frac{\partial^2 E}{\partial U_2^2} \mu_2(U_2) + \frac{\partial^2 E}{\partial V_2^2} \mu_2(V_2)]\end{aligned}$$

$$\begin{aligned}\mu_2(E) = & (\frac{\partial E}{\partial U_1})^2 \mu_2(U_1) + (\frac{\partial E}{\partial V_1})^2 \mu_2(V_1) + (\frac{\partial E}{\partial T_1})^2 \mu_2(T_1) \\ & + (\frac{\partial E}{\partial U_2})^2 \mu_2(U_2) + (\frac{\partial E}{\partial V_2})^2 \mu_2(V_2) + (\frac{\partial E}{\partial T_2})^2 \mu_2(T_2) \\ & + \frac{\partial E}{\partial U_1} \frac{\partial^2 E}{\partial U_1^2} \mu_3(U_1) + \frac{\partial E}{\partial V_1} \frac{\partial^2 E}{\partial V_1^2} \mu_3(V_1) \\ & + \frac{\partial E}{\partial U_2} \frac{\partial^2 E}{\partial U_2^2} \mu_3(U_2) + \frac{\partial E}{\partial V_2} \frac{\partial^2 E}{\partial V_2^2} \mu_3(V_2)\end{aligned}$$

$$\begin{aligned}\mu_3(E) = & (\frac{\partial E}{\partial U_1})^3 \mu_3(U_1) + (\frac{\partial E}{\partial V_1})^3 \mu_3(V_1) + (\frac{\partial E}{\partial T_1})^3 \mu_3(T_1) \\ & + (\frac{\partial E}{\partial U_2})^3 \mu_3(U_2) + (\frac{\partial E}{\partial V_2})^3 \mu_3(V_2) + (\frac{\partial E}{\partial T_2})^3 \mu_3(T_2)\end{aligned}$$

Algorithm Evaluation

In order to determine whether the AMC method described above produces estimates which are realistic enough to have practical application, the algorithm was implemented in a computer program. Since the production costing codes available to us do not produce estimates other than expected generation, a simple Monte Carlo model was implemented to produce estimates of expected generation, variance, and skewness of generation. This Monte Carlo model provided the values for comparison to those produced by the AMC algorithm. The two models will be briefly discussed and then some results exhibited.

The Monte Carlo model is a load duration curve approach based on a discrete convolution algorithm which included interpolation to accommodate units of various capacities. Each station is modeled as containing n identical units with each unit's outages assumed independent of the other unit's outages. Each station has an EFOR distribution which is applied to each unit independently as they are loaded under the ELDC. Table II exhibits the EFOR distributions used in the runs to be discussed later. In the EFOR distributions shown, each EFOR is equally likely.

The data used both by the simple Monte Carlo model and the AMC method are shown in Tables I and II. The capacities of units and number of units at a station is shown in Table I. The EFOR distributions are shown in Table II. Note that some distributions

consist of one number, others have various numbers of EFOR representing the distribution. For each unit, the probability of a particular EFOR occurring is the reciprocal of the number of EFORs listed, i.e., the EFORs listed are equally likely. Another point to be noted is that for this study all plants are treated as thermal plants. There are no assigned energy plants and no special treatment of hydro generation. The loading order of the plants is fixed by the input data as shown in Table I.

Of course, both models require load data; both use load duration curves but with different representations. For the Monte Carlo model, the discrete representation of the load duration curve was done with a 50 MW step size. The algorithm contains an interpolation scheme which allows for loading of units with capacities which are not multiples of the selected step size. Five load forecasts, with probability of occurrence shown in Table III, were used in all the examples to be shown later. Of course, during the execution of the Monte Carlo model the load forecast for each iteration was chosen at random in accordance with this distribution.

The load duration curve representation used by the AMC algorithm consists of nine numbers referred to as the distributions of the moments. Three numbers describe the distribution of each moment (the mean, variance, and third moment) of the various forecasts. In Tables III, IV, and V, the mean, variance, and third moment of the five load forecasts (computed from the hourly data) for various months are shown with the resulting distributions of the moments for input to the AMC algorithm.

Table I

System Characteristics

Plant	No. Of Units	Unit Capacity (MW)
HYDRO BASE	1	700
NUCLEAR A	3	1,065
NUCLEAR B	2	1,128
FOSSIL A	1	870
FOSSIL B	2	614
FOSSIL C	2	272
FOSSIL D	1	1,018
FOSSIL E	2	248
FOSSIL F	4	190
FOSSIL G	2	1,245
FOSSIL H	3	265
FOSSIL I	4	190
FOSSIL J	1	488
FOSSIL K	4	139
FOSSIL L	2	486
FOSSIL M	5	188
FOSSIL N	4	122
FOSSIL O	6	114
FOSSIL P	6	125
FOSSIL Q	10	145
FOSSIL R	4	56
PEAKING A	1	3,668
PEAKING B	4	383
PEAKING C	1	1,573

Table II

Plant EFOR Distributions
(Equally Likely Values Shown in %)

HYDRO BASE	0.0											
NUCLEAR A	35.0											
NUCLEAR B	35.0											
FOSSIL A	4.6	5.3	5.4	6.3	9.9	11.4	11.7	14.3	14.6	16.8	20.9	28.3
FOSSIL B	18.4	19.3	20.6	27.3	29.7	29.9	32.2	33.4	36.5	36.6	36.8	36.9
FOSSIL C	5.8	6.6	7.5	7.8	8.1	9.6	10.7	11.0	12.7	21.6	38.1	41.6
FOSSIL D	18.4	19.3	20.6	27.3	29.7	29.9	32.2	33.4	36.5	36.6	36.8	36.9
FOSSIL E	5.8	6.6	7.5	7.8	8.1	9.6	10.7	11.0	12.7	21.6	38.1	41.6
FOSSIL F	1.4	1.9	2.5	3.3	1.8	4.8	6.6	6.6	8.5	8.7	9.9	14.9
FOSSIL G	21.9	28.3	31.3	33.1	35.1	41.4	42.6	45.8	53.3			
FOSSIL H	21.3	24.4	24.9	25.0	26.3	31.2	32.6					
FOSSIL I	3.0	5.0	13.0	17.0	24.0	25.9	28.4	28.9	29.0	39.5	41.5	41.8
FOSSIL J	3.0	5.0	13.0	17.0	24.0	25.9	28.4	28.9	29.0	39.5	41.5	41.8
FOSSIL K	5.3	5.6	5.8	6.9	10.1	17.2	18.0	19.5	19.6	23.2	23.4	23.7
FOSSIL L	19.6	20.9	21.6	21.6	25.4	32.4	37.9	40.0	42.4	42.9	48.3	51.5
FOSSIL M	5.0	5.0	6.0	6.0	7.0	7.0	8.0	9.0	10.0	12.0	13.0	13.0
FOSSIL N	5.0	5.0	6.0	6.0	7.0	7.0	8.0	9.0	10.0	12.0	13.0	13.0
FOSSIL O	5.3	5.6	5.8	6.9	10.1	17.2	18.0	19.5	19.6	23.2	23.4	23.7
FOSSIL P	3.8	5.1	7.1	7.5	9.0	9.0	9.6	11.1	12.1	14.6	14.7	20.9
FOSSIL Q	2.4	3.3	1.1	1.7	4.7	5.2	6.4	8.7	10.8	10.9	11.3	18.1
FOSSIL R	8.0											
PEAKING A	0.0											
PEAKING B	20.0											
PEAKING C	0.0											

Table III

The July Forecasts and Distribution of Moments

Forecast	Weight (%)	1	Moments 2	3
Ultra High	10	15,513	10,953,165	-0.5467E10
High	20	15,405	10,939,084	-0.5448E10
Medium	40	15,273	10,839,529	-0.5182E10
Low	20	15,164	10,882,020	-0.5125E10
Very Low	10	14,999	10,721,046	-0.4996E10

Distribution of Moments

	1 (U)	2 (V)	3 (T)
Mean (μ_1)	15274	0.1086745E8	-0.52340E10
Variance (μ_2)	19063	0.4258700E10	0.23753E17
Third (μ_3)	-549963	-0.1854436E15	-0.15995E25

Table V

The April Forecasts and Distribution of Moments

Forecast	Weight (%)	1	Moments 2	3
Ultra High	10	12,270	3,516,192	0.1334E10
High	20	11,992	3,489,728	0.1314E10
Medium	40	11,672	3,348,041	0.1225E10
Low	20	11,380	3,339,363	0.1218E10
Very Low	10	10,963	3,170,392	0.1040E10

Distribution of Moments

	1 (U)	2 (V)	3 (T)
Mean (μ_1)	11666	0.337369E7	0.1234E10
Variance (μ_2)	123461	0.935544E10	0.6141E16
Third (μ_3)	-0.107504E8	-0.253293E15	-0.5331E24

Table IV

The January Forecasts and Distribution of Moments

Forecast	Weight (%)	1	Moments 2	3
Ultra High	10	15,737	3,565,704	-0.4282E9
High	20	15,441	3,561,069	-0.4308E9
Medium	40	15,042	3,403,834	-0.4289E9
Low	20	14,760	3,513,065	-0.1581E9
Very Low	10	14,206	3,313,433	-0.2832E9

Distribution of Moments

	1 (U)	2 (V)	3 (T)
Mean (μ_1)	15051	0.346427E7	-0.3605E9
Variance (μ_2)	165934	0.711529E10	0.1211E17
Third (μ_3)	-0.212221E8	-0.122567E15	0.1475E25

The results of executing the Monte Carlo and the AMC models for three different months are compared in Tables VI, VII, and VIII. The months chosen, January, April, and July, represent the three various load shapes of a year occurring during the peak seasons of winter and summer and the offpeak seasons of spring and fall.

There are some interesting points to note in Tables VI, VII, and VIII. One of the first things that is noticeable is that the expected generation for January and April is approximated well by the AMC approach but that it experiences difficulties with the July loads. The most probable explanation for this is that the summer loads are more difficult to fit by the Gram-Charlier series than the other load shapes. (It seems the summer load shapes exhibit a bimodal behavior). The approximation to the standard deviation provided by the AMC algorithm seems to be quite adequate.

ate in all cases but, again, performance is worst in the July case. The approximation to the skewness appears to be acceptable for plants that are base loaded or near base loaded. However, for most of the plants the skewness approximation from an overall point of view is not acceptable. It may be possible to improve the skewness approximation including additional terms in the skewness approximation.

Conclusion

The results obtained with this simple version of the AMC algorithm are encouraging. Although there are some problems with the technique, the very fact that the variation of the generation can be efficiently and correctly estimated without running a Monte Carlo program, indicates that the technique is promising.

One basic problem which must be considered in order to obtain more useful results from the AMC algorithm is the treatment of assigned energy units. This has not been addressed at all here, but some ideas have been broached. One of these is to consider the generation by the assigned energy units (especially the pumped storage) as being distributed over several load levels. This is particularly appealing since pumped storage operates both on the peak and shoulder of the loads. Also, since with AMC several load forecasts may be involved (or at least it may

be thought of in this manner), distributing both the generation and the payback may be the correct modeling technique to use. It may be possible to do this by some transformation of the distribution of moments before the thermal plants are dispatched.

The AMC technique is not limited to the particular realization described here. It is possible and may be necessary to consider better representations of the load duration curve to improve performance. However, given the variability involved and the likely smoothing effects of considering a whole year as opposed to a month, the current algorithm may prove adequate.

In any case, the results obtained thus far show enough promise to warrant our continued investigation of this family of algorithms.

REFERENCES

- 1) G. J. Hahn and S. S. Shapiro, Statistical Models in Engineering, John Wiley and Sons, New York, 1967.
- 2) J. P. Stremel, "Production Costing for Long-Range Generation Expansion Planning Studies," IEEE actions on Power Apparatus and Systems, vol. PAS-101, No. 3, 1982.

Table VI

Comparison of Model Results for July

Station	Number Unit	Unit Capacity	Expected Generation (GWh)			Standard Deviation (GWh)			Skewness		
			MC	AMC	Error	MC	AMC	Error	MC	AMC	Error
HYDRO BASE	1	700	520.8	520.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NUCLEAR A	3	1,065	1,545.1	1,544.9	0.2	0.0	0.0	0.0	0.0	0.0	0.0
NUCLEAR B	2	1,128	1,091.0	1,090.2	0.8	0.0	0.0	0.0	0.0	0.0	0.0
FOSSIL A	1	870	566.7	565.5	1.2	45.3	44.1	1.2	-0.88	-0.85	-0.03
FOSSIL B	2	614	641.4	638.9	2.5	44.1	43.1	1.0	0.42	0.40	0.02
FOSSIL C	2	272	344.1	341.5	2.6	32.6	33.5	-0.9	-0.95	-1.01	0.06
FOSSIL D	1	1,018	533.0	526.1	6.9	50.1	50.2	-0.1	0.55	0.57	-0.02
FOSSIL E	2	248	312.0	308.7	3.3	30.6	30.2	0.4	-0.88	-1.01	0.13
FOSSIL F	4	190	529.1	520.8	8.3	10.9	10.9	0.0	-0.38	-0.37	-0.01
FOSSIL G	2	1,245	1,116.1	1,109.3	6.8	113.0	111.6	1.4	-0.06	-0.10	0.04
FOSSIL H	3	265	396.2	399.5	-3.3	12.7	12.2	0.5	-0.22	-0.22	0.0
FOSSIL I	4	190	369.8	380.6	-10.8	30.7	31.6	-0.9	-0.04	0.14	-0.18
FOSSIL J	1	488	228.2	237.2	-9.0	38.5	39.5	-1.0	0.32	0.29	0.03
FOSSIL K	4	139	284.3	297.8	-13.5	13.4	13.6	-0.2	0.28	0.07	0.21
FOSSIL L	2	486	363.8	384.5	-20.7	43.1	45.5	-2.4	-0.10	-0.06	-0.04
FOSSIL M	5	188	451.7	478.9	-27.2	14.6	16.0	-1.4	0.13	-0.02	0.15
FOSSIL N	4	122	218.3	229.4	-11.1	7.7	9.1	-1.4	0.05	-0.02	0.07
FOSSIL O	6	114	267.6	277.0	-9.4	12.1	14.9	-2.8	0.16	0.02	0.14
FOSSIL P	6	125	285.1	287.6	-2.5	12.2	15.3	-3.1	0.08	-0.02	0.10
FOSSIL Q	10	145	491.0	468.7	22.3	24.3	30.0	-5.7	-0.09	-0.01	-0.08
FOSSIL R	4	56	65.0	59.0	6.0	3.9	4.4	-0.5	-0.06	-0.01	-0.05
PEAKING A	1	3,668	666.1	586.0	80.1	70.3	57.5	12.8	0.08	-0.01	0.09
PEAKING B	4	383	53.2	60.2	-7.0	12.5	8.9	3.6	0.45	-0.01	0.46
PEAKING C	1	1,573	21.3	35.3	-14.0	6.7	6.1	0.6	0.69	-0.01	0.70

Table VII

Comparison of Model Results for January

Station	Number	Unit Capacity	Expected Generation (GWh)			Standard Deviation (GWh)			Skewness		
			MC	AMC	Error	MC	AMC	Error	MC	AMC	Error
HYDRO BASE	1	700	520.8	520.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NUCLEAR A	3	1,065	1,545.1	1,545.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NUCLEAR B	2	1,128	1,091.0	1,091.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FOSSIL A	1	870	566.7	566.6	0.1	45.3	44.2	1.1	-0.88	-0.84	-0.04
FOSSIL B	2	614	641.4	641.4	0.0	44.1	43.3	0.8	0.42	0.40	0.02
FOSSIL C	2	272	344.1	343.6	0.5	32.6	33.7	-1.1	-0.95	-1.01	0.06
FOSSIL D	1	1,018	533.0	531.7	1.3	50.1	50.8	-0.7	0.55	0.57	-0.02
FOSSIL E	2	248	312.3	313.2	-0.9	30.6	30.7	-0.1	-0.88	-1.01	0.13
FOSSIL F	4	190	531.3	531.6	-0.3	11.0	11.1	-0.1	-0.38	-0.37	-0.01
FOSSIL G	2	1,245	1,156.4	1,158.3	-1.9	118.9	117.9	1.0	-0.05	-0.10	0.05
FOSSIL H	3	265	426.0	424.9	1.1	12.8	13.0	-0.2	-0.38	-0.23	-0.15
FOSSIL I	4	190	407.4	409.2	-1.8	33.9	34.6	-0.7	-0.07	0.13	-0.02
FOSSIL J	1	488	256.7	257.1	-0.4	43.5	43.2	0.3	0.32	0.28	0.04
FOSSIL K	4	139	323.4	324.1	-0.7	16.4	16.8	-0.4	0.06	0.01	0.05
FOSSIL L	2	486	418.1	419.1	-1.0	51.2	52.1	-0.9	-0.09	-0.06	-0.03
FOSSIL M	5	188	514.7	517.5	-2.8	32.8	33.9	-1.1	-0.60	-0.18	-0.42
FOSSIL N	4	122	241.8	242.9	-1.1	20.4	21.2	-0.8	-0.51	-0.19	-0.32
FOSSIL O	6	114	284.4	285.4	-1.0	29.7	31.5	-1.8	-0.30	-0.17	-0.13
FOSSIL P	6	125	282.0	282.4	-0.4	36.3	37.4	-1.1	-0.30	-0.19	-0.11
FOSSIL Q	10	145	413.1	410.2	2.9	70.7	71.9	-1.2	-0.10	-0.20	0.10
FOSSIL R	4	56	46.3	45.4	0.9	9.8	9.7	0.1	0.05	-0.21	0.26
PEAKING A	1	3,668	328.9	320.0	8.9	96.3	88.5	7.8	0.37	-0.21	0.58
PEAKING B	4	383	11.1	12.0	-0.9	6.1	5.0	1.1	1.12	-0.22	1.34
PEAKING C	1	1,573	3.0	3.9	-0.9	2.1	1.8	0.3	1.52	-0.23	1.75

Table VIII

Comparison of Model Results for April

Station	Number	Unit Capacity	Expected Generation (GWh)			Standard Deviation (GWh)			Skewness		
			MC	AMC	Error	MC	AMC	Error	MC	AMC	Error
HYDRO BASE	1	700	504.0	504.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NUCLEAR A	3	1,065	1,495.3	1,495.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NUCLEAR B	2	1,128	1,055.8	1,055.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FOSSIL A	1	870	548.4	548.3	0.1	43.8	42.8	1.0	-0.88	-0.85	-0.03
FOSSIL B	2	614	620.5	619.8	0.7	42.6	41.8	0.8	0.42	0.40	0.02
FOSSIL C	2	272	332.2	331.0	1.2	31.4	32.4	-1.0	-0.95	-1.01	0.06
FOSSIL D	1	1,018	510.1	507.6	2.5	47.9	48.4	-0.5	0.54	0.57	-0.03
FOSSIL E	2	248	295.1	295.8	-0.7	28.9	29.1	-0.2	-0.86	-1.00	0.14
FOSSIL F	4	190	490.5	491.9	-1.4	13.2	13.3	-0.1	-0.38	-0.22	0.16
FOSSIL G	2	1,245	949.8	957.7	-7.9	100.7	100.5	0.2	-0.02	-0.09	0.07
FOSSIL H	3	265	309.3	309.4	-0.1	23.7	25.2	-1.5	-0.33	-0.13	-0.2
FOSSIL I	4	190	268.2	269.0	-0.8	31.8	33.4	-1.6	0.02	-0.03	0.05
FOSSIL J	1	488	152.5	151.8	0.7	31.3	30.1	1.2	0.41	0.13	0.28
FOSSIL K	4	139	176.6	175.4	1.2	23.6	24.0	-0.4	-0.06	-0.13	0.07
FOSSIL L	2	486	194.4	192.6	1.8	36.7	36.5	0.2	0.18	-0.09	0.27
FOSSIL M	5	188	197.8	196.1	1.7	37.6	36.3	1.3	0.07	-0.14	0.21
FOSSIL N	4	122	75.4	74.8	0.6	16.9	15.9	1.0	0.20	-0.14	0.34
FOSSIL O	6	114	74.6	74.1	0.5	18.7	17.4	1.3	0.33	-0.14	0.47
FOSSIL P	6	125	59.5	59.2	0.3	16.9	15.3	1.6	0.40	-0.14	0.54
FOSSIL Q	10	145	61.9	61.6	0.3	20.5	18.2	2.3	0.55	-0.14	0.69
FOSSIL R	4	56	5.1	5.0	0.1	1.9	1.7	0.2	0.68	-0.14	0.82
PEAKING A	1	3,668	23.7	23.5	0.2	10.6	8.7	1.9	0.88	-0.14	1.02
PEAKING B	4	383	0.2	0.3	-0.1	0.2	0.1	0.1	1.70	-0.15	1.85
PEAKING C	1	1,573	0.0	0.0	0.0	0.0	0.0	0.0	2.17	-0.15	2.32