and

$$M^{(2)} = \prod_{k=n/2+1}^n M_k \quad (14b)$$

(13) may be written as

$$M = M^{(1)}M^{(2)}. \quad (15)$$

Equations (13), (14), and (15) form the basis of the divide-and-conquer technique based algorithm for computing M . The task of computing the product of n transformation matrices is reduced to two tasks of computing the product of $n/2$ transformation matrices. Using analogous arguments, each of $M^{(1)}$ and $M^{(2)}$ could be decomposed further into product of $n/4$ transformation matrices and so on. Also, each element of $M^{(1)}$ and $M^{(2)}$ is a polynomial of degree $n/2$. Therefore, M is obtained from $M^{(1)}$ and $M^{(2)}$ in 8 polynomial multiplications and 4 polynomial additions, where the polynomials have degree at most $n/2$. The product of two polynomials of degree k each can be formed in $\alpha k \log_2 k$ MULT and $\beta k \log_2 k$ ADD using the FFT algorithm, where α and β are constants [3]. Let $M(n)$ and $A(n)$ denote the number of MULT and ADD respectively, for computing the transformation matrix M using the divide-and-conquer technique based algorithm. Then, we have the following relation:

$$M(n) = 2M(n/2) + 8\alpha(n/2) \log_2(n/2) \quad (16a)$$

and

$$A(n) = 2A(n/2) + 8\beta(n/2) \log_2(n/2) \quad (16b)$$

or,

$$M(n) = 2\alpha n \log_2^2 n \quad (17a)$$

and

$$A(n) = 2\beta n \log_2^2 n. \quad (17b)$$

Here, the lower-order terms have been neglected. Also, a more computationally efficient scheme for computing the product of (2×2) matrices is Strassen's algorithm, which requires 7 MULT [4]. If Strassen's algorithm is incorporated into the above described algorithm, then the multiplicative and additive complexities can be further reduced to $1.75\alpha n \log_2^2 n$ and $1.75\beta n \log_2^2 n$, respectively. The classical algorithm based on the recursive computation of (4) requires n^2 MULT and n^2 ADD. Thus, the algorithm presented here is computationally superior to the classical algorithm asymptotically. It is worthwhile noting that for real asymmetric and symmetric, and hermitian TMs, the elements of the transformation matrices (and, therefore, the coefficients of the various polynomials computed using the FFT) are real. This fact may be incorporated into the FFT algorithm to reduce the value of the constants α and β .

A comparison of the algorithm described above (let us call it A_1) to the $O(n \log_2^2 n)$ complexity algorithm described in [1] (let us call it A_2), is in order here. A_1 is obtained by exploiting the mathematical properties of the three-term recurrence relation, while A_2 is obtained by exploiting the structure of the TM and its submatrices. The derivation of A_2 is considerably more complicated than A_1 . Also, A_1 is applicable to the real asymmetric and symmetric, and hermitian TMs, while A_2 is applicable to symmetric TMs only. Finally, A_1 and A_2 require approximately the same number of computations (the number of arithmetic operations for A_1 as stated in [1] is twice the actual values).

We end this section by stating that the algorithm described here may be employed to compute the three-term recurrence relations in $O(n \log_2^2 n)$ computations, arising in a number of other mathematical and engineering situations.

III. CONCLUSIONS

A new algorithm is described for computing the characteristic polynomial of a tridiagonal matrix. This algorithm is based on the application of divide-and-conquer technique to the evaluation of a three-term recurrence relation. It requires $O(n \log_2^2 n)$ arithmetic operations as compared to the classical algorithm that requires $O(n^2)$ arithmetic operations.

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Realization of Multiplierless Coherent Detection Scheme in Digital Domain

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In this letter, a digital NTSC decoder which adopts the multiplierless coherent detection scheme has been proposed for the feasibility of digital composite/component interface. The proposed scheme which has been realized in real-time shows a good performance.

I. INTRODUCTION

Since the late 1970s, there has been much research in processing the composite video signal in digital domain. The techniques for separating a composite video signal into digital component signals have been found in such applications as predictive coding [1], digital TV system [2], etc. It is well known that the composite color video signal consists of the luminance and chrominance components in the same frequency band. The chrominance is a multiplexed signal which results from the quadrature-amplitude modulation. An experimental digital NTSC decoder was developed for the component-coded VTR system [3]. Recently, video signal processors have been reported for the digital color decoding and encoding [4]. But they all used digital multipliers for color demodulation. In this letter we propose a color detection scheme which detects the color components without using digital multipliers.

II. PRINCIPLES AND SYSTEM

The color signals $R-Y$ and $B-Y$ are modulated by the subcarrier of 3.579545 MHz and orthogonal in phase diagram [5], where the color burst leads the $R-Y$ in phase by 90° and the $R-Y$ leads the $B-Y$ by 90° .

In analog representation a chrominance signal is expressed as

$$C = (R - Y) \cos 2\pi f_{sc} t + (B - Y) \sin 2\pi f_{sc} t \quad (1)$$

where $R-Y$ and $B-Y$ denote the color difference signals. R and B denote red and blue components, respectively, and f_{sc} is the frequency of subcarrier (3.579545 MHz).

To detect color components $R-Y$ and $B-Y$, (1) should be multiplied respectively by $\cos 2\pi f_{sc} t$ and $\sin 2\pi f_{sc} t$ and filtered by low-pass filters. In digital domain, the same procedure is also required to detect the digital color components. This implies that high-speed digital multipliers will be required which are very expensive and complicated to implement.

From (1), $\cos 2\pi f_{sc} t$ and $\sin 2\pi f_{sc} t$, used as synchronous signals, which are phase-locked with the color burst signal, can be sampled

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and replaced by the $4f_{sc}$ clock signal. For the $4f_{sc}$ sampling rate, the sampled $\cos 2\pi f_{sc}t$ and $\sin 2\pi f_{sc}t$ signals become $\cos(n\pi/2)$ and $\sin(n\pi/2)$ in digital domain. As the integer n increases, three values 1.0, 0.0, and -1.0 alternate between two sequences.

We can use these sampled signals as control clock for detecting two color components, $R-Y$ and $B-Y$, to avoid the multiplication process. When the control logic $c(n)$ for the scheme as shown in Fig. 1 is "1", the input chrominance signal from the digital bandpass filter is passed and for the logic "0", the input is stopped. In the other case, the input is passed and complemented for the arithmetical operation. This multiplierless digital color detection scheme can be implemented as shown in Fig. 1.

Fig. 1. Proposed digital color detector.

When the control clock is phase-locked with the color burst, $R-Y$ signal lagged by 90° can be detected and $B-Y$ signal also detected by one sample delay z^{-1} .

III. RESULTS

A digital chrominance signal is obtained from the output of the digital bandpass filter (BPF) whose input is an A/D-converted NTSC video signal sampled at $4f_{sc}$ rate. The composite video signal (top) and the band pass filtered chrominance signal (bottom) are shown in Fig. 2(a).

Detected color components $R-Y$ (top) and $B-Y$ (bottom) are shown in Fig. 2(b).

Fig. 2. (a) Input/Output signals of BPF. (b) Detected color components.

IV. CONCLUSION

The quadrature-multiplexed color components $R-Y$ and $B-Y$ modulated by the subcarrier signal have been easily demodulated by the phase-locked $4f_{sc}$ clock signal without resorting to digital multipliers. We have confirmed that the detection process can be realized in real-time via the hardware implementation.

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Comments on "A New Approach to Recursive Fourier Transform"

MICHAEL UNSER

The above letter¹ calls for two comments; the first relating to the newness of some of the results and the second relating to the practicality of the proposed algorithms. First of all, the recursive structure of the running Fourier transform has been investigated by a number of authors [1]-[6], none of whom is quoted by Amin. Furthermore, the main idea behind the generalization presented in the third section stems from the properties of a yet more general class of features that can be computed using the same recursive structure. This result is expressed by the following theorem to be found in [6].

Theorem: A feature $g(n)$, being a function of the sample values $x_n, \dots, x_{n+N-1}$, satisfies the first order recursion condition

$$g(n) = w \cdot g(n-1) + w_+ \cdot G(x_{n+N}) + w_- \cdot G(x_{n-1}) \quad (1)$$

where w, w_+ and w_- are complex values and $G(\cdot)$ is an arbitrary function, iff

$$g(n) = \sum_{m=0}^{N-1} w^{-m} \cdot G(x_{n+m}) \quad \text{and} \quad \begin{cases} w_+ = w^{-N+1} \\ w_- = -w. \end{cases} \quad (2)$$

Examples of quantities sharing this property are the Fourier coefficient $F(n, \omega)$, the local mean value, the q th order moment, and the z transform (value $z = z_0$) (cf. [6], table 1). In particular, the exponentially weighted Fourier coefficient $F(n, \omega, \gamma)$ described by Amin is the z -transform evaluated at $z_0 = \gamma e^{-j\omega}$.

When discussing the issue of computational complexity, the author does not take into account the fact that the use of a windowing function produces running Fourier coefficients that are bandlimited and that there is no major loss of information when $F(n, \omega)$ is sub-sampled at a rate of $N/2$, which can result in a substantial saving in the number of operations when using a nonrecursive algorithm.

The author also considers the general form of a weighted running Fourier coefficient and suggests expressing the weighting function as a sum of $2M+1$ geometric series and treating each term separately. There are two major drawbacks with this approach. First, there is generally no guarantee of the existence of such a decomposition. Second, as stated by the author, this method turns out to be quite impractical for large values of M .

When dealing with an arbitrary weighting function, there is an alternative and generally simpler approach which evaluates a given

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¹M. G. Amin, *Proc. IEEE*, vol. 75, no. 11, pp. 1537-1538, Nov. 1987.

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