Potential Uses of the Differential Propagation Phase Constant to Estimate Raindrop and Hailstone Size Distributions

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Abstract—One of the Doppler radars operated by the National Severe Storms Laboratory has a new capability to measure the difference between propagation phase constants at horizontal and vertical polarization. This study examines the uses of this parameter K_{DP} in addition to the reflectivity factor Z_H and the differential reflectivity Z_{DR} , to obtain new information about rain and hail. It is shown from theory and experiments that a third parameter of the drop size distribution, obtained from K_{DP} , may be used to support Z_{DR} measurements, and/or to point out mixed phase hydrometeors. Quantitative information on hail size distribution may be obtained for small-size hailstones in cases when their major axes are nearly vertically aligned, giving rise to negative Z_{DR} and K_{DP} values.

Keywords—Differential propagation phase, drop size distribution, hail size distribution, remote sensing of hail.

I. INTRODUCTION

DOPPLER RADAR capable of transmitting and receiving horizontally and vertically polarized fields measures the reflectivity factors Z_H and Z_V at two orthogonal polarizations. Their ratio, the differential reflectivity Z_{DR} , is related to shape and degree of common orientation of the precipitating hydrometeors. Raindrops are oblate and form a highly oriented medium, in which each drop's shape is uniquely related to size; therefore, with two reflectivity factors, both parameters of a two-parameter drop size distribution model may be calculated, and used for a better rain rate estimation [1], [2]. Also, due to the difference in shape and falling behavior between raindrops and hailstones, considerable success in hail detection has been documented [3]. This was achieved by noting that Z_{DR} values in hail regimes are low, close to zero, while the reflectivity is relatively high. Additional information on rain and hail may be determined from measurements of the propagation phase constants at horizontal and vertical polarizations. The differential propagation phase constant K_{DP} is an independent quantity which, like Z_{DR} , is a measure of the hydrometeors' shape and orientation. Although there have been no studies concerning the behavior of K_{DP} through hail regions, it has been shown that

the rainfall rate is correlated with the differential phase constant [4], or alternately, that K_{DP} may be related to the liquid water content and the mass weighted mean axis ratio [5].

Experimental study of K_{DP} properties in rainfall and hail regimes is now possible with the National Severe Storms Laboratory's (NSSL) dual-polarized Doppler radar. Analysis of the statistical error in K_{DP} estimates has been done by Sachidananda and Zrnic' [6], who also showed theoretically that the direct relationship between differential propagation phase constant K_{DP} and the rain rate R was less sensitive to drop size distribution (DSD) variations than the reflectivity factor versus R relationship. They suggested that K_{DP} could replace Z in rain rate estimation. Analysis of some data sets showed [7] that K_{DP} measurements were generally consistent with measurements of reflectivity Z_H and differential reflectivity Z_{DR} . However, the expected accuracy of inferred rain rate was not achieved at low rain rates, presumably because of signal contamination through antenna sidelobes [7]. We show here that the observed scatter of K_{DP} at medium and higher reflectivities is likely due to physical causes. Part of the error in rain rate estimation is from mixed-phase particles that would be interpreted as raindrops, therefore, improvement is possible by identifying these.

It is suggested that a third measured parameter K_{DP} may be used to solve for a third parameter of a three parameter drop size distribution (DSD) model, which may lead to a better rain rate estimation, whereas its dependency on the orientation of the hydrometeors may lead to better hail discrimination. The purpose of this study then is to examine how K_{DP} measurements may add information about rain characteristics and hail identification, when used in conjunction with reflectivity and differential reflectivity. Theoretical considerations are based on the assumption of Rayleigh scattering at a wavelength of 10 cm, and radar data of rain and hail storms are analyzed and compared with theory.

II. COMPUTATION OF POLARIZATION PARAMETERS

The phase of the backscattered electric field depends on the hydrometeors' size, shape, and quantity, their distance from the radar, and the polarization of the signals. Differential phase shift is experienced in cases of non-

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spherical particles with a preferred orientation. Based on Van de Hulst's method [8], the complex phase constant at vertical or horizontal polarization may be calculated, assuming a particular shape and size distribution of the hydrometeors. It is assumed that the hydrometeors may be modeled as oblate spheroids. This assumption has been supported by observations of raindrops, and to a lesser extent, is reasonable for hailstones [9].

The horizontal and vertical complex propagation constants of the medium $K_{H,V}$ may be expressed as [10]

$$K_{H,V} = k_0 \int_0^{D_m} Q_{H,V}(e, m) D^3 N(D) dD \quad (1)$$

where D is the diameter of a sphere with volume equal to that of the spheroid, a and b are the semi-minor and semi-major axes, $N(D)$ is the drop size distribution, D_m is the maximum diameter of scatterers, and k_0 is the free space propagation constant. The eccentricity e is a function of the axis ratio, a/b , $e^2 = 1 - a^2/b^2$. It is further assumed that the spheroidal raindrops have zero canting angle, whereas spheroidal hailstones are canted at 90° (i.e., their minor axis is horizontally oriented). With the assumption of Rayleigh scattering, $Q_{H,V}$ are functions of the eccentricity e and refractivity m of the hydrometeors as follows [10]:

$$Q_{H,V} = \frac{\pi(m^2 - 1)}{12[1 + (m^2 - 1)A_{H,V}]}$$

where

$$A_H = \frac{1 - e^2}{2e^2} \left\{ (1 - e^2)^{-1/2} e^{-1} \cdot \tan^{-1} [e/(1 - e^2)^{1/2}] - 1 \right\}$$

$$A_V = 1 - 2A_H.$$

The two-way differential propagation phase constant K_{DP} is defined as the real part of the difference between the horizontal and vertical complex propagation constants [10]

$$K_{DP} = \text{Re}(K_H - K_V) (360/\pi) (\text{deg m}^{-1}). \quad (2)$$

The reflectivity factors at horizontal and vertical polarizations Z_H and Z_V are defined by [11]

$$Z_{H,V} (\text{mm}^6 \text{m}^{-3}) = \frac{1}{|K|^2} \int_0^{D_m} S_{H,V}(e, m) D^6 N(D) dD \quad (3)$$

where $S_{H,V}$ are functions of the eccentricity and refractivity of the drops or the hailstones

$$S_{H,V} = \frac{16 \pi^2 |m^2 - 1|^2}{9 [4\pi + (m^2 - 1)P_{H,V}]^2}$$

where

$$P_V = (4\pi/e^2) \left\{ 1 - [(1 - e^2)/e^2]^{1/2} \sin^{-1} e \right\}$$

$$P_H = (4\pi - P_V)/2; |K|^2 = |m^2 - 1|/|m^2 + 2|.$$

The differential reflectivity Z_{DR} (in dB) between these two reflectivity factors for horizontal and vertical polarizations is [1]

$$Z_{DR} = 10 \log_{10} (Z_H/Z_V). \quad (4)$$

Both K_{DP} and Z_{DR} depend on the anisotropy of the medium, and have positive values in rain. Also, they depend on the integral of drop size distribution, with different weighting functions; the integrand of K_{DP} is a function of D^3 compared to the D^6 dependence of Z_H and Z_V . Furthermore, K_{DP} is obtained from the difference, whereas Z_{DR} is obtained from the ratio of such integrals.

Z_{DR} from time series data is calculated using square law power estimates [12]

$$Z_{DR}(\text{dB}) = 10 \log_{10} \left[\frac{1}{M} \sum_{i=1}^M |H_{2i}|^2 / \frac{1}{M} \sum_{i=1}^M |V_{2i+1}|^2 \right] \quad (5)$$

where H and V are complex voltages corresponding to alternately received electric fields at a horizontal and vertical polarization. In our experiment, the number of samples M was 256. Estimation of K_{DP} is done as follows [6]:

$$K_{DP} = [\phi_{DP}(r_2) - \phi_{DP}(r_1)]/[r_2 - r_1] \quad (6)$$

where $\phi_{DP}(r)$ is the phase shift at a range r and is estimated, for each range, from M samples

$$\phi_{DP} = 0.5 \arg [R_a R_b^*]$$

$$R_a = (2/M) \sum_{i=1}^{M/2} H_{2i}^* V_{2i+1}$$

$$R_b = (2/M) \sum_{i=1}^{M/2} V_{2i+1}^* H_{2i+2}. \quad (7)$$

Note that whereas Z_{DR} is measured within radar resolution volumes, K_{DP} is computed along a path between two spaced resolution volumes. Also, in order to reduce the ambiguity resulting from occasional noisy information, K_{DP} data (6) are averaged in range over 16 consecutive range cells as follows:

$$K_{DP_j} = \left[\sum_{i=j+1}^{j+8} \phi_{DP_i} - \sum_{i=j-7}^j \phi_{DP_i} \right] / [r_{j+8} - r_j]. \quad (8)$$

The separation $r_2 - r_1$ over which K_{DP} is calculated, is matched to the averaging interval, which varies from 1.2–2.4 km.

III. DROP SIZE DISTRIBUTION

For the purpose of interpreting dual-polarization radar measurements, raindrops may be modeled as oblate spheroids, with minor axes vertically oriented. The equations for Z_H , Z_{DR} , and K_{DP} (1–4) include several free parameters, which in the case of raindrops are the DSD parameters only. The refractivity m , which for raindrops undergoes slight changes with temperature, may be assumed constant, and the axis ratio of the drops a/b can be uniquely related to the equivolume diameter D [13], [14].

With these assumptions, relatively simple computations are needed to solve (2) and (4) for the parameters of a two parameter drop size distribution $N(D)$. A distribution frequently used is the exponential distribution $N(D) = N_0 e^{-\Lambda D}$. With only one free parameter Λ , the value of N_0 is chosen as $8000 \text{ mm}^{-1} \text{ m}^{-3}$ for the DSD of Marshall-Palmer [15], and Λ may be computed from the reflectivity factor to obtain the rain rate. But in practice an appropriate $Z - R$ relationship must be chosen for each storm. When both Z_H and Z_{DR} measurements are available, N_0 , which can vary significantly even within a short period of time, may also be calculated [16]. As there is evidence that the number of drops with diameter smaller than 1 mm is depleted, it seems that a gamma distribution of the form $N(D) = N_0 D^\mu e^{-\Lambda D}$, with μ between 2 and 3, would fit observational results better [17]. Variations between empirical relations and theoretical calculations may also be reduced by truncating the DSD at an upper limit [18]. When three independent parameters are measured, there is a possibility of estimating a third parameter of a DSD, either the exponent μ of a gamma distribution, or the maximum diameter of the drops D_m . Alternately, two parameters of the DSD may be examined against the additional information provided by the third measured parameter, perhaps to increase the accuracy of the calculated values or to point out the presence of other types of hydrometeors in the medium.

Calculations of Z_H , Z_{DR} , and K_{DP} , based on (2)–(4), are presented in Figs. 1 and 2 for a radar wavelength of 10 cm. Generally, K_{DP} follows similar trends as Z_{DR} . This is because larger drops are more oblate; hence, horizontal fields (aligned with the major axis) experience more phase shift when high concentration of big drops is present. It should be noted here that the three measurables, Z_H , Z_{DR} , and K_{DP} , are independent. However, whenever the DSD can be characterized by an exponential distribution with two unknowns or a gamma distribution with known μ , the variables Z_H and K_{DP} are no longer independent. In such situations, K_{DP} may be expressed in terms of Z_H as $K_{DP} = \alpha Z_H^\beta N_0^{1-\beta}$, where α and β are functions of D_m and Λ . In the case of exponential DSD with $D_m = 8 \text{ mm}$, these values are: $\alpha = 4 \times 10^{-5}$ and $\beta = 0.86$, and when $\mu = 3$ they are 3×10^{-5} and 0.88, respectively; K_{DP} is in deg km^{-1} , Z_H in dBZ, and N_0 in $\text{mm}^{-1} \text{ m}^{-3}$. Therefore, measurements of Z_H , Z_{DR} , and K_{DP} could play a significant role, either when the DSD cannot be characterized by less than three parameters, or when the precipitation medium contains mixed phase hydrometeors. Variations of K_{DP} with D_m (Fig. 1) are more pronounced when $Z_{DR} > 3 \text{ dB}$ and μ is small, indicating the presence of big drops. Also, the sensitivity of K_{DP} to variations in μ is larger than its sensitivity to variations in D_m in the lower ranges of Z_{DR} , which is generally associated with small drops. If both Z_{DR} and Z_H are perfectly measured, the accuracy of K_{DP} measurements required to distinguish between $D_m = 5$ and $D_m = 10 \text{ mm}$, or between $\mu = 0$ (exponential) and $\mu = 3$, is about 5–10 percent. Statistical analysis of K_{DP} shows [6] that the standard error of K_{DP}

Fig. 1. Relationships between K_{DP} , Z_{DR} , Z_H , and DSD parameters for exponential and gamma distributions. Radar wavelength is 10 cm, and in our convention D^μ is normalized to mm^μ to have dimensions of N_0 invariant with μ .

Fig. 2. Theoretical curves of K_{DP} (deg km^{-1}), R (mm h^{-1}), and N_0 ($\text{m}^{-3} \text{mm}^{-1}$) in a Z_H - Z_{DR} plane, for exponential DSD with $D_m = 8$ mm. Rain boundaries, separating liquid from frozen hydrometeors, are 1) from [20] and 2) from [21].

measurements is of the order of 0.5 deg km^{-1} . Therefore, if such an error is practically achievable, estimation of a third parameter of the DSD may be possible. If both N_0 and Λ are accurately known, the addition of a third parameter would reduce the error in R from 20–15 percent, as the contribution of variations in μ or D_m to the rainfall rate is secondary to that due to changes in Λ and N_0 [19]. However, one may use K_{DP} to increase confidence in the two parameters of the distribution, as K_{DP} may replace Z_H in the Z_H - Z_{DR} method to achieve additional, independent evaluation of N_0 , but with reduced spatial resolution.

In Fig. 2, curves of iso- K_{DP} values, for exponential DSD with $D_m = 8$ mm, are presented in a Z_H - Z_{DR} plane. Also presented are N_0 and rain rate curves. High Z_H generally indicates relatively high rainfall rate, associated with large drops (high Z_{DR} values), or a high number of medium-size drops (medium Z_{DR} , high N_0). Although different types of rain may result in various Z_H - Z_{DR} combinations, not all Z_H - Z_{DR} combinations are physically possible; data from rain have a preferential region in a Z_H - Z_{DR} plane. This has been confirmed from both radar measurements [20] and Z_H - Z_{DR} computations based on disdrometer data [21]. The boundaries established by these two studies are in fairly good agreement; both have an upper Z_H limit of 60 dBZ, but, as seen in Fig. 2, there are differences at lower values of Z_H and Z_{DR} . Z_H - Z_{DR} pairs on the boundary correspond to very high, unrealistic water contents that would seldom be present in nature. For instance, the calculated liquid water content for $Z_H = 60$ dBZ and $Z_{DR} = 1.3$ dB is 100 g/kg . $Z_{DR} = 2$ dB would

reduce the water content to a more reasonable, but still high, value of 50 g/kg . Lower Z_H corresponds to lower water content for similar Z_{DR} values; for instance, $Z_H = 50$ dBZ and $Z_{DR} = 1.3$ dB are produced by exponentially distributed raindrops with water content of 10 g/kg . High Z_H and low Z_{DR} are therefore signals for non-rain hydrometeors, presumably hail [21]. K_{DP} within the rain boundary in Fig. 2 is between 0 and about 15 deg km^{-1} , and for pure rain must be consistent with Z_{DR} and Z_H . When substantial inconsistencies among K_{DP} , Z_{DR} , and Z_H occur, the ensemble of hydrometeors is less likely to be pure rain. This is a basis for possible discrimination among rain and other hydrometeors, using the three measured parameters.

IV. HAIL SIZE DISTRIBUTION

Remote discrimination between hail and rain using dual-polarization measurements is based on the difference in shape, orientation, and size distribution between raindrops and hailstones. As noted previously, typical Z_{DR} values in regions of rainfall range from 0.5–4 dB. The larger Z_{DR} generally corresponds to larger Z_H . However, radar measurements indicate that when hail is likely, Z_H is high and Z_{DR} is low [3]. This has led Aydin *et al.* [21] to define a function of Z_H and Z_{DR} that they call "hail signal," H_{DR} . H_{DR} is the difference between the reflectivity Z_H (for a given Z_{DR}) and the reflectivity for the same Z_{DR} at a boundary between rain and hail (see Fig. 2). Dual-polarization measurements indicate Z_{DR} values from hail are close to zero [3]. This is characteristic of relatively isotropic media. Zero Z_{DR} , together with high Z_H , are due to relatively symmetric or irregular but tumbling hailstones, with no mean preferred orientation. At present there is no conclusive experimental evidence regarding either the orientation of free-falling hailstones, or their mean shape; both may have large variability. Theory and measurements of falling hailstones suggest various orientations, one of which is random, due to tumbling [22], the other of which has aligned spheroids rotating about their minor axes, which is gyrating about the horizontal [23].

If hailstones could be modeled as oblate spheroids that fall with major axis vertically oriented (canting angle 90°), the scattering characteristics would lead to negative Z_{DR} and K_{DP} ((1) and (5)). In that case, K_{DP} measurements would provide a means of obtaining a two-parameter hailstone size distribution, which shall be discussed next. Note that this applies to Rayleigh scattering, which at S-band frequencies occurs when hailstones have relatively small diameters ($< 1 \text{ cm}$). For large wet hail, Mie scattering may result in both positive and negative Z_{DR} values [24]. Another assumption is that the axis ratio is independent of the size. This has been measured by Barge and Isaac [25] in Alberta, and Matson and Huggins [9] in Colorado, who report axis ratios with a narrow distribution between 0.73 and 0.85. Knight [26] recently observed similarity of the shapes of small hailstones in Oklahoma and Alberta. Both exhibit axis ratios that have larger

variability than previously reported; b/a ranges from 0.95 (for 1- to 5-mm hail), to 0.7 (for 21- to 25-mm hail). The difference between experimental results may be attributed to natural variability of hailstones, and to some difference in experimental methods.

In our calculations we assume Rayleigh scattering, and oblate hailstones with canting angle of 90° and eccentricity independent of size. Then Z_{DR} becomes a function of axis ratio and refractive index only (3)–(5)

$$Z_{DR} = 10 \log_{10}(S_H/S_V). \quad (9)$$

It is possible to obtain a rough estimate of a two-parameter hail size distribution, as follows. From Z_{DR} one can obtain the axis ratio b/a for a known m , or alternately, m for known b/a . From Equations (1), (2), and (4) one finds that Z_H and K_{DP} are related by

$$\begin{aligned} Z_H [\text{mm}^3]/K_{DP} [\text{deg m}^{-1}] \\ = F_1(b/a, m)/F_2(\Lambda, D_m, \mu) \end{aligned} \quad (10)$$

where

$$F_1 = \pi S_H / [360 k_0 \text{Re}(Q_H - Q_V)] \text{ (m deg}^{-1}\text{)}$$

and

$$F_2 = \int_0^{D_m} D^3 N(D) dD \int_0^{D_m} D^6 N(D) dD.$$

These relations are presented in Fig. 3 for dry, and 10-percent wet hail of density 0.5 g/kg. The refractivity m in S_H was set to a value for liquid water, because that is used for automatic calculations of Z_H in a calibrated radar.

Although the Rayleigh–Gunn theory of scattering should be used for sizes less than 1 cm, we also show curves with D_m of 2 and 3 cm. For sizes up to 3 cm, errors in Z_H , Z_{DR} , and K_{DP} of individual scatterers are less than 1.5 dB, 0.4 dB, and 3.4° ; when sizes are integrated over drop size distributions the average errors are further reduced. Thus, presented curves are not strictly exact, but serve to illustrate general trends and provide quantitative results that have less uncertainty than radar measurements.

The hail size parameters are then estimated as follows: the axis ratio, and therefore F_1 , are obtained from Z_{DR} measurements, then F_2 may be calculated from Z_H and K_{DP} measurements to obtain Λ , using exponential or gamma DSD for an assumed, relatively small D_m . Once Λ is known, N_0 may be calculated from measurements of either Z_H (3) or K_{DP} (1). This method may be used in the case of small hail, when both Z_{DR} and K_{DP} measurements yield negative values. The assumption that axis ratio is independent of size is not necessary in order to solve for the hail size parameters. In fact, if the dependence of the axis ratio on diameter can be calibrated from measurements, and it is found to be similar for certain storm types, then K_{DP} and Z_{DR} may be used to solve for an additional parameter, like the liquid-ice ratio of hailstones or their maximum diameter.

Fig. 3. Dependence of functions F_1 and F_2 on hail parameters: (a) F_1 versus the axis ratio b/a for dry and 10-percent wet hail of density 0.5 g/kg. (b) F_2 versus Λ and D_m as parameters for exponential and gamma distributions.

V. RADAR MEASUREMENTS

The data presented in this study are from rain showers on June 4 and 10, 1986, and from a hail storm on June 2, 1985. They were collected with a radar having a 10-cm wavelength and the capability of switching polarization from pulse-to-pulse. Simultaneous collection of time series data from 16 consecutive range locations was possible and the spacing could be adjusted in powers of 2, from 1 to 16 μs . 512 alternately polarized samples from each range location were processed, as explained in Section II. Theoretically computed standard error of K_{DP} estimates is about 0.6 deg km^{-1} when the resolution is 1.2 km and 0.3 deg km^{-1} when the resolution is 2.4 km [6]. There are no system induced biases in the K_{DP} estimates, but in regions of weak reflectivities near strong cores, contamination by

Fig. 4. Scattergrams for a rainfall on June 10, 1986, from the elevation of 1° , with a curve for $N_0 = 10^5 \text{ mm}^{-1} \text{ m}^{-3}$: (a) Z_H - Z_{DR} and (b) K_{DP} - Z_H , and the rain boundaries, marked (1) from [20] and (2) from [21].

echoes through sidelobes is possible. This was not the case in the analyzed data. Standard errors of reflectivity estimates are better than 1 dB, and biases are kept below 1 dB. Standard errors of differential reflectivity are less than 0.1 dB, and there were no systematic biases [27].

In Fig. 4 results of the radar measurements on June 10, 1986 are shown. These were obtained from a sector of 30 degrees, and between the ranges of 37 and 63 km, at an elevation of 1° . Polarization parameters were calculated from time series data that were spaced in range by 150 m. For comparison, Z_H - Z_{DR} boundaries for rain [20], [21] are marked, along with the calculated curve for $N_0 = 10^5 \text{ mm}^{-1} \text{ m}^{-3}$ of an exponential DSD with $D_m = 8 \text{ mm}$. Scattergrams show that the majority of Z_H - Z_{DR} data points (all measured at 1° elevation) lie in the range considered typical for rain. Some data points show Z_{DR} close to zero at relatively high reflectivity, and hence outside the rain boundary, presumably indicating the presence of partially melted hailstones, or drops with ice cores (the freezing level was at a 4-km height). Z_H - Z_{DR} points indicate large variability in DSD (see also Fig. 2), with N_0 from 10^3 to $10^6 \text{ mm}^{-1} \text{ m}^{-3}$, and Λ larger than 2 mm^{-1} (Z_{DR} values are relatively small, generally below 2.5 dB). There is also a corresponding high variability in rainfall rate, from less than 1 mm h^{-1} (drizzles, where reflectivity is lower than 25 dBZ) to 200–300 mm h^{-1} and over, corresponding to high reflectivity close to the rain boundary. K_{DP} data (Fig. 4(b)) are correlated with Z_H as expected for raindrops. There are, however, many points with negative K_{DP} values, which may be partially explained in terms of statistical uncertainty of K_{DP} measurements. On the other hand, K_{DP} values corresponding to higher reflectivities are consistently lower than expected. For instance, for $Z_H = 55 \text{ dBZ}$ and $Z_{DR} = 1.5$ to 2 dB, K_{DP} is

expected to be over 10 deg km^{-1} . The measured values, 7 to 8 deg km^{-1} , would fit the theoretical ones (for the same Z_H) if Z_{DR} were 3.5 to 4 dB. Assuming an N_0 of 10^4 to $10^5 \text{ m}^{-3} \text{ mm}^{-1}$, this conclusion is nearly independent of D_m (see Fig. 1(c)) and also holds for the examined gamma distributions (Fig. 1(d)). Perhaps some less realistic DSD would fit these data, or a simpler explanation is possible? A source for relatively low K_{DP} values could be a mixed population of rain and ice pellets.

To illustrate this point, data from another rain case (June 4, 1986) are presented in Fig. 5. These data were obtained from a stationary beam above Norman and are from ranges 38–40 km and at an elevation of 0.7° . At that elevation, reflectivity is relatively high, with only a few points below 45 dBZ. Only rain was observed on the ground. About 70 percent of the Z_H , Z_{DR} data are within the rain boundary and have a rain rate in excess of 100 mm h^{-1} . As in the previous case, there are no negative Z_{DR} values in this high reflectivity region, although some are rather low. K_{DP} data are positive, as is expected for rain, with only a very few scattered data points below zero. This may then represent pure rainfall; in effect, no observations of ice phase hydrometeors have been reported. A more careful examination of K_{DP} data shows that, whereas the mean N_0 values based on Z_H - Z_{DR} data and exponential distribution are about $10^5 \text{ mm}^{-1} \text{ m}^{-3}$, from Z_H - K_{DP} measurements, one gets N_0 closer to $10^4 \text{ mm}^{-1} \text{ m}^{-3}$. Most Z_H - K_{DP} data points (those that correspond to Z_H - Z_{DR} data within the rain boundary, and the others outside) are within the range of $K_{DP} = 1$ to 4 deg km^{-1} , for $Z_H = 45$ to 55 dBZ . However, Z_H - K_{DP} data points corresponding to Z_H - Z_{DR} within the rain boundary (below 60 dBZ) include in addition, some higher K_{DP} values. This group has more data points of medium reflectivity values than the other one, which cor-

Fig. 5. Scattergrams for a storm on June 4, 1986, from the elevation of 0.7° : (a) Z_H - Z_{DR} data are presented with curves of equal N_0 (in $\text{mm}^{-1} \text{m}^{-3}$) and the rain boundaries (1) [20] and (2) [21]. (b) K_{DP} - Z_H data subdivided into groups within the rain boundary and outside ($H_{DR} > 0$), with theoretical curves for two N_0 's. (c) K_{DP} - Z_{DR} data for $H_{DR} < 0$. Circles indicate data with reflectivities between 59 and 61 dBZ, whereas crosses are for data with Z_H between 49 and 51 dBZ. Theoretical graphs for exponential DSD ($D_m = 8$ mm) at reflectivities of 45, 50, and 55 dBZ, exponential DSD ($D_m = 4$ mm) at 50 dBZ, and gamma DSD ($D_m = 8$ mm and $\mu = 3$), at 55 dBZ are superposed.

responds to Z_H - Z_{DR} data outside the rain regime. These data points are further examined in Fig. 5(c) via a K_{DP} - Z_{DR} scattergram. Data are grouped into two categories. One set contains all K_{DP} - Z_{DR} pairs with Z_H between 49 and 51 dBZ; the other set has points with Z_H between 59 and 61 dBZ; both are within the rain boundary. The correlation of K_{DP} with Z_H and with Z_{DR} may clearly be seen. A comparison with theoretical K_{DP} - Z_{DR} values for an exponential DSD with $D_m = 8$ mm shows that such distribution is possible for data corresponding to $Z_H = 50$ dBZ, although the theoretical Z_H values seem to be biased high by 2 to 3 dBZ, compared to the mean measured values. A DSD with $D_m = 4$ mm yields a better fit to the measured data. Thus, the addition of K_{DP} measurements points toward a narrower DSD, which is consistent with other variables, but cannot be independently verified. The data with $Z_H = 60$ dBZ (and others above 55 dBZ, not included in Fig. 5(c)), show K_{DP} lower by 5 deg km^{-1} , or Z_{DR} lower by 2 dB than expected for rain (see Fig. 2). If gamma distribution is considered, K_{DP} data would be lower by 0.5–1 deg km^{-1} only. The consequence is that the regions of higher reflectivity values, though within the rain boundary according to Z_H - Z_{DR} measurements, may not represent rain of either exponential or gamma distribution. Rain may be mixed with other hydrometeors, whose scattering properties are more isotropic than those of rain alone.

Results of the measurements in rainfall regimes are therefore consistent with theoretical calculations, pointing out the possibility that some of the rain, mainly in the range of high reflectivity, could be mixed with ice phase hydrometeors. Data in these cases show positive Z_{DR} . This is not so in the case of June 2, 1985, as is shown in Figs. 6 and 7, where some high reflectivity data have substantial negative Z_{DR} from -1 to -2.2 dB at a height of 0.2 km. Hail signal values, based on criteria of Aydin *et al.* [21], are over 30 dB (Fig. 7). Hail was verified on the ground by the University of Oklahoma observers, who saw hailstones 2–3 cm in diameter and a mixture of hail and rain in several locations. K_{DP} data also show some negative values, generally -0.5 to -3 deg km^{-1} , associated with negative Z_{DR} and high H_{DR} . There are also some data points of negative Z_{DR} , with positive K_{DP} , and vice versa. Positive Z_{DR} values are less than 1.7 dB. This is lower than expected for rain in regions of high reflectivity (above 50 dBZ). When related to Z_H (Fig. 6(a)), they imply N_0 of over $10^6 \text{ mm}^{-1} \text{m}^{-3}$. On the other hand, K_{DP} - Z_H pairs require rather low N_0 of only $100 \text{ mm}^{-1} \text{m}^{-3}$, if interpreted as originating from rain. Therefore, it appears that the dominant hydrometeors were in the form of hail.

This case is rather exceptional with regard to Z_{DR} data reported by other investigators, which are generally close to zero for hail [3]. The largest negative Z_{DR} value reported so far was an isolated point of -1.4 dB [28]. The measured negative Z_{DR} and K_{DP} of the June 2 storm could be attributed to scattering from small hail falling, on the average, with major axis vertically oriented. Another pos-

Fig. 6. Scattergrams from a hail storm on June 2, 1985; the elevation is 0.2° ; (a) Z_H - Z_{DR} with the rain boundaries (1) [20] and (2) [21] and curves of constant N_0 values. (b) K_{DP} - Z_H with two curves of constant N_0 superposed.

Fig. 7. Cross sections of Z_H , Z_{DR} , and K_{DP} , and calculated H_{DR} values following [21], of a hail storm on June 2, 1985. Beam center is at 0.2° in elevation, but the lower part of the beam is blocked. Azimuthal limits of these sectors are 235° - 250° , the range from the radar is 81-94 km, and only data that have reflectivities larger than 30 dBZ are displayed.

sibility could be that contributing hailstones are, on the average, horizontally oriented, but the dominant sizes are such that Mie scattering produces stronger reflection at vertical polarization [24], [29]. Because of simplicity we consider further only the first possibility. In that case, one may apply the method just presented to get some quantitative information on the two parameter hail size distribution. The June 2 hail storm had high Z_H values of 60–65 dBZ. Z_{DR} of -1 to -2 dB indicate mean axis ratio of 0.77–0.88 for wet hail, similar to observations in a Colorado storm [9]. Because high reflectivity and negative Z_{DR} extend through a depth of 10 km, contamination via antenna sidelobes can be excluded. A nearby ridge that partially blocks the beam also eliminates all ground clutter from the range beyond 50 km. Calculated Λ ranges from 0.05 to 1.1 mm^{-1} , depending on K_{DP} (which ranges from -0.5 to -3 deg km^{-1}), the distribution (exponential or gamma), and the maximum assumed size, 10 to 30 mm (see Fig. 3). The estimated N_0 are typically between 2 and $300 \text{ mm}^{-1} \text{ m}^{-3}$, although lower values, 0.01, for higher K_{DP} , and higher, 600, for dry hail, were also calculated. The classic measurements by Douglas [30] had $\Lambda = 0.3 \text{ mm}^{-1}$ and $N_0 = 3.1 \text{ mm}^{-1} \text{ m}^{-3}$, for hail size over 5 cm. Other studies [31] suggest $\Lambda = 0.4 \text{ mm}^{-1}$ and $N_0 = 12 \text{ mm}^{-1} \text{ m}^{-3}$. Hail study of an Oklahoma storm [32] indicates gamma distributions with $\Lambda = 0.05\text{--}0.1 \text{ mm}^{-1}$ and $N_0 = 0.005\text{--}0.01 \text{ mm}^{-1} \text{ m}^{-3}$. A wider range of Λ and N_0 has been measured for wet hail [33]: $\Lambda = 0.1\text{--}1.5 \text{ mm}^{-1}$ and $N_0 = 0.01\text{--}3000 \text{ mm}^{-1} \text{ m}^{-3}$. Although our deduced values are within the range of previous measurements, it is premature to draw positive inferences. Verification of the falling characteristics of the hail, and combined radar and *in situ* measurements, are needed before a conclusion can be made on the usefulness of this method for hail size estimation.

VI. CONCLUSIONS

Measurements of the differential phase constant K_{DP} are examined in conjunction with the reflectivity and differential reflectivity with the aim of obtaining additional information on rain and hail characteristics. Although more effort is needed to improve K_{DP} measurements, this parameter adds information that is unavailable from dual-polarization measurements of reflectivity alone. The range resolution of K_{DP} estimates is lower than the inherent resolution possible, but it is compatible with the operational 1-km resolution of reflectivity estimates. It is shown that K_{DP} may be used to support Z_{DR} measurements with regard to the hydrometeor phase, or alternately to detect the presence of hydrometeors, other than raindrops, when Z_H and Z_{DR} data are within the range typical for rain. Calculations based on Rayleigh scattering suggest that with K_{DP} measurements, one could solve for a third parameter of DSD, which may be either D_m , when exponential distribution is assumed, or μ , in the case of a gamma distribution. Assuming that small hailstones fall with their major axes aligned vertically, the use of K_{DP} together with Z_H and Z_{DR} may enable remote quantitative estimation of

their size distributions. We presented data from one hailstorm in which a predominant orientation of hailstones resulted in negative Z_{DR} and K_{DP} . Observed negative Z_{DR} (minimum -2.2 dB) and K_{DP} (minimum -3 deg km^{-1}) in regions of high reflectivity (over 55 dBZ) suggest that these hailstones in the freefall could, on the average, be vertically oriented. However, horizontal orientation (like raindrops) of certain hail sizes may, in the Mie regime of scattering, produce similar results [24]. This possibility needs further exploration. The effectiveness of K_{DP} measurements increases with increasing reflectivity, as the standard error of the measurements is constant (less than 0.6 deg km^{-1}) and K_{DP} increases exponentially with Z_H . K_{DP} may principally be used for rain rate estimation when reflectivity is larger than 40 dBZ. Furthermore, our study shows that the K_{DP} measurements may be used to identify mixed phase regimes, for which Z_H and Z_{DR} are insufficient or ambiguous. Although independent confirmation, using *in situ* measurements, is needed to positively identify the type of precipitation above the ground, polarimetric measurements in storms of the Great Plains imply that hydrometeors of mixed phase are often present aloft.

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