

2) the sum of the mean function and the amplitude with phase function. However, if the physics of the problem of interest is understood so that modeling one or both of the smooth functions with as few coefficients is possible, a generalization of Ksienski's algorithm using least squares to minimize with respect to the known functions could be much more accurate and effective.

REFERENCES

- [1] D. A. Ksienski, "A method of resolving data into two maximally smooth components," *Proc. IEEE*, vol. 73, pp. 166-168, Jan. 1985.

Comments on "Fast Interpolation Algorithm Using Fast Hartley Transform"

CHAU-YUN HSU, TENG-PIN LIN, AND
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Certain errors in the aboved titled letter¹ are pointed out and corrected.

In the above letter,¹ Agbinya proposed a fast interpolation algorithm using fast Hartley transform which reduces the computational load and the mean-square error as compared with the FFT approach. But unfortunately, there are several errors in the original letter as indicated in the following.

The gain of the low-pass filter for the interpolation $H(mT_f)$ should be

$$H(mT_f) = \text{rect}(W) = \begin{cases} S+1, & |W| < \pi/T \\ 0, & \pi/T < |W| < \pi/T_f. \end{cases}$$

Further, (8) should be also corrected as

$$\begin{aligned} Z_k &= (S+1) F_k, & k &= 0, 1, 2, \dots, (N/2) - 1 \\ Z_k &= 0, & k &= N/2, (N/2) + 1, \dots, P - (N/2) - 1 \\ Z_k &= (S+1) F_k, & k &= P - (N/2), P - (N/2) + 1, \dots, P - 1. \end{aligned}$$

The original (8) is used for decimation not for interpolation.

And the desired points $Z(m)$ of the new sequences are obtained by getting the inverse DHT of (8) as

$$Z(M) = 1/P \sum_{k=0}^{P-1} Z_k \text{cas}(b/P), \quad m = 0 \text{ to } P - 1.$$

That is, the transform kernel of (9) should be $\text{cas}(b/P)$ instead of $\text{cas}(b/N)$ as described in the original letter, where $P = N + S^*(N - 1)$. Therefore, the complexity analysis is questionable, for the misusing of transform kernel.

REFERENCE

- [1] J. I. Agbinya, "Fast interpolation algorithm using fast Hartley transform," *Proc. IEEE*, vol. 75, no. 4, pp. 523-524, Apr. 1987.

Author's Reply²

The suggested corrections [1] to the gain of the low-pass filter and the kernel of the cas function (9) respectively of [2] are in order.

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¹J. I. Agbinya, *Proc. IEEE*, vol. 75, no. 4, pp. 523-524, Apr. 1987.

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However, Hsu *et al.* have introduced a further error into (9) by using $Z(M)$ instead of $z(m)$. The correct version is

$$z(m) = 1/P \sum_{k=0}^{P-1} Z_k \cdot \text{cas}(b/P), \quad m = 0 \text{ to } P - 1.$$

Their form of (8) is usually used for decimation and leads to large errors, more noticeable when N is small in both FFT and DHT applications. The use of (8) of [2] in (9) lessens serious errors and requires restoring scaling factors to the amplitudes of $z(m)$.

The complexity analysis expressions in [2] are not dependent on P as Hsu *et al.* have implied in [1], since no additions and multiplications need to be done for the trivial cases of $Z_k = 0$ in the range $k = N/2$ to $P - (N/2) - 1$ in both (8) and (9). By such considerations, it is infact possible to further reduce both addition and multiplication counts respectively down to

$$\text{Addition} = 2N - 2 + (2N - 4) \text{Int}(\log_4 N)$$

and

$$\text{Multiplication} = 1 + 2 \cdot N + (N - 4) \cdot \text{Int}(\log_4 N).$$

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- [1] C.-Y. Hsu, T.-P. Lin, and J.-L. Wu, "Comments on 'Fast interpolation algorithm using fast Hartley transform'," *Proc. IEEE*, this issue.
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On the Number of Costas Arrays as a Function of Array Size

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A conjecture that the number of Costas arrays is a monotonic increasing function of array size is disproved by extension of the known values to $n = 17$. A probabilistic estimation formula is developed which predicts the peak at $n = 16$ and tracks the known values to (typically) 5-6 percent.

I. INTRODUCTION

A Costas array or "constellation" [1] is a pattern of n dots on an $n \times n$ grid, one dot per row and one per column, in which the $n(n - 1)/2$ vectors between the dots are all distinct. Such patterns, first identified by J. P. Costas, provide a template for generating radar and sonar signals with ideal ambiguity functions [2]-[4] and also have potential utility for problems in physical alignment and synchronization.

While no construction for generating a Costas array valid for general n has been found, Golomb and Taylor [5] present a variety of constructions for special n values based on the properties of primitive roots of finite fields. In a list of open questions on the number of Costas array $C(n)$, they include the conjectures: 1) $C(n) \geq 1$ for all $n \geq 1$, and 2) $C(n)$ is monotonic increasing. We report here a further extension to $n = 17$ of the known values of $C(n)$ [6]. Our results disprove the second conjecture and cast doubt on the first. Further, we derive a simple probabilistic estimation formula with one free parameter which predicts the peak in $C(n)$ at $n = 16$ and tracks the known values of $C(n)$, which vary over four orders of magnitude, to (typically) 5-6 percent.

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