Near-Field Detection of Buried Dielectric Objects

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Abstract—The plane-wave scattering-matrix method is used to compute the response of a detector to a buried dielectric scatterer. The Born approximation is used to derive the scattering matrix for scatterers of small dielectric contrast, but the general theory is not limited to such cases. Specific numerical results are generated for a UHF dipole detector swept over a buried dielectric cube. The maximum response is obtained when the detector is located at the air-earth interface, and the response decays rapidly with detector height. The sweep curves are symmetrical in the horizontal direction and have a null when the detector is directly over the object. An experimental curve for a free-space environment has the same qualitative features.

I. INTRODUCTION

THE analysis of scattering by buried objects [1]–[3] is complicated by the presence of the air-earth interface. The problem becomes even more complicated when the characteristics of the transmitting and receiving antennas are included [4], [5]. The plane-wave scattering-matrix method [6] provides a convenient means of combining the source, receiver, scatterer, and interface effects in a self-consistent formulation [7].

In this paper we use the plane-wave scattering-matrix method to study the detection of buried objects by a detector that is swept over the earth's surface at a low height. Numerical results are obtained for a UHF detector that consists of oppositely directed transmitting dipoles [8] and a single receiving dipole. The buried dielectric target is a homogeneous rectangular box of arbitrary dimensions [9]. The scattering matrix for this particular target is derived by using the Born approximation for a low dielectric contrast.

The organization of this paper is as follows: In Section II we review the plane-wave scattering-matrix theory for the detection application [7] and obtain an approximate expression for the received signal as a function of detector position. In Section III we derive the required plane-wave transmitting and receiving characteristics for the UHF dipole detector. In Section IV we carry-out the scattering-matrix calculations and present sweep curves for the UHF detector for various parameters. An experimental sweep curve is also shown for a free-space environment. Conclusions and recommendations for further work are contained in Section V.

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II. PLANE-WAVE SCATTERING-MATRIX THEORY

The geometry of a detector in air and a buried scatterer is shown in Fig. 1. The air has permittivity ϵ_0 , the earth has permittivity ϵ' , and the permeability is μ_0 everywhere. The detector can consist of either a single antenna or separate transmitting and receiving antennas, and it is located at a height h above the earth. The scatterer is located at a depth d and a horizontal offset P .

The general notation and the formal scattering-matrix solution for the fields and the received signal are contained in [7], and here we will only extract the results that are necessary for our detector application. The column matrices $\hat{a}_1$ and $\hat{b}_1$ contain the coefficients of the upgoing and downgoing plane waves in the region between the detector and the interface. Similarly, the column matrices $\hat{b}'_2$ and $\hat{a}'_2$ contain the coefficients of the upgoing and downgoing plane waves in the region between the scatterer and the air-earth interface.

The receiving and transmitting characteristics of the detector are specified by

$$b_0 = \hat{S}_{01}\hat{a}_1 \quad \text{and} \quad \hat{b}_1 = \hat{S}_{10}a_0 \quad (1)$$

where a_0 and b_0 are coefficients of incoming and outgoing modes at the detector feed, $\hat{S}_{01}$ is a row matrix representing the receiving characteristics of the detector, and $\hat{S}_{10}$ is a column matrix representing the transmitting characteristics of the detector. Here we have assumed that the reflection coefficient at the detector feed and the scattering matrix of the detector are negligible.

The scattering characteristics of the buried object are specified by

$$\hat{b}'_2 = \hat{S}'_{22}\hat{a}'_2 \quad (2)$$

where $\hat{S}'_{22}$ is a square matrix representing the scattering characteristics of the buried object. The results in [7] apply to general scatterers, but here we assume that the scatterer is homogeneous and has complex permittivity ϵ_s which does not depart markedly from ϵ' . For this case the scattering matrix $\hat{S}'_{22}$ has been derived using the Born approximation [9]. This weak-scattering case is applicable to detection of plastic mines in dry soil [10].

The general solution for the received signal b_0 in [7] contains the effects of multiple reflections between the scatterer and the interface. For our case of weak scattering we neglect multiple reflections, and the result in [7] reduces to

$$b_0 = \hat{S}_{01}\hat{R}'_{11}\hat{S}_{10}a_0 + \hat{S}_{01}\hat{T}'_{12}\hat{S}'_{22}\hat{T}'_{21}\hat{S}_{10}a_0. \quad (3)$$

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Fig. 1. Geometry for a detector over a buried dielectric object.

The first term in (3) represents reflection of the air-earth interface and is independent of the buried scatterer. Most detectors are designed to null out this ground-reflected signal either by placing the receiving element at a null of the field [8] or by using a cross-polarized receiving element [11]. Consequently, we will assume that the first term in (3) is zero, and the ratio of the received to the incident signal is

$$b_0/a_0 = \hat{S}_{01} \hat{T}_{12}' \hat{S}_{22}' \hat{T}_{21}' \hat{S}_{10}. \quad (4)$$

The right side of (4) has a simple physical interpretation. $\hat{S}_{10}$ represents transmission from the detector into air, $\hat{T}_{21}'$ represents transmission from air into the earth, $\hat{S}_{22}'$ represents scattering from the buried object, $\hat{T}_{12}'$ represents transmission from the earth into air, and $\hat{S}_{01}$ represents reception at the detector.

The square diagonal matrices $\hat{T}_{21}'$ and $\hat{T}_{12}'$ have diagonal elements of the following form for $\exp(-i\omega t)$ time dependence:

$$T_{21}' = T_{21} e^{i(\gamma h + \gamma' d + K \cdot P)} \quad \text{and} \quad T_{12}' = T_{12} e^{i(\gamma h + \gamma' d - K \cdot P)}. \quad (5)$$

Here T_{21} and T_{12} are Fresnel transmission coefficients, and the superscript indicates translation as given by the exponential factors. The transverse wavenumber K is given by

$$K = k_x e_x + k_y e_y \quad (6)$$

where e_x , e_y , and e_z are unit vectors. The horizontal offset P of the scatterer is given by

$$P = P_x e_x + P_y e_y. \quad (7)$$

The z components of the wavenumbers are given by

$$\gamma = (k^2 - k_x^2 - k_y^2)^{1/2} \quad \text{and} \quad \gamma' = (k'^2 - k_x^2 - k_y^2)^{1/2} \quad (8)$$

where $k^2 = \omega^2 \mu_0 \epsilon_0$ and $k'^2 = \omega^2 \mu_0 \epsilon'$.

Detectors are normally moved at a constant height above the earth. To generate sweep curves (detector response as a function of horizontal position) we need to calculate b_0/a_0 from (4) for varying values of the horizontal offset P . This calculation involves multiplication of large matrices, but fortunately $\hat{S}_{10}$, $\hat{S}_{22}'$, and $\hat{S}_{01}$ are independent of P . Thus only $\hat{T}_{21}'$ and $\hat{T}_{12}'$ need to be recal-

Fig. 2. Geometry for a dipole detector over a buried dielectric box.

culated for each value of P . The numerical evaluation of (4) will be discussed in more detail in Section IV.

III. DIPOLE DETECTOR

A UHF dipole detector of particular interest in detection of buried objects at shallow depths is shown in Fig. 2. The transmitting antenna consists of two x -directed dipoles of length L fed in opposition and separated by a distance s . The x -directed receiving dipole of length L is located at the center of the detector ($y = z = 0$) which is in the null of the transmitting antenna. Thus the detector will respond to inhomogeneities that are asymmetrical with respect to the xz plane, but not to a homogeneous or horizontally stratified earth.

A. Transmitting Antenna

The transmitting antenna has been analyzed previously [8] for a sinusoidal current assumption. It would be possible to solve for the current distribution on a pair of wires over the ground by the method of moments [12], but the classical sinusoidal current approximation [13] is adequate for our detector calculations. A major advantage of the sinusoidal current assumption is that it allows us to obtain an analytical expression for the transmission matrix $\hat{S}_{10}$.

The coefficients of the outgoing plane waves $b_1(m, K)$ are given by [8, eq. (6)]. For linear antennas it is more convenient to normalize the transmission matrix to the input current I_0 rather than a mode coefficient a_0 as indicated in Fig. 1. Thus we can define the elements $S_{10}(m, K)$ of the transmission matrix $\hat{S}_{10}$ as

$$S_{10}(m, K) = b_1(m, K)/I_0 \quad (9)$$

where $b_1(m, K)$ is given by [8, eq. (6)]. In (9), m takes on values of 1 for transverse magnetic (TM) polarization, and values of 2 for transverse electric (TE) polarization. The transverse wavenumbers k_x and k_y in K take on real values from $-\infty$ to ∞ .

B. Receiving Antenna

We assume that the receiving dipole also has a sinusoidal current [13]:

$$I(x) = \frac{\sin \{k[(L/2) - |x|]\}}{\sin(kL/2)}, \quad |x| < L/2. \quad (10)$$

For an incident field E^i , the open-circuit received voltage V_r can be determined from the induced EMF method [13]:

$$V_r = - \int_{-L/2}^{L/2} \mathbf{e}_x \cdot \mathbf{E}^i|_{y=z=0} I(x) dx. \quad (11)$$

To determine the elements $S_{01}(m, \mathbf{K})$ of the receiving matrix $\hat{S}_{01}$, we let the incident field in (11) be either a TM or a TE plane wave. Thus we have

$$\mathbf{e}_x \cdot \mathbf{E}^i|_{y=z=0} = \frac{\mathbf{e}_x \cdot \boldsymbol{\kappa}_m}{2\pi} e^{ik_x x}. \quad (12)$$

The transverse unit vectors $\boldsymbol{\kappa}_m$ are [6]

$$\boldsymbol{\kappa}_1 = (k_x \mathbf{e}_x + k_y \mathbf{e}_y)/K \quad \text{and} \quad \boldsymbol{\kappa}_2 = (k_y \mathbf{e}_x - k_x \mathbf{e}_y)/K \quad (13)$$

where $K = |\mathbf{K}|$. For linear antennas we find it more convenient to define the receiving matrix in terms of the received voltage V_r rather than a mode amplitude. Thus we set $S_{01}(m, \mathbf{K}) = V_r$. If we substitute (10), (12), and (13) into (11) and carry out the x integration, we obtain the final result

$$S_{01} \begin{pmatrix} 1, \mathbf{K} \\ 2 \end{pmatrix} = \frac{-kk_x}{\pi K(k^2 - k_x^2) \sin(kL/2)} \cdot [\cos(k_x L/2) - \cos(kL/2)]. \quad (14)$$

When $k_x^2 = k^2$, (14) is indeterminate and the limiting result is

$$S_{01} \begin{pmatrix} 1, \mathbf{K} \\ 2 \end{pmatrix} = -k_x L / (4\pi K). \quad (15)$$

In (14) and (15), the x subscript on k corresponds to $m = 1$, and the y subscript corresponds to $m = 2$.

IV. NUMERICAL AND EXPERIMENTAL RESULTS

In this section we present numerical results for the geometry shown in Fig. 2. We simulate sweep curves by varying P_y while holding constant the detector height h , the scatterer depth d , and the x offset P_x . The computed quantity is the ratio of the received voltage V_r to the input current I_0 :

$$V_r/I_0 = \hat{S}_{01} \hat{T}_{12}^t \hat{S}_{22}^t \hat{T}_{21}^t \hat{S}_{10}. \quad (16)$$

The result in (16) is the same as (4) except that $\hat{S}_{01}$ and $\hat{S}_{10}$ have been defined in terms of voltage and current as indicated in the previous section. The quantity V_r/I_0 is the mutual impedance between the transmitting antennas and receiving antennas and has units of ohms. It is zero in the symmetrical environment of free space or a homogeneous half-space, but is unbalanced by the presence of any asymmetries.

The buried target is modeled as a rectangular box (parallelepiped) of dimensions $2a \times 2b \times 2c$ and complex dielectric constant ϵ_s . As indicated in Fig. 2, the faces of

the box are parallel to the xy , yz , and xz planes. The scattering matrix $\hat{S}_{22}$ for this target has been derived for the low contrast case [9], and we will not reproduce the result here. The bulk of the computing in evaluation of (16) is the filling of $\hat{S}_{22}$ and the associated matrix multiplications. $\hat{S}_{22}$ is a square ($N \times N$) matrix where N is determined by the sampling in k_x and k_y . If N_k is the number of samples required for k_x (and k_y), then $N = 2N_k^2$ where the 2 factor is due to the two polarizations (TM and TE). This means that $\hat{S}_{22}$ contains $N^2 = 4N_k^4$ elements, which is typically a very large number. Fortunately, $\hat{S}_{01}$ is a row matrix, $\hat{S}_{10}$ is a column matrix, and both matrices contain only $2N_k^2$ elements. Also, both $\hat{T}_{12}^t$ and $\hat{T}_{21}^t$ are diagonal matrices that contain only $2N_k^2$ nonzero elements. A computer program was written to evaluate (16), and the convergence was studied by varying the step size and the maximum values of k_x and k_y .

The magnitude of V_r/I_0 versus P_y is shown in Figs. 3–5 for a number of different parameters. All dimensions are normalized to the free space wavelength λ , and the actual frequency of operation for this type of detector is typically 380 MHz, which corresponds to a wavelength of 78.9 cm. The detector dimensions are $L = s = 0.2735 \lambda = 21.6$ cm. For ground constants we choose $\epsilon'/\epsilon_0 = 3.5 + i0.1$, which represents fairly dry soil. For target dimensions we choose $2a = 2b = 2c = 0.126 \lambda$, which equals 10 cm. The complex dielectric constant is $\epsilon_s/\epsilon_0 = 3.0 + i0.05$, which represents a fairly low-loss material such as plastic or wood.

In Fig. 3 we show the effect of offset distance P_x on the sweep curves. All curves have a null at $P_y = 0$ and are symmetrical in P_y , and the response falls off as P_x is increased. In Fig. 4 we show the effect of the scatterer depth d on the sweep curves. For $d/\lambda = 0.063$, the top surface of the cube is flush with the air-earth interface. For increasing d , the response decreases. In Fig. 5 we show the effect of detector height h on the sweep curves, and the response decreases rapidly with increasing h . All of the results in Figs. 3–5 are consistent with the scattering results in [9] and the antenna results in [8]. For soil with a higher moisture content, both the real and imaginary parts of ϵ' will be larger, and some results for the increased attenuation of the transmitted field have been given in [8].

Some experimental work was performed for the UHF detector in Fig. 2 for the same physical dimensions ($L = s = 21.6$ cm). The detector suffered from lack of sensitivity and an imperfect balance. However, its performance improved somewhat when the operating frequency was increased from 380 to 430 MHz. An experimental curve for a spherical target in a free-space environment ($\epsilon'/\epsilon_0 = 1$) is shown in Fig. 6. The sphere diameter was 9.85 cm, and the sphere material was nylon ($\epsilon_s/\epsilon_0 \approx 3.0$). The height of the detector above the surface of the sphere was 1.2 cm. Since the Born approximation is not valid for such a large contrast ($\epsilon_s/\epsilon' \approx 3.0$), we did not attempt a direct comparison with the theory. However, the main features of the theoretical and experimental

Fig. 3. Sweep curves for various offset distances P_x . Parameters: $h/\lambda = 0.1$ and $d/\lambda = 0.126$.

Fig. 4. Sweep curves for various target depths d . Parameters: $h/\lambda = 0.1$ and $P_x = 0$.

Fig. 5. Sweep curves for various detector heights h . Parameters: $d/\lambda = 0.126$ and $P_x = 0$.

Fig. 6. Experimental sweep curve for a dielectric sphere in free space. Parameters: frequency = 432 MHz, $\epsilon_s/\epsilon_0 \approx 3.0$, and $P_x = 0$.

curves are the same, and the locations of the peaks are approximately the same ($P_y \approx \pm 8$ cm). The experimental curve is not quite symmetrical and the unbalance of

the detector results in a baseline value (large $|P_y|$) that is not zero. The null value for $P_y = 0$ is actually lower than the baseline value.

V. CONCLUSIONS

We have used the plane-wave scattering-matrix formulation to compute the response of a dipole detector to a buried dielectric scatterer. The computations involve a large number of evaluations of the scattering matrix of the buried rectangular box, but the evaluation of Sommerfeld integrals usually associated with antennas near an interface [12] is avoided.

The numerical results in Figs. 3–5 indicate that the sweep curves are symmetrical and have a null directly over the buried target. Experimental results in Fig. 6 show the same qualitative features. The results in Fig. 5 show that the detector should be located as close as possible to the earth for maximum response.

A number of extensions to this work would be useful. The importance of multiple reflections between the detector, the buried target, and the air-earth interface could be studied using the complete formulation in [7]. Also, a quantitative study of the errors caused by truncating the range of the horizontal wavenumbers k_x and k_y would be useful for further numerical work. Larger dielectric targets with higher contrast could be treated and the effects of target resonances [14] on the frequency response could be studied. Other antenna configurations could be analyzed and the effect of the ground on the input impedance could be studied as a function of antenna height [15]. A related problem is the effect of surface roughness and volume clutter on the detector response. A more accurate experimental study of the UHF detector over various earths and targets would also be very useful and would provide a more complete comparison with the theory.

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